Technical Note: Determination of Time Spent in a Backlog

(Or Similar Queue)

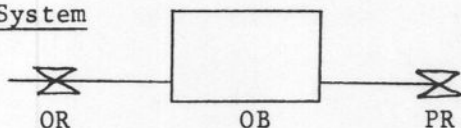By

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Problem

Given a simple system with orders flowing into a backlog at a varying rate of OR units per unit time and being executed at a varying rate PR units per unit time - to determine the length of time τ which has been spent in the backlog by the order currently being executed.

The System

OR   OB   PR

Mathematical Solution

If a new order enters the order backlog at time T when the backlog has a value OB(T) then this order will be executed at time $T + \tau$ when all orders in front of it have been executed.

τ is therefore defined by

$$OB(T) = \int_{T}^{T+\tau} PR(t)dt \qquad\qquad (i)$$

Shifting the time origin to $T+\tau$

$$OB(-\tau) = \int_{-\tau}^{0} PR(t)dt \qquad\qquad (ii)$$

But, generally for the system

$$OB(0) = OB(-\tau) + \int_{-\tau}^{0} OR(t)dt - \int_{-\tau}^{0} PR(t)dt \qquad\qquad (iii)$$

Substituting (ii) into (iii)

$$OB(0) = \int_{-\tau}^{0} OR(t)dt$$

defining $\quad TOR(x) = \int_{\text{Start of run}}^{x} OR(t)dt$

then $\quad OB(0) = TOR(0) - TOR(-\tau)$

so $\quad TOR(-\tau) = TOR(0) - OB(0) \ldots\ldots\ldots\ldots \quad$ (iv)

Computing Method

OB(0), the current order backlog of the system is known, and TOR(0), the total orders received by the system since the start of the simulation can be calculated by the addition of an extra level variable to the system.

The difference Δ between these can then be obtained.

The problem of calculating τ then reduces to that of finding the absolute value of time at which TOR was equal to Δ. The difference between this figure and the current value of time is τ.

In the problem in which this method was used, a current value of TOR was held and in addition a number of previous values of TOR were stored in an array. From this array the value of τ was obtained by linear interpolation.

In general if this type of approach is used for solution of equation (iv), the oldest value of TOR held in the array should be sufficiently old to allow calculation of the maximum expected value of τ. The time interval between successive values of TOR held in the array controls the accuracy at which τ is calculated and should therefore be sufficiently small for the accuracy required.

At the start of the simulation it is necessary to initialise both the current and stored values of TOR. The initialisation must be such that the current value of TOR is greater than the initialised value of OB. This ensures a positive starting value for τ and that all subsequent values are also positive.