

# A Dimensional Analyser for System Dynamics Models

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## Abstract

The reasons that led to the identification of a need for a package that could carry out the dimensional analysis of a System Dynamics model are discussed. A dimensional analysis package has been developed for use with the DYSMAP compiler and examples of the way it is used are given. A preliminary assessment of the usefulness of such a package to System Dynamics models is also made.

Beginners often have problems in formulating System Dynamics equations. In the course of teaching at Bradford it was noticed that students with a good background in engineering or the physical sciences made far fewer obvious errors in modelling. Eventually it was realized that the way a System Dynamics model is constructed ensures that the model variables and constants correspond to some identifiable real entity, and therefore have associated with them 'dimensions'. Thus a constant such as 'Average Output per Man' would be measured in Units/Man/Week. Though these dimensions do not have the same fundamental significance as those of the physical sciences, it became clear on examination of the way in which experienced modellers worked that they had subconsciously carried over the ideas of dimensions from the modelling of physical systems, and automatically rejected equations that were dimensionally inconsistent.

Figure 1 is a small DYSMAP program showing two common beginners mistakes. In the first (line 2), the user has forgotten to convert months into years, so that the interest paid, instead of being 15% is 12 times as large. This is also

a dimensional error, since:

$$\begin{aligned} [\text{INTER}] &= [\text{F} * \text{MONTH}^{-1}] \\ \text{and } [\text{LOAN}] * [\text{IR}] &= [\text{F} * \text{YEAR}^{-1}] \end{aligned}$$

The second (line 6) shows the difficulty that many users have in understanding the continuous character of a System Dynamics model. Obviously, the difference between desired and current stocks (DS-S) should have been spread out over time and this is shown by the fact that the dimensions of F are inconsistent with those of S and DS.

Even from these simple examples it is clear that errors of this type can be very hard to find from an examination of the output. The obvious cure to this would be to check the equations for dimensional consistency, while building the model. However, this assumes the user to be familiar with the technique of Dimensional Analysis which is not necessarily the case. Besides this, except for very small models, it is a very tedious task and further study showed that even experienced modellers sometimes make dimensional errors. Though these are scarcely ever serious errors of understanding they can give rise to a great deal of unnecessary work at the model-debugging stage and may even occasion embarrassment if spotted by the client rather than the modeller.

It was, therefore, decided to introduce Dimensional Analysis as an additional facility of DYSMAP so that these checks could be made automatically. The method for doing this was suggested by A. K. Ratnatunga.

The information required by the DYSMAP Dimensional Analyzer for the examples given is shown in the documentation sector of Figure 1. The dimensions of the variables are entered between parentheses on the D cards which are also used by DYSMAP documentor. The D cards thus serve a dual purpose. They can be placed anywhere in the program. In this case they have been grouped together for ease of reference. The purpose of the DIM card is used to inform the DYSMAP Compiler that the dimensional analysis of the model is required.

Dimensional Analysis takes place in two stages. The D cards are first analyzed to ensure that the syntax is correct and that dimensions have been given for each variable. If a variable has been incorrectly dimensioned or if no D card has been supplied for it an error message is printed.

The Dimensional Analyzer output for this model is shown in Figure 2, and it can be seen that the equation

$$A \text{ INTER.K} = \text{LOANS} * \text{IR}$$

is rejected because the dimensions of INTER, that is (£/MONTH), are incompatible with the dimension of the right hand side, that is (£/YEAR). The equation

$$A \text{ DP.K} = \text{DS} - \text{S.K} + \text{F.K}$$

is rejected because the dimensions of F are not compatible with the dimensions of S and DS.

A further facility of the Dimensional Analyzer not shown here is the ability to check that the arguments of DYSMAP special functions are consistent with the definition of the functions.

The Dimensional Analyzer is written in FORTRAN and requires a core area of 16K or 18K, of 24 bit words, depending on the size of the model to be analyzed.

The central processor time required for dimensional analysis is of the order of a few seconds even for models containing several hundred equations.

Though it has only been in use for a few months the Dimensional Analyzer has already proved its worth in both teaching and research work. It has encouraged new students to understand and apply the ideas of dimensional analysis from the start of their studies. In actual research projects it has been extremely useful in detecting obscurities and inconsistencies in the naming and dimensioning of variables.

It is noticeable also that the existence of the Dimensional Analyzer has encouraged the more precise use of dimensions within the Bradford group. Since the dimensional analyzer distinguishes between £ sterling at constant value (dimensioned, as for example, £1970) it has also detected errors such as the omission of price indices from individual equations in financial models dealing with inflation. Also it has become feasible and even desirable to subdivide certain model dimensions. Thus the dimension MEN can be divided into TECHNICAL and MANAGERIAL staff and requirements for the two types of manpower can be separately accounted for.



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DIM
* EXAMPLE OF DIMENSIONAL ERRORS
A INTER.K=LOAN*I+1 INTEREST PAID IN E PER MONTH
C IR=0.15 INTEREST RATE PER YEAR
C LOAN=10000 LOAN IN E PER MONTH
NOTE
A DP.K=DS-S.K+F DESIRED PRODUCTION RATE IN UNITS PER MONTH
C F=500 FORECAST MONTHLY DEMAND
A S.K=STO CURRENT STOCK LEVEL IN UNITS
C STO=200
C DS=1000
C DT=1
C LENGTH=24
NOTE
NOTE DOCUMENTATION SECTOR
D LOAN=(E) LOAN
D INTER=(E/MONTH) INTEREST PAID
D IR=(YEAR*(-1)) INTEREST RATE
D DP=(UNITS/MONTH) DESIRED PRODUCTION RATE
D DS=(UNITS) DESIRED STOCK LEVEL
D F=(UNITS/MONTH) FORECAST DEMAND
D STO=(UNITS)
D S=(UNITS) STOCK LEVEL
D DT=(MONTH)
D LENGTH=(MONTH)
D TIME=(MONTH)
RUN

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FIGURE 1

PHASE 2 : DIMENSIONAL ANALYSIS OF THE EQUATIONS

\* EQUATION LOCATED AT 2 \*

A INTER,K=LOAN\*IR INTEREST PAID IN F PER MONTH

DIMENSIONS OF RIGHT AND LEFT HAND SIDES INCOMPATIBLE.CARD REJECTED.

INTER	D CARD LOCATED AT	16
LOAN	D CARD LOCATED AT	15
IR	D CARD LOCATED AT	17

\* EQUATION LOCATED AT 6 \*

A DP.K=DS-S.K+F DESIRED PRODUCTION RATE IN UNITS PER MONTH

OPERANDS OF ADDITION ARE DIMENSIONALLY INCONSISTENT  
 VARIABLES/NUMBERS PART OF TWO OPERANDS ARE

FIRST OPERAND:

S D CARD LOCATED AT 22

SECOND OPERAND:

F D CARD LOCATED AT 20

FIGURE 2