

Three Strategies of Pollution Control -
A Heuristical Optimization Model.

By

Tero Ansio and Erkki Mattila
Helsinki School of Economics

Abstract

Three alternative strategies for pollution control are studied. From the results of this paper one of these is recommended as the best strategy. The method of this study is system dynamics which has been used in three ways. First, an analytic approach has been used in formulating some decision rules for the system. Secondly, the heuristically optimizing SDRDYN has been used so that the optimal path of one SD variable has been found. Thirdly, SDRDYN is used in sensitivity analysis when strategies are compared.

We proceed as follows: First, three different strategies for pollution control are introduced, then the model and the analytic solution of production and pollution is presented. In the third section the method for finding the optimal path in system dynamics is introduced and after that the results of simulation are given. The sensitivity of the results for parameter changes is studied and conclusions and suggestions for further research are given.

1. Introduction

The economics of pollution and pollution control is a relatively new branch of economic science. The economic interest of these problems is due to the fact that most pollution is caused by economic activity (production or consumption) and that different economic means can be used to control and restrict polluting behaviour. The increasing difference between the diminishing supply of and the growing demand for unpolluted environment have increased the importance of economic answers to the problem.

In this paper we study alternative strategies that can be used to restrict pollution caused by economic activity to some specified level. The three strategies are as follows. The first one is to use the decision variable of the pollution control policy in a way to achieve the specified level of pollution in a minimum time. We call this strategy MAX-strategy because it will mean that the policy parameter is most of the time in its maximum (or highest possible) value. The second strategy, which we call CONSTANT-strategy, is to set the decision variable of the control policy on its long-term equilibrium value and keep it constant over time. The third alternative is to use the decision variable in order to minimize the sum of environmental, pollution and control costs. This strategy we call MIN-strategy.

In the following we try to find out the advantages and disadvantages of the strategies. Some recommendations and valuations concerning the strategies and their implications are also made.

In pollution control different alternative means can be used (for a general discussion about the alternatives, see e.g. Pearce 1976). The most common ones are pollution taxes, some kind of regulation, subsidies, licence markets and some combination of these (combination of policies is suggested e.g. by Roberts & Spence 1976 and Bawa 1975). In every alternative policy, however, there is a decision variable which can be used by the authority to control the level of pollution. E.g. in pollution tax policy the decision variable of the authority is the level of the tax, in regulation the amount of pollution allowed etc. In every policy the decision variable can be used according to the three strategies defined earlier (MAX, CONSTANT, MIN).

In the following we build a model of environmental pollution and see what impacts on the behaviour of the model the use of different strategies during alternative policies has.

2. The model

The simulation model consists of three sectors: the production sector, the environment and the authority which are linked by interdependencies, as can be seen in figure 1. These links can be summarized as follows: the reduction of control causes pollution which causes a deterioration in the quality of the environment. The authority reacts to the observed quality of

the environment which has some target level and imposes some control policy the production sector to achieve the target. There are also some lags in the flows between the sectors. The study has been carried out with water pollution in mind.

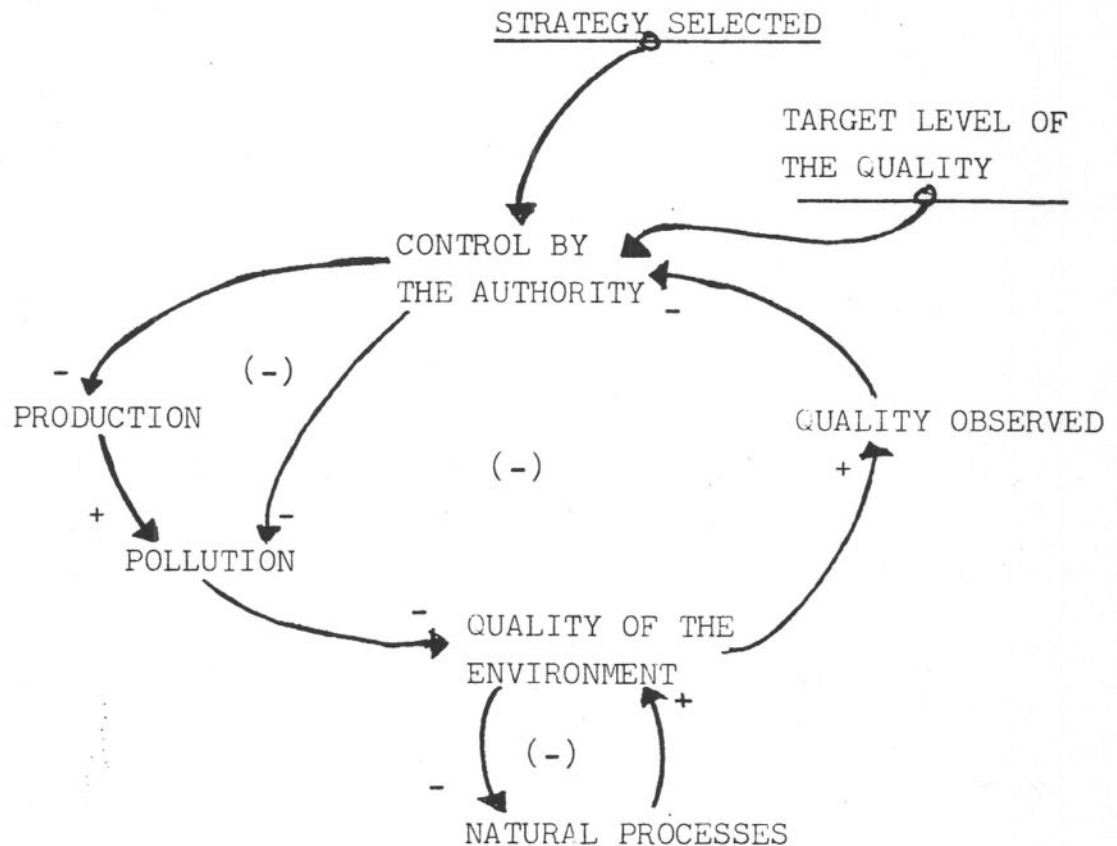


Figure 1. The structure of the simulation model.

The production sector is considered to be a unit which is assumed to maximize its profit at each individual period during the simulation. The model of this sector is as follows.

The price p is assumed constant. To assure the existence of a finite value of profit-maximizing output we assume increasing total and marginal production costs (see e.g. L.-S. Fau and B.R. Froehlich 1972 and Ethridge 1973). As a simple representative of a cost function with these properties we have

$$C = cq^2, \text{ where}$$

C = total costs of production

q = quantity produced

c = constant

This kind of cost function is used e.g. in models of linear decision rules (see for instance the classical article by C.C. Holt, F. Modigliani and H.A. Simon 1955).

The net revenue function of the firm has decreasing marginal revenue and the profit maximizing quantity produced can be found.

The rate of waste production is assumed to be proportional to the rate of pollution

$$E = eq$$

E = rate of waste production

e = constant

The producers can reduce the rate of pollution without reducing the level of production by reducing the amount of waste before it is released to the environment. Costs caused by this practice are expressed

$$P = reqx^2$$

P = costs of waste reduction

r = constant

x = percent of waste reduced from the waste initially produced,
if $x=1$ all the waste is reduced, if $x=0$ no waste at all is reduced.

The production sector maximizes its profit V

V = total income - production costs - environmental costs.

The last term of the equation (environmental costs) is dependent on the pollution control policy used by the authority. So, if e.g. pollution tax is used

$$V = pq - cq^2 - reqx^2 - t(1-x)eq, \text{ where}$$

t = tax per unit of pollution caused.

We are now able to find out the optimal behaviour of the production sector, that is the optimal values of q and x as functions of t . This could also be done if any other policy was used but let us first see what comes out with the tax policy.

The first order conditions for optimality are:

$$\frac{\delta V}{q} = p - 2cq - rex^2 - t(1-x)e = 0$$

$$\frac{\delta V}{x} = 2reqx + teq = 0 \quad ,$$

then the optimal x (percent of waste reduction) is

$$x' = \frac{t}{2r} \quad ,$$

which we introduce to the first equation and have for optimal q

$$q' = \frac{p - et(1 - \frac{t}{4r})}{2c} \quad (1)$$

The waste released to the environment, s , is

$$s = (1-x)eq \quad ,$$

and we get optimal waste released by introducing optimal x to the formula above:

$$s' = (1 - \frac{t}{2r})eq \quad . \quad (2)$$

The waste not reduced by industry is released to the environment and this part of the waste increases the stock of waste in the receiving water. Some amount of the stock of waste is released every unit of time and reduces the quality of the environment. The environmental quality is, on the other hand, increased by the natural processes of the water. Pollution and quality of water may be assessed by BOD (biochemical oxygen demand) and DO (dissolved oxygen) measures.

The authority is considered to keep an eye on the quality of environment and to make a decision about the pollution control decision variable by the observed level of quality. As mentioned, the authority has three alternative strategies to follow which will have different consequences on the system. We want to find out by a simulation study which strategy should be selected by the authority which tries to keep the environmental quality in the equilibrium state.

By an equilibrium we mean in this system a state where the level of the environmental quality stays constant. Preferably this constant value should be the target value of the quality. When solving the optimal equilibrium value of the environmental quality (by this we mean the value which minimizes the total costs due to pollution) an analytic approach leads to difficulties due to the algebraic formulations and time lags of the model. That is why the SDRDYN-algorithm is used (for SDRDYN, see Keloharju 1977).

3. Simulation

In order to study the dynamic properties of the system and to find the 'best' strategy simulation experiments are performed (simulation is also used in pollution control study e.g. by Downing & Watson 1976 and Gates & Males & Walker 1970). The method used is system dynamics where dynamic tendencies of any complex system are assumed to arise from its causal structure, where every decision is made within a feedback loop. The decision controls action which alters the system levels which influence the decision.

In traditional system dynamics decision rules are formulated mainly using intuition, some kinds of rules of thumb, common practice etc. In this paper the analytical solutions derived in section 2 are used as decision rules for production and waste released to the environment (equations 1-2). We assume that in every period the production sector makes these decisions by these rules using the information currently available. So our assumption is that the production sector does not maximize the total profit over the whole simulation period, but maximizes separately the profit in each subperiod. We simulate the model in order to find the equilibrium where the environmental quality stays constant at a level which causes minimum costs.

In finding the MIN-policy we have a problem: In the ordinary Dynamo-language there are no possibilities of optimizing. That is why a method, called SDRDYN is used. In SDRDYN a system dynamics model can be heuristically optimized as follows: a model builder defines the objective function, which is one of the system dynamics equations. Any Dynamo-parameters can be defined as SDR-variables. These parameters are given their upper and lower bounds by the user, and SDRDYN will seek the parameter values that optimize the objective function.

But using SDRDYN gives rise to a new problem: SDR-variables are constants in the system dynamics model during the simulation. In this context our aim is to find an optimal path for the decision variable of the control policy. We have come to a conclusion that third order equations are often very useful for approximating optimal paths of variables desired (see also the method of Specific Optimal Control by Sage & White 1976).

In many cases the time path of some variable can be formulated in the following manner:

$$\text{Decision variable} = a + by + cy^2 + dy^3 \quad ,$$

where

a, b, c, d = SDR-variables

y = some variable of system dynamics model, e.g. time.

This formulation gives a large variety of different time paths depending on the values a, b, c and d. The task of the SDRDYN-algorithm is to find such a combination of a, b, c and d that optimizes the objective function.

Numerical values used in the basic simulation runs are as follows :

e = 0.5 (waste production constant)

p = 20 (unit price)

c = 0.01 (production cost constant)

r = 15 (waste reduction cost constant)

The simulation was performed with several combinations of parameter values. The effect of these values on the results derived is more deeply analyzed in section 5.

The equilibrium quantity produced by industry when no pollution control exists is determined by the values of p and c . The values above give an optimal quantity $q = 1000$. An increase in c or a decrease in p would decrease the quantity produced and vice versa. Changes in c or p have an impact on the equilibrium values of the simulation runs but do not change the ordering of policies.

The quality of the environment is measured by a scale 0 - 100 and initial target level is 80. The function for the environmental costs due to pollution is defined:

$$pc = g(100 - \text{quality}) + f(100 - \text{quality})^2, \text{ where}$$

pc = pollution costs per period

g = constant = 100

f = constant = 2

At the beginning of the simulation the environmental quality is assumed to be 60 BOD.

4. The results of simulation

Time series of simulation runs under different strategies are presented in figures 2 - 4. Figure 5 describes differences between MIN-, CONSTANT- and MAX-strategies: in the optimal policy (MIN) the tax is set between MAX- and CONSTANT-values as was expected.

Some of the information received from the simulation is given in table 1. We can see that there are no great differences between the total costs of different strategies. The costs are only a few percent higher in the two other strategies than with the MIN-strategy. However, if no pollution control were introduced at all (i.e. tax = 0) the total costs would be more than doubled from the MIN-strategy.

Strategy	Total costs		Periods to achieve target level of environmental quality
	mk	% more than MIN	
CONSTANT	193000	6	24
MAX	185000	1.6	8
MIN	182000	-	12

Table 1. Total costs and time needed to achieve target level of environmental quality in different strategies.

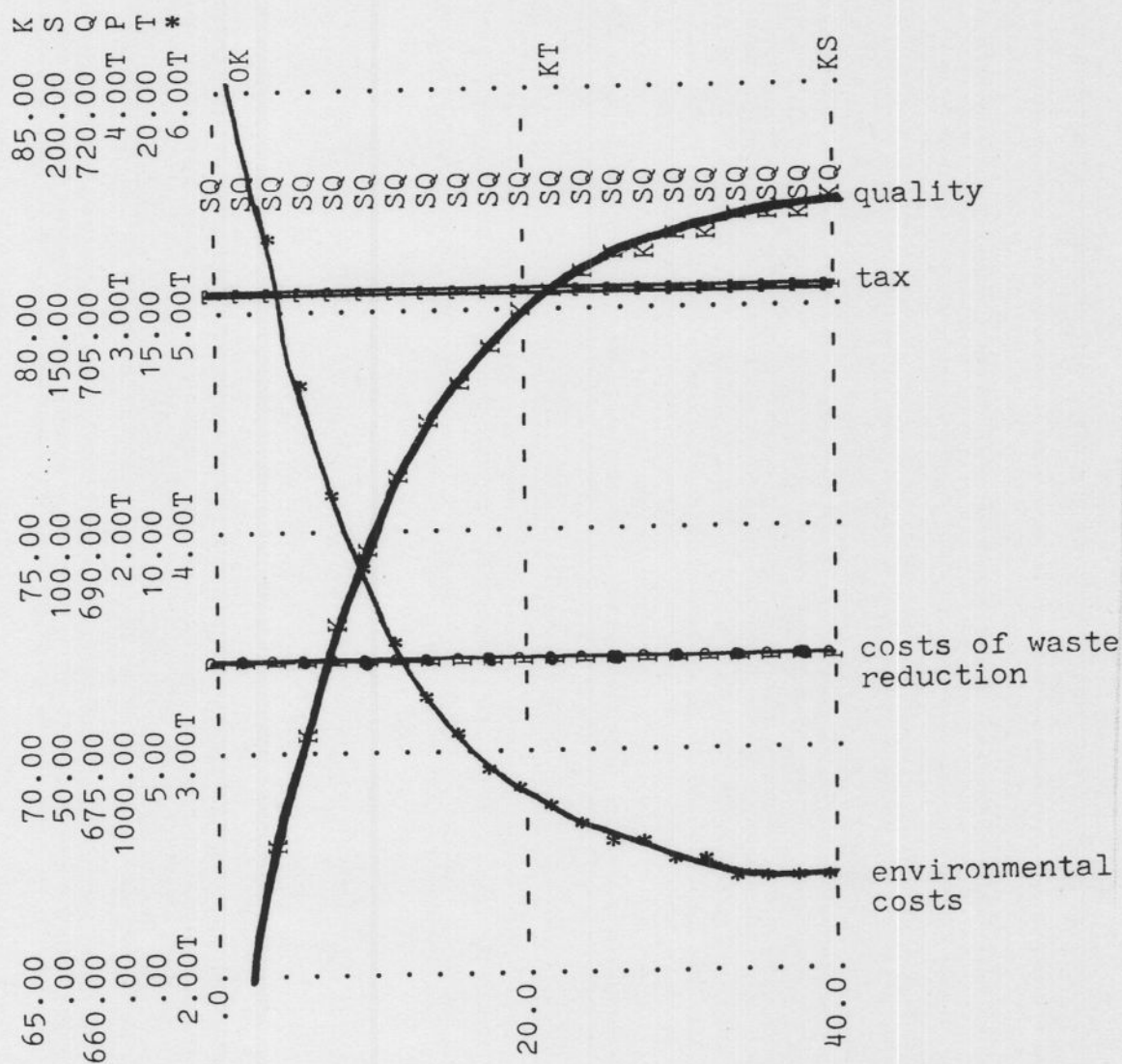


Figure 2. CONSTANT-strategy

The time that will be taken to achieve the target level of environmental quality is lowest for the MAX-strategy as was expected. This time is longest for the constant-strategy, it is about three times as long as the time taken during the MAX-strategy. The difference between strategies MAX and MIN is not very large in this respect either. The disadvantage of the MIN-strategy is that when the policy variable (tax) is constantly changing to minimize costs, also all the other variables of the system change constantly and this fact brings more oscillation with this alternative than with the others. For instance the level of the production in the production sector has the following maximum and minimum values during the different strategies:

Strategy	Minimum q	Maximum q	Difference (max-min) per cent from the equilibrium value of q (714)
CONSTANT	714	714	0 %
MAX	625	714	89 %
MIN	625	719	94 %

Table 2. Variation of the production with different strategies.

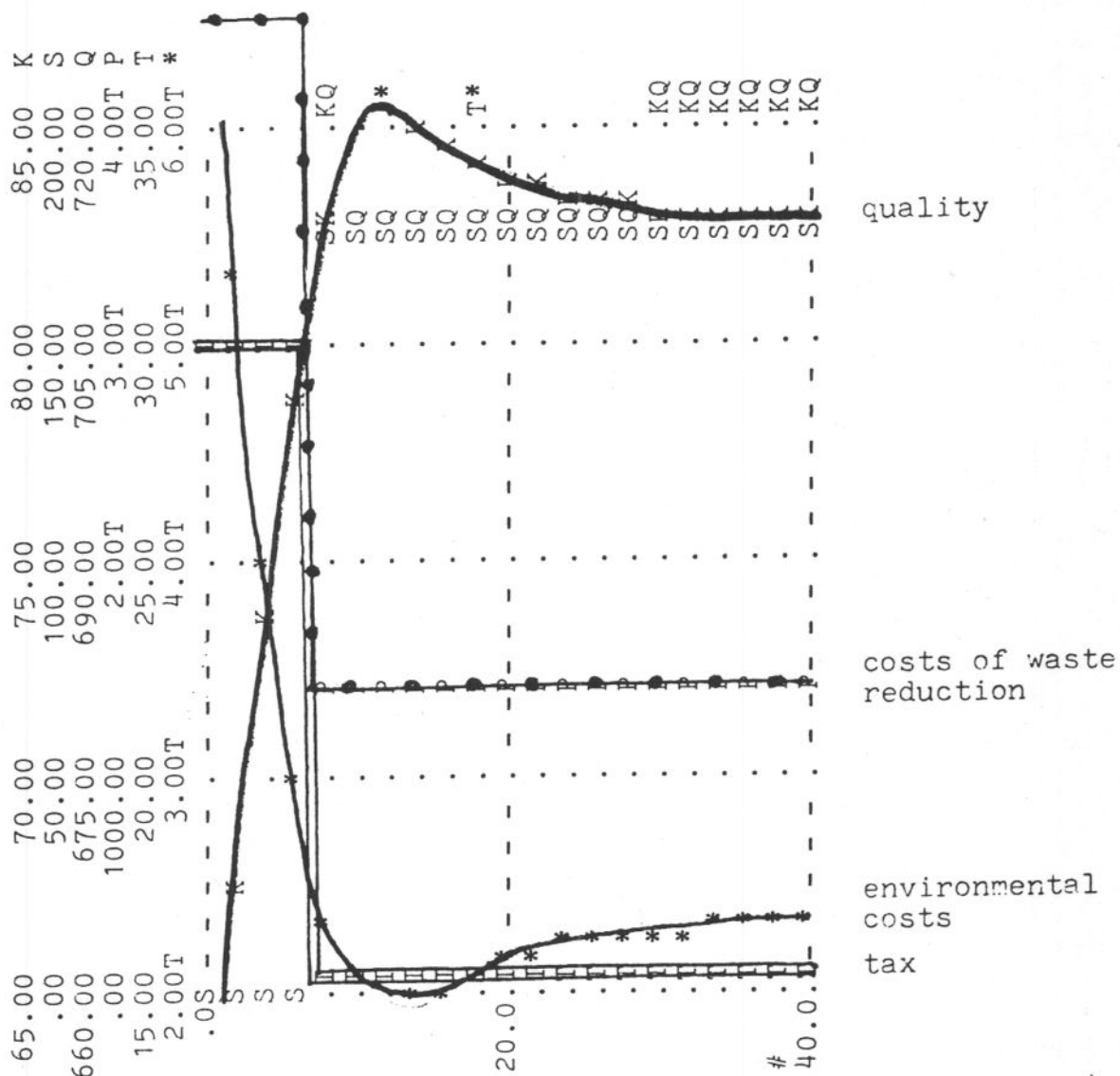


Figure 3. MAX-strategy

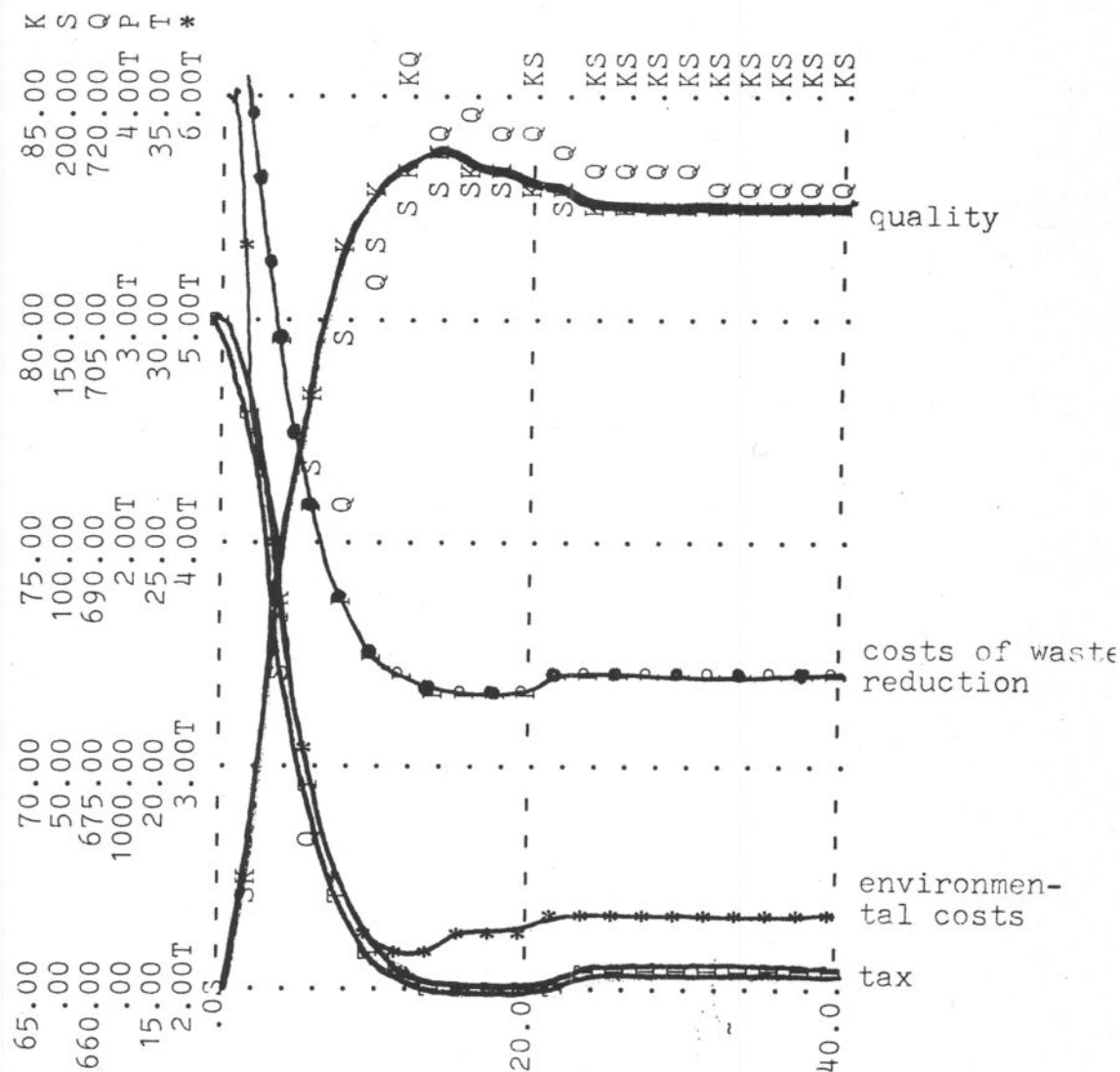


Figure 4. MIN-strategy

If we think of the situation in practice the continuously changing tax could cause some difficulties to polluting firms (see e.g. Førsund 1975). For example investments and decisions concerning pollution control technology which are made for many years would most likely nonoptimal. The organizational costs and costs of implementation would also be highest in the MIN-strategy due to the increased work of the authority.

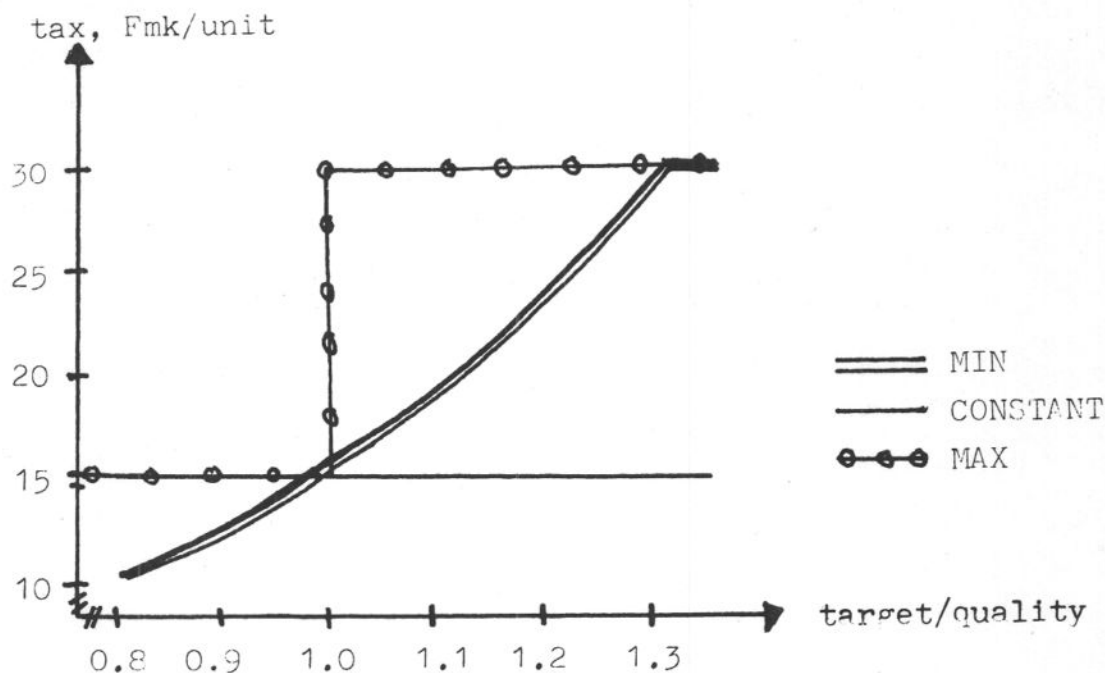


Figure 5. The tax as a function of target quality/quality in different strategies

As a result we can say that the CONSTANT-strategy has some relative advantages which compensate for the only disadvantage of this alternative namely the longer time that will be needed to achieve the target level. In the CONSTANT-strategy the system approaches the equilibrium in a balanced way where the changes take place in the environment sector and the production sector stays in equilibrium through all periods. The stability of the production sector is achieved with little relative cost.

An interesting feature of the system is the development of the cumulative costs of the strategies. The best strategy in this respect depends on the planning period, as can be seen in the figure 6.

If the planning period is shorter than 13 periods, the CONSTANT-strategy gives the minimum costs and the second is MIN-strategy. If the planning period is between 13 and 17 periods, the ordering of strategies is MIN, CONSTANT and MAX. After period 17 the ordering remains unchanged: MIN-strategy gives the minimum costs and the second best is MAX-strategy. However, as mentioned before, the differences are not very significant.

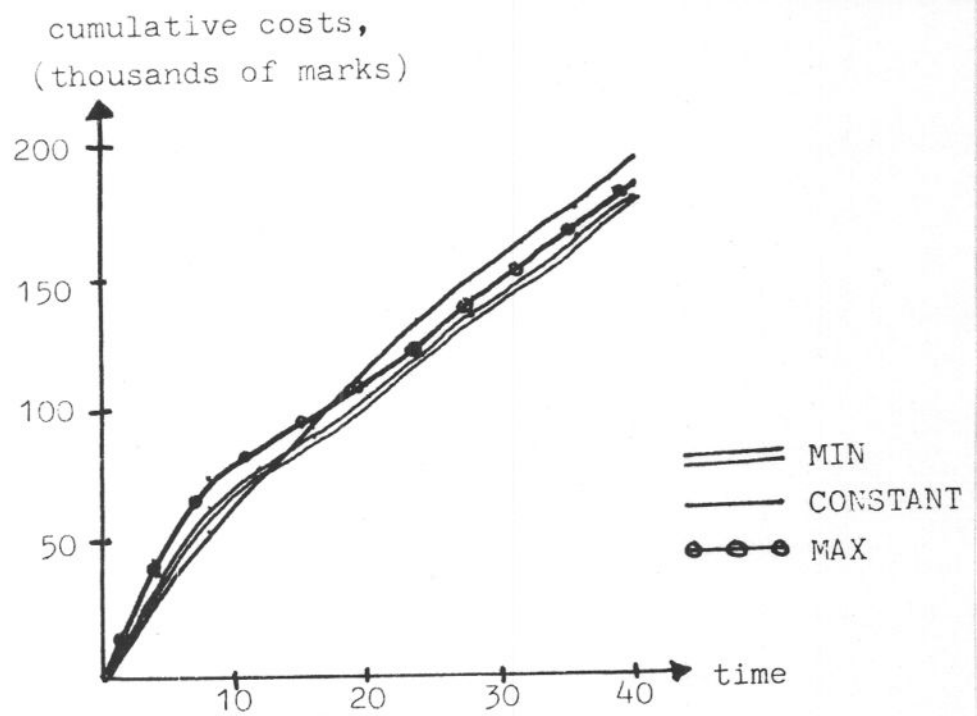


Figure 6. Cumulative costs in different strategies

5. Sensitivity analysis

5.1 Introduction

We have defined a system f , behaviour of which depends on two things: the pollution control strategy ($|const, max, min|$) of the authority and the parameter combination k of the model. In the previous study we used the set of parameters defined in section 3.

However, the results achieved can be thought to depend on the parameter set used and so we want to study the sensitivity of the results as to the changes in the values of the parameters.

The aim of the study was to find for some objective function and parameter set the best strategy x_0

$$f(x_0, k_e) \leq f(x_j, k_e) \quad (1)$$

$$x_0 \in X, \quad x_j \in X$$

where X = the set of strategies (const, max, min) and k_e = values of parameters used. Formula (1) says that the behaviour of the system for

parameter set k_e using strategy x_0 is better than with the other strategies. However, the results are dependent on values of parameters and the problem is whether the following statement is valid for x_0 :

$$f(x_0, k) \leq f(x_j, k) \quad (2)$$

$$x_0 \in X, \quad x_j \in X, \quad k \in K$$

where K = set of all possible parameter combinations. In other words we want to see if the results hold with different parameter values too.

From the authority's point of view the problem is as we see it the following. Some strategy (const) is selected after careful study of the system and its parameters. However, some parameters might be given wrong estimates because of several reasons and the authority is interested to know if the results achieved are sensitive to changes in these values. The authority wants to know if the constant-strategy should be selected even if the true parameter set is different from the one estimated.

5.2 The principle of the sensitivity analysis (see also Keloharju 1977)

The comparison of different strategies is very common in system dynamics. A new method for sensitivity analysis in strategy comparison is presented. The sensitivity analysis proceeds through the following steps:

- 1° Solve the model (1) with estimated parameter values and different strategies and find x_0 which fulfils the criterion:

$$f(x_0, k_e) \leq f(x_j, k_e)$$

$$x_0 \in X$$

$$x_j \in X$$

- 2° Define K

3° Solve the sensitivity analysis model: find k so that

$$\text{Max } \{f(x_0, k) - f(x_j, k)\} = Z$$

$$k \in K$$

for every $j \in J$

If the optimal $Z \leq 0$, then the equation (2) stands and general conclusions about the strategies can be drawn.

4° To get more information about the sensitivities, solve the secondary sensitivity analysis model:

$$\text{Min } \{f(x_0, k) - f(x_j, k)\} = E$$

$$k \in K$$

for every $j \in J$

Now one has minimum and maximum differences between strategies x_0 and x_j 's. This kind of analysis can be done not only between the best and other strategies but also between all strategies.

5.3 Sensitivity analysis of the pollution control model

The following parameters which we have considered to be the most complex to be estimated were assumed to be biased: c , r , g , f . The maximum deviations were thought to be $\pm 20\%$ from the original values. The sensitivity analysis models were run pairwise with strategies: CONST-MAX, CONST-MIN, MAX-MIN. The results of these runs are in tables 3 and 4.

Run	Original difference (from table 1)	Variations through sensitivity analysis
CONST - MAX	8000	911 - 17950
CONST - MIN	11000	5779 - 25614
MAX - MIN	3000	-2158 - 51068

Table 3. Variations through sensitivity analysis

parameter	orig. value	Parameter values in the sensitivity run					
		CONST	-	MAX	CONST	-	MIN
		minimum difference		maximum difference	minimum difference		maximum difference
c	0.01	0.012		0.0096	0.012		0.008
r	15	12		17.36	14.56		18
g	100	80		120	80		120
f	2	1.6		2.4	1.6		2.4

Table 4. Parameter values in different sensitivity runs

The outcome of the system seems not to be sensitive to changes in parameter values.

The CONSTANT strategy is seen to result in the highest total costs for any actual parameter set in the range specified but the difference in costs between it and the other strategies is, however, less than 10 % for any parameter combination. The other two strategies result in greater variations in quantity produced and the other variables. The time periods taken to achieve the target level of environmental quality are similar to those in table 1.

Even after the sensitivity analysis the results and implications derived earlier seem to be valid.

6. Conclusions

Three different strategies for pollution control were studied. As a whole, the best strategy seems to be the CONSTANT-strategy, where the system approaches the equilibrium in a balanced way. This result was not sensitive to changes in the values of the parameters of the model.

Using the analytic approach in defining decision rules of a system dynamics model as well as SDRDYN in finding the optimal paths of some SD variables seem to give new perspectives for system dynamics model building: the analytic approach gives a good theoretical basis for equation formulation and SDRDYN with third order equations makes it possible to optimize a simulation model. SDRDYN has even proved to be a useful tool in sensitivity analysis when strategies are compared to each other. Much further research should be done in this area. The assumptions of this paper like constant price could be loosened. It is also possible to study impacts of alternative means of pollution control like regulation, subsidies and so on on the system.

References.

- Bawa, V.S. On Optimal Pollution Control Policies, Management Science Vol 21, no: 9, May 1975, pp 1397-1404.
- Downing, P. B. & A Simulation Study of Alternative Pollution Control
Watson, W.D. Enforcement Systems, Management Science Vol 22,
no: 5, January 1976, pp 558-569.
- Ethridge, D. The Inclusion of Wastes in the Theory of the Firm
Journal of Political Economy Vol 81, no: 6, 1973,
pp 1430-1441.
- Fau, L.S. & Pollution Control and the Behaviour of the Firm -
Froehlich, B.R. A Technical Note, Engineering Economist Vol 17,
no: 4, 1972.
- Førsund, F.R. The Polluter Pays Principle and Transitional Period
Measures in a Dynamic Setting, The Swedish Journal
of Economics Vol 7, no: 1, 1975, pp 56-68.
- Gates, W.E. & Application of Industrial Dynamics Concepts to Decision
Males, R.M. & Making in Environmental Management, Water Resources
Walker, J.F. Research Vol 6, no: 6, December 1970, pp 1549-1560.
- Holt, C.C. & A Linear Decision Rule for Production and Employment
Modigliani, F. & Scheduling, Management Science Vol 11, no: 2,
Simon, H.H. October 1955, pp 1-30.
- Keloharju, R.J. Multi-objective Decision Models in System Dynamics,
Dynamica Vol 3, Part I and II.
- Pearce, D.P. Environmental Economics, Longman Group Ltd, London 1976.

../..

Roberts, M.J. &
Spence, M.

Effluent Charges and Licences under Uncertainty,
Journal of Public Economics, 5, 1976, pp 193-208.

Sage & White

Optimum Systems Control, Prentice Hall, 1976.

Wenders, J.T.

Methods of Pollution Control and the Rate of
Charge in Pollution Abatement Technology,
Water Resources Research Vol 11, no: 3,
June 1975, pp 393-396.