

The Moving Average Filter

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A problem frequently encountered in systems modelling is to ensure a negligible response to disturbances of a specific period. This need often arises, for instance in Production systems where orders show strong seasonal components of 12 months, 6 months, 3 months, etc. In other systems it may be similarly required to reduce response to Business Cycle fluctuations with a four to five year period. The standard way of achieving this in a System Dynamics model is to make use of the SMOOTH function to reduce the effect of disturbances which the system should not respond to. Unfortunately the SMOOTH function is a rather crude instrument for this purpose and at least for systems subject to seasonal variation tends to lead to worse performance than is necessary. Thus a SMOOTH with a 6 month Smoothing Time has an output/input amplitude ratio for a sine wave input of period 12 months of about 1:3.3. Where the input has a strong seasonal variation of period 12 months this degree of attenuation may not be sufficient. Greater amplitude reduction can of course be recurred by using a longer Smoothing Time. Thus a 12 month SMOOTH has an output/input amplitude ratio for a 12 month sine wave of 1 : 6.4. At the same time, however, the use of SMOOTH's with longer Smoothing Times certainly degrades system response and may lead to stability problems.

In business systems it is common to use moving averages to overcome this problem. Thus a 12 month moving average removes all seasonal affects, i.e. disturbances with periods in months that divide into 12 exactly. A moving average (M) of length TAU is defined by the equation

$$M = \frac{I(t) - I(t-TAU)}{TAU} \quad (1)$$

where I(t) is the current input and t denotes time.

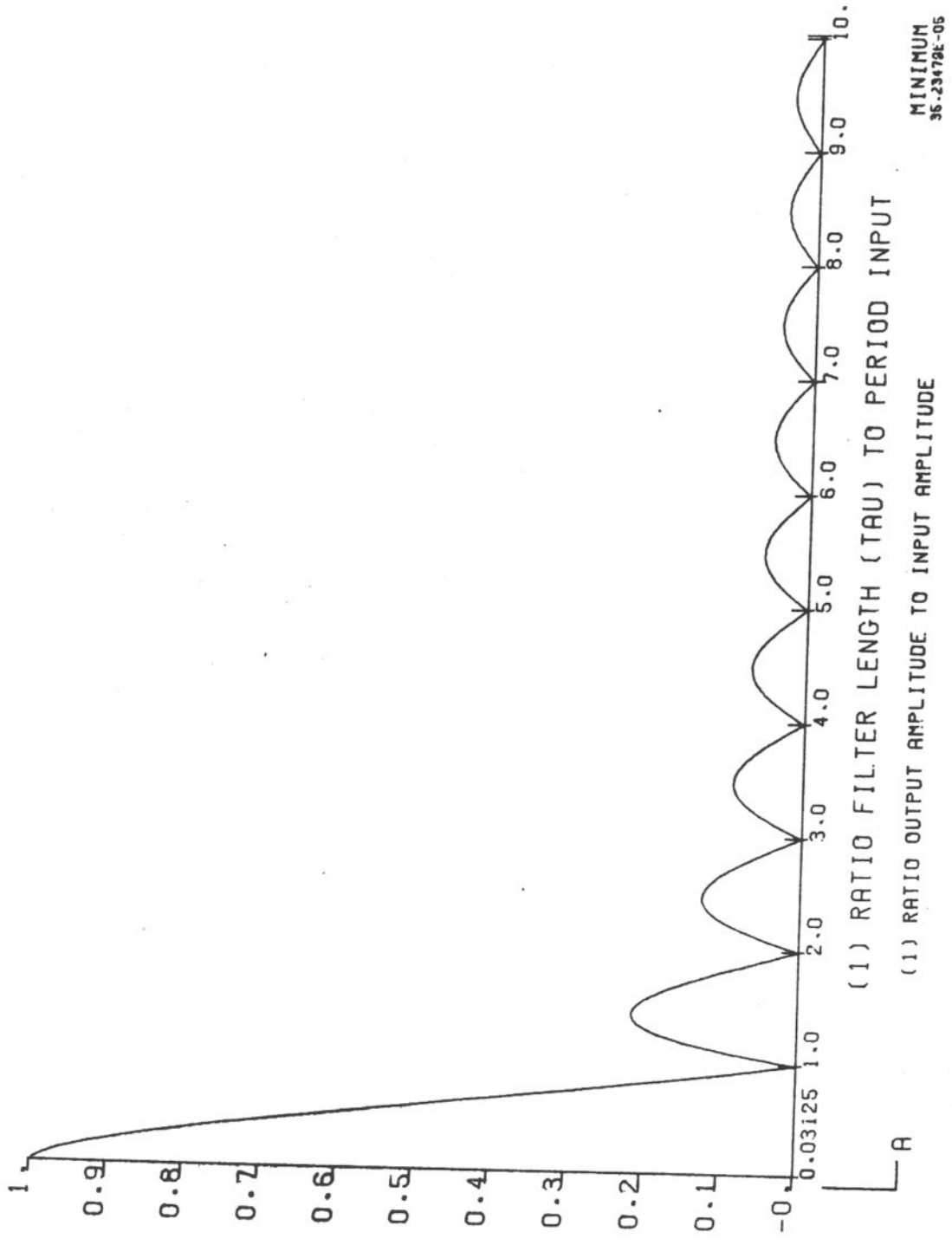


FIGURE 1
RESPONSE MOVING AVERAGE FILTER

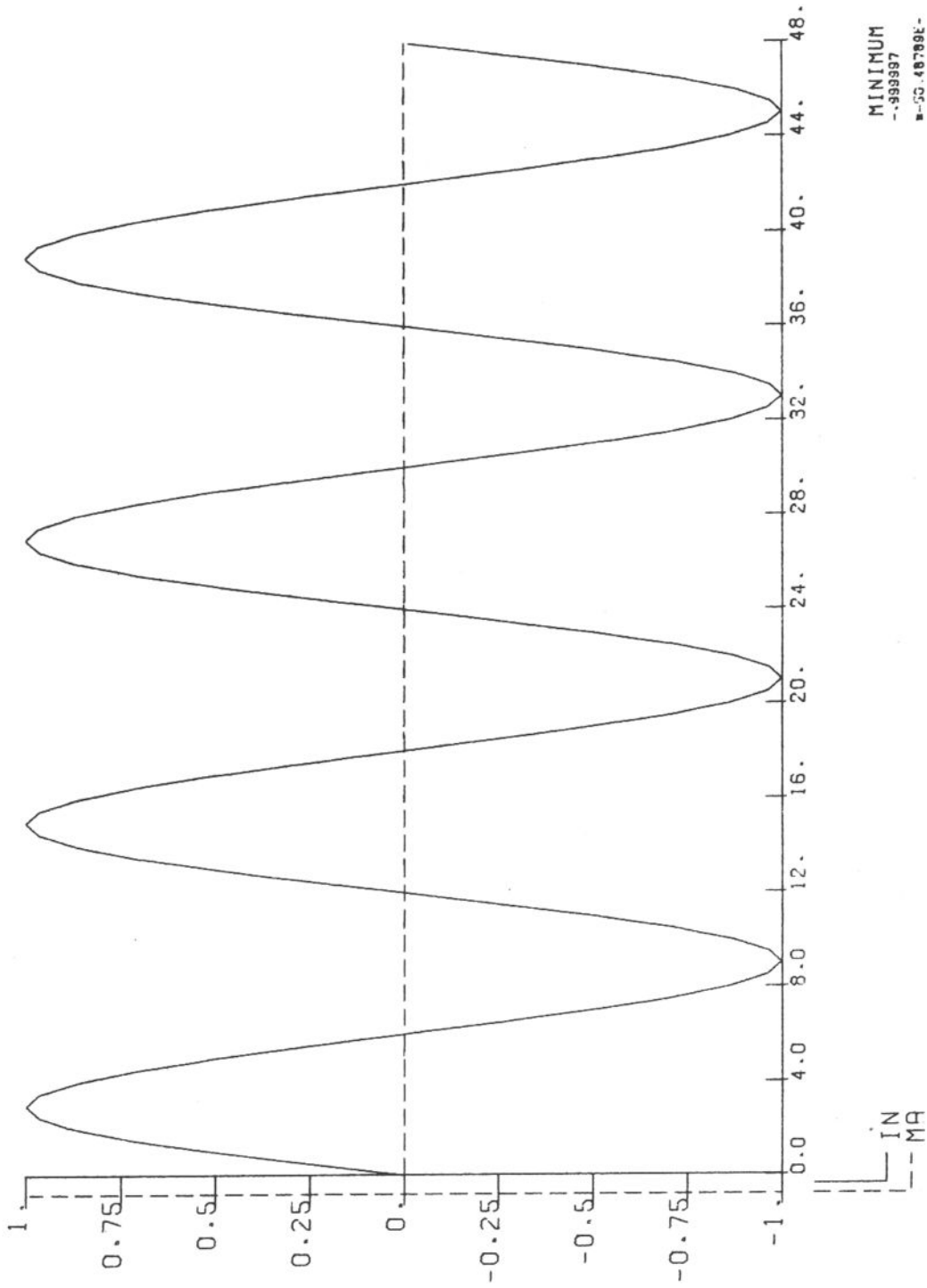


FIGURE 2
 OUTPUT 12 MONTH MOVING AVERAGE FILTER

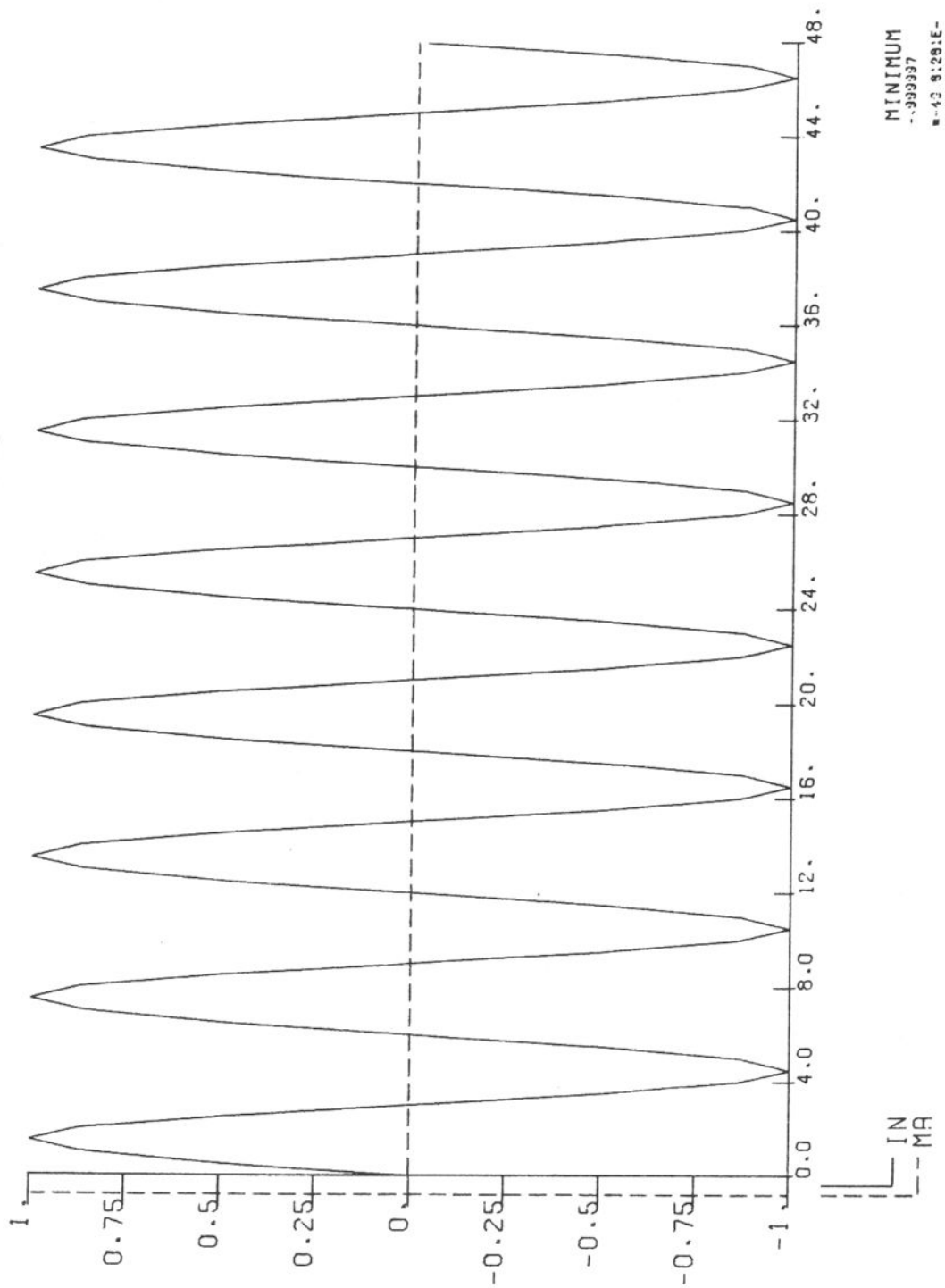
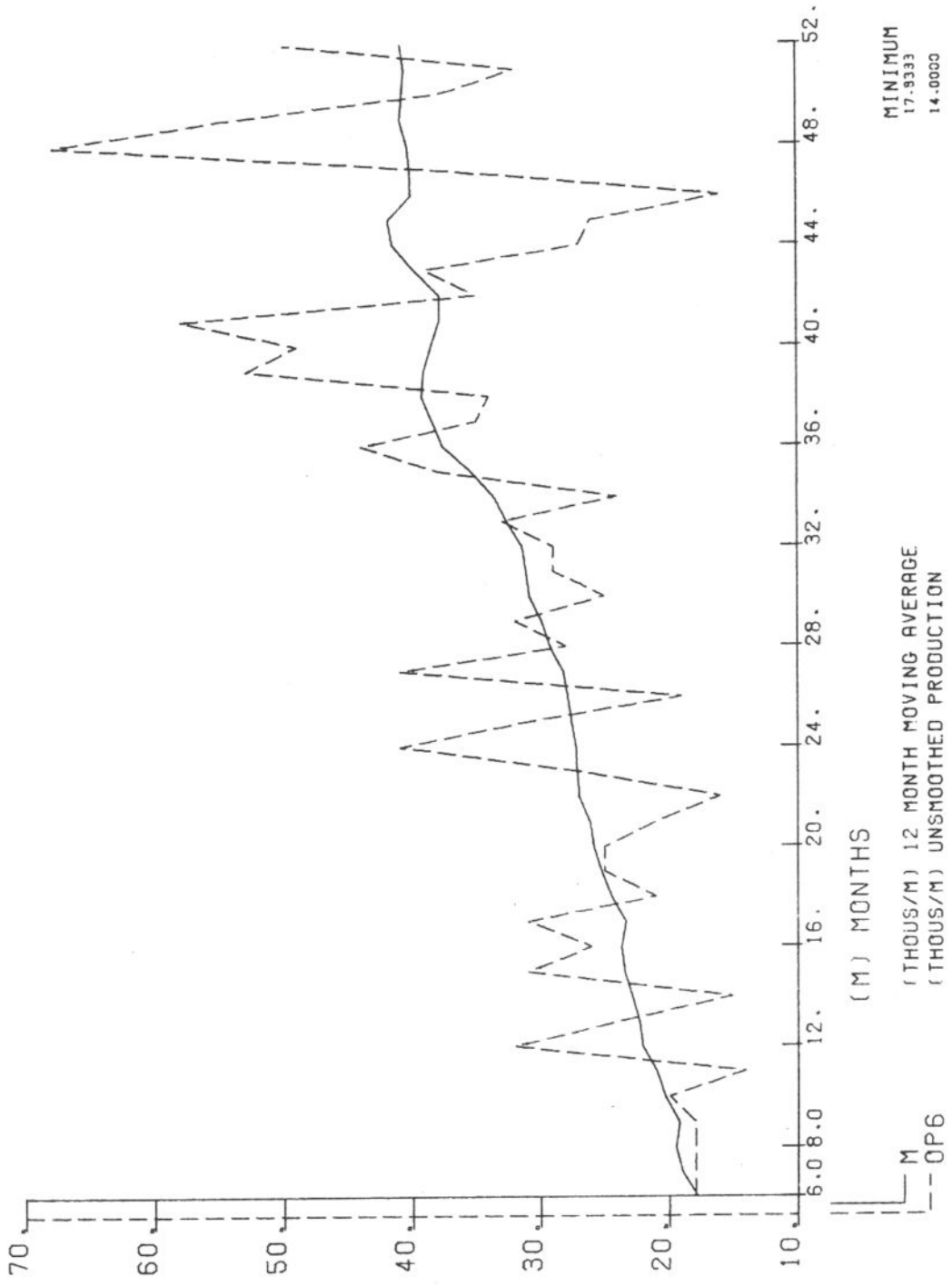


FIGURE 3
 OUTPUT 12 MONTH MOVING AVERAGE FILTER



MAXIMUM
 41.9333
 59.0000

MINIMUM
 17.9333
 14.0000

FIGURE 4
 PRODUCTION DATA TREND FITTING

The only problem that arises in programming the Moving Average is that care is needed in initial conditioning. Unlike the SMOOTH function the effects of initial condition errors do not die out as the simulation progresses. It is therefore necessary to ensure that the initial value assigned to the Moving Average is the same as that implied by the initial values of the Pipeline Delay.

As well as its use in eliminating seasonal variation, the Moving Average is useful for trend determination. It frequently happens that a data series from which a time trend is to be derived for use say in a model TABLE contains a great deal of short period variation, due to seasonal or business cycle effects. One convenient way of removing these effects is to pass the data series through a filter such as a SMOOTH. The problem in using a SMOOTH for this purpose, however, is that except for very long period disturbances the time lag between input and output is not constant. The desired trend cannot therefore recover by shifting the output time series forward by some fixed amount. The Moving Average filter, though, as mentioned earlier, does have such a constant lag of $\text{TAU}/2$ for all periods. The desired trend is therefore easily recovered by shifting the Moving Average of the data forward by $\text{TAU}/2$. Figure 4 shows the result of applying a 12 month moving average to the highly seasonal data series PR and shifting the output (MA) of the moving average forward by 6 months. As can be seen this gives a good trend curve. Such an approach has been applied by Zepeda (1978) to eliminate business cycle effects in data for capital costs of generating plant and thus to determine the long term trend in these costs.

Reference

- Zepeda, E.A. Capacity Acquisition Policies in the Electricity Supply Industry,
Ph.D. Thesis, University of Bradford, to appear, 1978.