The Moving Average Filter

By

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A problem frequently encountered in systems modelling is to ensure a negligible response to disturbances of a specific period. This need often arises, for instance in Production systems where orders show strong seasonal components of 12 months, 6 months, 3 months, etc. In other systems it may be similarly required to reduce response to Business Cycle fluctuations with a four to five year period. The standard way of achieving this in a System Dynamics model is to make use of the SMOOTH function to reduce the effect of disturbances which the system should not respond to. Unfortunatel the SMOOTH function is a rather crude instrument for this purpose and at least for systems subject to seasonal variation tends to lead to worse performance than is necessary Thus a SMOOTH with a 6 month Smoothing Time has an output/input amplitude ratio for a sine wave input of period 12 months of about 1:3.3. Where the input has a strong seasonal variation of period 12 months this degree of attenuation may not be sufficient. Greater amplitude reduction can of course be recurred by using a longer Smoothing Time. Thus a 12 month SMOOTH has an output/input amplitude ratio for a 12 month sine wave of 1: 6.4. At the same time, however, the use of SMOOTH's with longer Smoothing Times certainly degrades system response and may lead to stability problems.

In business systems it is common to use moving averages to overcome this problem. Thus a 12 month moving average removes all seasonal affects, i.e. disturbances with periods in months that divide into 12 exactly. A moving average (M) of length TAU is defined by the equation

$$M = I(t) - I(t-TAU)$$
TAU
(1)

where I(t) is the current input and t denotes time.

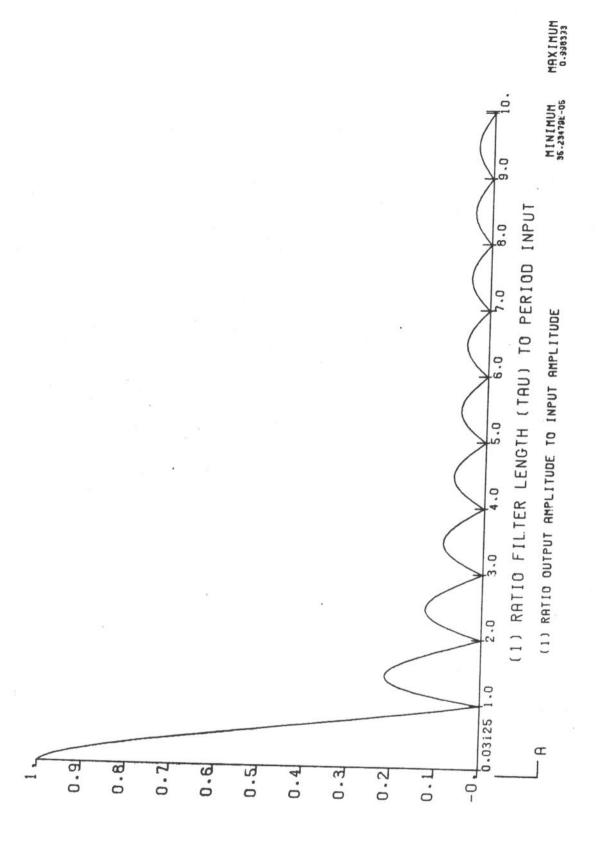


FIGURE 1 RESPONSE MOVING AVERAGE FILTER

The output/input amplitude ratio of a moving average filter of length TAU is easily computed by standard Laplace transform methods to be for an input sine wave of period TS

$$\frac{\text{Output Amplitude}}{\text{Input Amplitude}} = \frac{\text{TS SIN}(\pi \text{TAU/TS})}{\pi \text{TAU}} \tag{2}$$

or

$$\frac{SIN(T)}{T} \qquad \text{where } T = \frac{\pi TAU}{TS}$$

Another useful property of the moving average is that for any period of sine wave input the output lags the input by the TAU/2.

The form of equation (2) is shown in Figure 1. For inputs with period much greater than TAU (i.e. Low values of the ratio T) the response is similar to a SMOOTH with smoothing time TAU/2. Where T takes on an integer value, however, the value of the ratio is zero.

The Moving Average is easily programmed using a Pipeline Delay. The following DYSMAP program shows an example of a 12 month Moving Average. Figure 2 shows the response of the Moving Average to a sine wave input with period 12 months and Figure 3 the response to an input sine wave with 6 month period.

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O FAMAC
            2 1 999
1 DLPIPE
2 END
3 DUC
4 * OUTPUT 12 MONTH HOVING AVERAGE FILTER
6 A DIN. K=DLPIPE(TAL, IN. K)
7 D OIN= (UNITS/LONTH) VALUE INPUT 12 HOUTHS AGO
8 A IN.K=SIN(6.28*TIME.K/PERD)
9 D IN= (UNITS/MONTH) CURRENT INPUT
O C PERDE12
1 D PERD= (MONTH) INPUT PERIOD
2 L MA.KEMA.J+DT+(III.J-UIN.J)/12
13 N MA=U
14 D MA= (UNITS/MONTH) 12 MONTH MOVING AVERAGE
5 N TIME =- 12
6 A PLTPER K=STLP(0.5,0)
7 D TIME= (MUNTH) SILULATION TIME
18 SPEC DT=0.5/LENGTH=00
19 PLOT IN=1, MA=1 (-1,1)
20 RUN FIGURE 2
21 C PERD=6
22 RUN FIGURE 3
23 +
24 ****
25
```

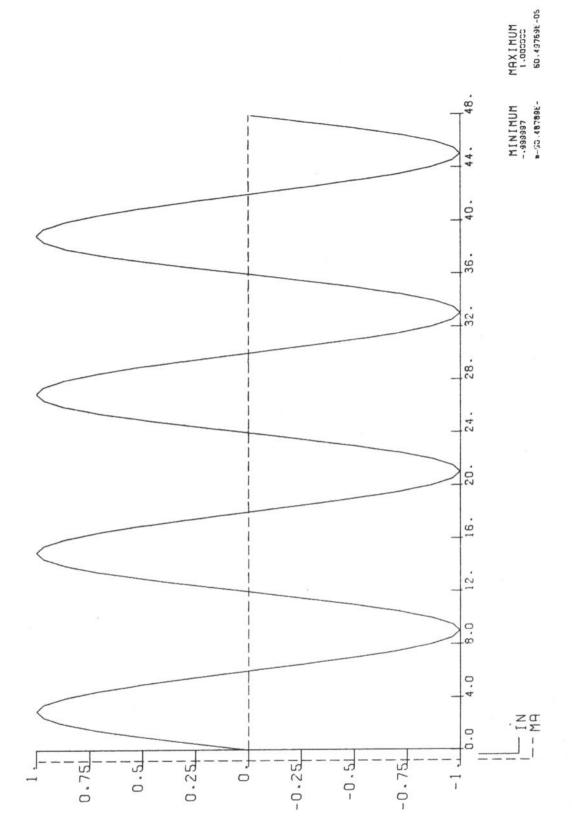


FIGURE 2 OUTPUT 12 MONTH MOVING AVERAGE FILTER

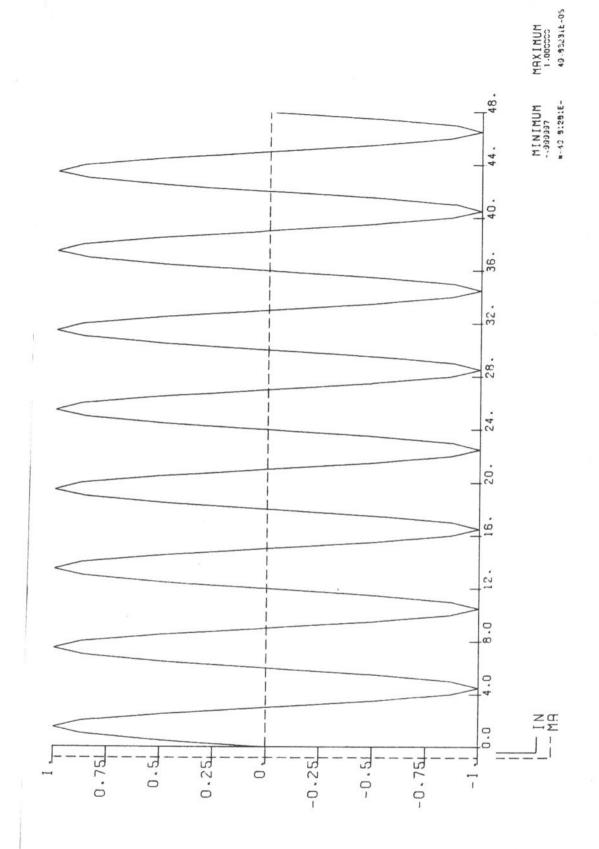


FIGURE 3 OUTPUT 12 MONTH MOVING AVERAGE FILTER

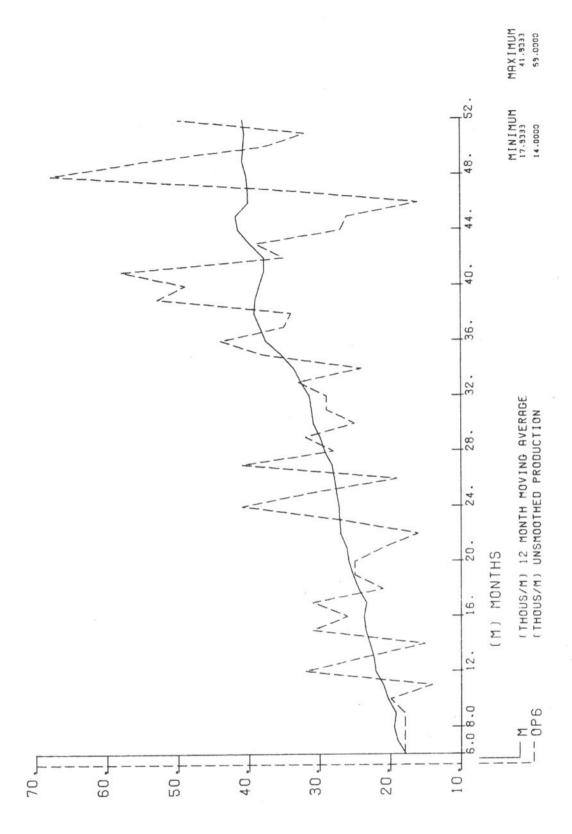


FIGURE 4 PRODUCTION DATA TREND FITTING

The only problem that arises in programming the Moving Average is that care is needed in initial conditioning. Unlike the SMOOTH function the effects of initial condition errors do not die out as the simulation progresses. It is therefore necessary to ensure that the initial value assigned to the Moving Average is the same as that implied by the initial values of the Pipeline Delay.

As well as its use in eliminating seasonal variation, the Moving Average is useful for trend determination. It frequently happens that a data series from which a time trend is to be derived for use say in a model TABLE contains a great deal of short period variation, due to seasonal or business cycle effects. One convenient way of removing these effects is to pass the data series through a filter such as a SMOOTH. The problem in using a SMOOTH for this purpose, however, is that except for very long period disturbances the time lag between input and output is not constant. The desired trend cannot therefore recover by shifting the output time series forward by some fixed amount. The Moving Average filter, though, as mentioned earlier, does have such a constant lag of TAU/2 for all periods. The desired trend is therefore easily recovered by shifting the Moving Average of the data forward by TAU/2. Figure 4 shows the result of applying a 12 month moving average to the highly seasonal data series PR and shifting the output (MA) of the moving average forward by 6 months. As can be seen this gives a good trend curve. Such an approach has been applied by Zepeda (1978) to eliminate business cycle effects in data for capital costs of generating plant and thus to determine the long term trend in these costs.

Reference

Zepeda, E.A.

Capacity Acquisition Policies in the Electricity Supply Industry,

Ph.D. Thesis, University of Bradford, to appear, 1978.