

Accounting for Depreciation

By

Dr. R. G. Coyle

Director System Dynamics Research Group  
University of Bradford

Abstract

This paper deals with the modelling of accounting depreciation on both a historic cost and an inflation adjusted basis.

Introduction

In corporate modelling capital investment and scrapping of obsolescent plant are key processes. It is usually necessary to model not only the physical processes involved, but also their impact on the published accounts of the company. In recent years, when inflation has been high, this requirement has become even more important. Accordingly, this paper deals with the modelling of both depreciation and asset revaluation.

Simple Depreciation Equations.....

The financial depreciation of capital equipment can be modelled in several ways. The basic idea is that there is a level equation, storing the "remaining value" of the plant, such as

$$L \text{ WDV.K} = \text{WDV.J} + \text{DT} * (\text{RAVP.JK} - \text{DEPR.JK}) \quad (1)$$

WDV = (\$)      Written-down value of Plant

RAVP = (\$/M)    Rate of Adding to Value of Plant, i.e. the money value of  
new plant being completed.

DEPR = (\$/M)    Financial Depreciation Rate.

We are not concerned here with RAVP, or with the use of DEPR as a source of cash flow for new investment. Our sole concern is with methods for the calculation of DEPR in cases where inflation can be ignored, perhaps because the model is in constant price terms. Later, we shall deal with the problem of plant revaluation.

We now describe equations for two different methods of depreciation.

### Exponential, or Reducing-Balance, Depreciation

This is by far the simplest. We simply put

$$R \text{ DEPR.KL} = \text{WDV.K} / \text{PLFD}$$

(2)

PLFD=(M) Plant Life for Depreciation.

There may be no connection between PLFD and the physical life of the plant, or even its life before it becomes technically obsolete. In general, PLFD is the smallest value the tax authorities will tolerate.

The drawbacks to equation (2) are

a) Because equations (1) and (2) act as a first order exponential delay of the recovery of RAVP, the actual time taken to 'recover' the investment will really be about three times PLFD, though, of course, the last part of the tail can be ignored.

The problem can be overcome by using a smaller PLFD. Thus, if the tax laws say that asset value can be written-off in 5 years, then PLFD=3 years (36 in equation 2) will be approximately correct. This should cause no problem with the accountants, because the method we are using is exactly the same as the Reducing-balance method described in most accounting textbooks.

b) The value of DEPR will at first be very large and will then tail away, as a first-order delay always does. This is not always bad, as it could be a good model of the phenomenon of accelerated early depreciation sometimes used by governments to stimulate industrial investment. It is, in any case, the purpose of the Reducing-balance method to produce such a pattern. The solution really is to look at some examples of the particular depreciation practice being modelled and, by trying various values of PLFD, see how close one can come. This has to be done with a PULSE for RAVP so that one compares a model of depreciation of a single plant with the accountant's calculations.

If the agreement between the model and the company practice is not very close, and if the difference really matters, given the purpose for which the model is intended, then we may have to use one of the following more involved methods, bearing in mind that they too will have disadvantages.

### Straight-Line Depreciation

This is an attempt to make sure that the plant is exactly and completely depreciated over PLFD. A careless reading of an accounting text book would lead one to write

$$R \text{ DEPR.KL} = \text{SAMPLE}(\text{WDV.K/PLFD}, \text{PLFD}, \text{WDV.K/PLFD}) \quad (3)$$

The trouble with equation (3) is that the new plant represented by RAVP would not even start to get depreciated until the next SAMPLE was taken. An even more careless reading could get us involved in trying to keep track of every single item of plant, which would usually be an unjustified complication to the model. We, therefore, go back to trying to model what the Finance Director does, because he most certainly does not follow each item of plant through its life, any more than the company accounting system does, no matter what is claimed for it.

We therefore introduce the parameter DAI, the Depreciation Assessment Interval, it being the period of time between decisions on how much to write down the plant. We have to allow for two alternatives - deciding the rate at which plant is to be written down during the coming DAI, and deciding the amount to be written off at the end of the DAI just ended. We call these Prospective and Retrospective Depreciation, respectively.

#### a) Prospective Straight Line Depreciation

This now provides

$$A \text{ DEPR.KL} = \text{SAMPLE}(\text{WDV.K/PLFD}, \text{DAI}, \text{WDV.K/PLFD}) \quad (4)$$

The only difference between equations 3 and 4 is the use of DAI, instead of PLFD, as the sampling interval in equation 4.

Equation 4 still does not allow for depreciation of plant, which will be completed during the coming DAI, but, if this refinement was needed, it can be provided by replacing WDV.K by  $(\text{WDV.K} + \text{ARAVP.K} * (\text{DAI}/2))$ . The new factor is the forecast average amount of plant completions in the coming DAI, ARAVP being the smoothed value of RAVP.

b) Retrospective Straight-line Depreciation

The situation where depreciation charges are assessed at the end of each DAI is easily handled by setting

$$R \text{ DEPR.KL} = \text{PULSE}((\text{WDV.K/PLFD}) * \text{DAI}) / \text{DT}, \text{DAI}, \text{DAI}) \quad (5)$$

WDV.K/PLFD is the rate of depreciation, \$/M on a straight-line basis.

Multiplying by DAI gives the amount in the period, and division by DT is the required method of making a rapid change in a level.

The forecast effect of depreciation next DAI on cash flow, which might be needed in the decision-making sector of the model, is, dropping the DYNAMO conventions,

$$\text{ENPD} = (\text{WDV} - \frac{\text{WDV} \times \text{DAI}}{\text{PLFD}}) \frac{\text{DAI}}{\text{PLFD}} + \text{ARAVP} \times \text{DAI} \times \frac{\text{DAI}}{\text{PLFD}} \quad (6)$$

ENPD=(\$) Expected Depreciation Next Period.

ENPD is the amount of money expected from depreciation at the end of the DAI, which is, as it were, about to start. The term in parentheses at the start is what remains after equation 5. The next factor of DAI/PLFD is the same as in equation 5. ARAVP x DAI is the expected new plant additions in the coming DAI. Again, we divide by PLFD to get a depreciation rate and multiply by the very last DAI to get a quantity.

General Comment on Equations 4 and 5

The more complicated methods produced in the last two sub-sections reduce to the method of equation 2, if DAI=DT. Generally, DAI will be 1 year (or DAI=12 as we have worked in months). Usually, one is interested in depreciation only in models where LENGTH is fairly large, perhaps twice as large as the physical life of the plant. Typically, therefore, one might have a model in which LENGTH=240 months, and such a model would probably have a DT of, perhaps, 3 months, unless one had got really confused about model purpose and had the micro-dynamics of production in the same model as the micro-dynamics of investment. This may sometimes be justified, but it will often happen that considerations of LENGTH, DT, and DAI (the last being our invented name for a real managerial parameter) mean that equation 2 will, after all, be perfectly adequate. It is always essential to question whether the detail in a model is really necessary, or has been put in to impress (or convince!) the client.

### Asset Revaluation and Depreciation

Inflation has had many effects, one being that the traditional methods of depreciation calculations used by accountants no longer provide enough money to replace the plant. After some debate, it is now widely accepted that something will have to be done, sometime.

The problem is far from new. Batty (1963) writes movingly of 'the chronic inflation and rising prices, not halted in 1962'. He quotes the 1961 revaluation of the retail shops owned by Montague Burton, from a book value of £14 million to £42 million. Rather more recently, the 1973 accounts of the UK company Associated Portland Cement Manufacturers' (APCM) contain the argument:

1. Cement plants have useful lives of 30 years.
2. A plant built in 1965 for £8 million would have cost £14 million in 1973. Depreciation based on original cost would not provide enough money by 1995 to replace it.
3. 'The Company has for 20 years periodically revalued its assets by taking their cost of replacement at today's values and reducing that figure by the proportion which its unexpired life bears to its total life'. The value placed on the 8 year old plant mentioned above would be  $\pounds(22/30) \times \pounds 14$  million or £10.26 million. This makes the depreciation charge 10.26/22 million, or £470,000 compared to the 8/30 million, £270,000, which it would have been without revaluation.
4. The company revalues its assets only every 5 years because of the work involved, using an annual correction based on price indices.
5. 'In industries where there are more rapid changes in technology it may be less important to deal with inflation in the accounts.'

### Method

Although there are several aspects of accounting for inflation, we shall provide in this example an approach tailored to the situation described in the APCM Accounts. Having dealt in the previous section with the calculation of the depreciation charge, we concentrate solely on the revaluation problem. We consider a typical situation where

$$L \text{ CAP.K} = \text{CAP.J} + \text{DT} * (\text{PCR.JK} - \text{PWR.JK}) \quad (7)$$

$$R \text{ PWR.KL} = \text{DELAY3}(\text{PCR.JK}, \text{PLT}) \quad (8)$$

CAP = (T/M) Productive Capacity

PCR = (T/M)/M Plant Commissioning Rate

PWR = ((T/M)/M) Plant Withdrawal Rate

PLT = (M) Plant Lifetime

It is not essential that the delay in equation 8 be of third order. Indeed, this may well be an instance where we may reasonably wish to incorporate a delay of higher order into our model.

If we are to revalue the plant to current model prices, we need to know

- 1) How much plant there is, i.e. CAP,
- 2) How old it is, i.e. APA, the Average Plant Age;
- 3) What it cost when we bought it and
- 4) How much prices have changed since then.

Equations to calculate the Average Plant Age (APA) have been given elsewhere (Coyle, 1976).

The third aspect is dealt with by the equations

$$L \text{ HCEP.K} = \text{HCEP.J} + \text{DT} * (\text{RAVP.JK} - \text{RRVP.JK}) \quad (9)$$

$$R \text{ RAVP.KL} = \text{PCR.KL} * \text{PRICE.K} \quad (10)$$

$$R \text{ RRVP.KL} = \text{DELAYn}(\text{RAVP.JK}, \text{PLT}) \quad (11)$$

HCEP = (\$) Historical cost of Existing Plant, i.e. the total paid, at the time it was built, for plant presently in operation.

RAVP = (\$/M) Rate of Adding to Value of Plant, i.e. cost of plant currently being installed, if any.

RRVP = (\$/M) Rate of Reducing Value of Plant. The rate of taking out of the historical cost 'book' the amounts paid when it was commissioned, for plant currently being withdrawn.

The delay in equation 11 must be the same as that in equation 8, using cascaded DELAY3's and dummy rates, or a DELAYX, if needed.

Note that RAVP will also feed the level for written-down value, but RRVP only affects the artificial variable HCEP.

If PCR is modelled as a delay of a Plant Order Rate, POR, and construction delay, CDEL, we might need to adapt equation 10, using a dummy rate, DR, to bring the plant into HCEP at the price paid for it when it was ordered, i.e.

$$R \text{ DR.KL} = \text{POR.KL} * \text{PRICE.K} \quad (12)$$

$$R \text{ PCR.KL} = \text{DELAYy}(\text{POR.JK}, \text{CDEL}) \quad (13)$$

$$R \text{ RAVP.KL} = \text{DELAYy}(\text{DR.JK}, \text{CDEL}) \quad (14)$$

The order,  $y$ , would be chosen to satisfy the dynamics of plant construction. If the price of plant is defined by a time series

$$A \quad \text{PRICE.K} = \text{BP} * \text{TABHL}(\text{INDEX}, \text{TIME.K}, t_1, t_2, t_3) \quad (15)$$

PRICE = (\$/(T/M)) Current Plant Price

BP = (\$/(T/M)) Base Price at time  $t_1$

INDEX = (1) Table of Values for an Inflation Index

Then Replacement Cost of Plant is given from

$$A \quad \text{RCP.K} = \text{HCEP.K} * (\text{I1.K} / \text{I2.K}) * ((\text{PLT} - \text{APA.K}) / \text{PLT.K}) \quad (16)$$

$$A \quad \text{I1.K} = \text{TABHL}(\text{INDEX}, \text{TIME.K}, t_1, t_2, t_3) \quad (17)$$

$$A \quad \text{I2.K} = \text{TABHL}(\text{INDEX}, \text{TIME.K} - \text{APA.K}, t_1, t_2, t_3) \quad (18)$$

The ratio  $\text{I1/I2}$  is the extent to which inflation has taken place during the last APA months. The factor  $(\text{PLT} - \text{APA}) / \text{PLT}$  reduces the revaluation according to the remaining useful life, exactly as described in point 3 in the quotation from APCM's annual report.

Equation 15 does not imply that there was no inflation before  $t_1$ , simply that  $t_1$  was the time of the last revaluation of plant. This does mean that the initial value of HCEP and WDV must be equal.

The first weakness of equation 16 is that it implies that all the plant currently in operation was bought APA months ago. If inflation has been fairly rapid recently, or there has been a recent major capacity expansion, then new plant will already have gone into HCEP at a fairly high cost, and will, therefore, tend to be rather overvalued by equation 16. The solution is to have, say, two categories of plant, Old and New, with lifetimes  $\text{TO}$  and  $\text{TN}$  such that  $\text{TO} + \text{TN} = \text{PLT}$ . Define a transition rate  $\text{TRNO}$  - Transition Rate from New to Old by

$$R \quad \text{TRNO.KL} = \text{DELAY3}(\text{PCR.JK}, \text{TN}) \quad (19)$$

$$R \quad \text{PWR.KL} = \text{DELAY3}(\text{TRNO.JK}, \text{TO}) \quad (20)$$

This automatically gives a 6th order delay and the method has an obvious extension to a 9th order delay. Now calculate  $\text{APAO}$  and  $\text{APAN}$  (c.f. Coyle, 1976), remembering to add  $\text{TN}$  to  $\text{APAO}$ . Replace HCEP by HCEPO and HCEPN, where the O and N throughout have their obvious meaning, and use an equation of the same form as 10, but twice the length, to add together the revaluations of the two categories of plant.

Finally, we may now write the revaluation and depreciation equations.



Revaluation is assumed to take place, as modelled in equation 16, at intervals of RVI months. We shall, for simplicity, suppose that the financial depreciation rate, DEPR, is calculated continuously, but the method is easily altered, as described earlier, if this is needed. We then have

$$L \quad WDV.K = WDV.J + DT * (RAVP.JK - DEPR.JK + PULSE((RCP.J - WDV.J) / DT, RVI, RVI)) \quad (21)$$

$$R \quad DEPR.KL = WDV.K / (PLT - APA.K + 1E-06) \quad (22)$$

The first two rates, RAVP and DEPR, are exactly as discussed earlier. The PULSE forces WDV back up to the value of RCP indicated by equation 10, at every revaluation occasion.

Note that, if no new plant was ever built, the effect of equation 21 would be to force WDV back up in line with prices, with WDV declining again under the influence of DEPR (the 1E-06 in equation 22 simply prevents division by zero). Eventually, as the plant gets older, the effect of  $(PLT - APA.K) / PLT$  in equation 16, and the reduction in HCEP caused by RRVP in equations 3 and 5, cause the revaluation effect to become negligible, as one cannot revalue plant which no longer exists. At the appointed time, cumulative depreciation should be fairly close to the amount needed to replace the plant, which is the best that one could hope for, in the real world, not merely in the synthetic existence of a model.

A final point is that, after the revaluation in equation 21, the accounts in the model must still balance. A common method of modelling corporate finance is to equate the sources and uses of funds by, for example, having

Shareholders' Capital	Cash
+	+
Cumulative Retained	Working Capital
Profits	
+	+
Debt	Written-down Value of
	Plant
=	=
Capital Employed	Capital Used

If we increase written-down value as in equation 21, the equality between Capital Employed and Capital Used (or Deployed) will no longer hold. We avoid this, as accountants would, by creating on the left hand side of the foregoing 'Capital Balance Sheet', a new level called the Revaluation Account,



REVA. This has the equation

$$L \text{ REVA.K} = \text{REVA.J} + \text{DT} * \text{PULSE}((\text{RCP.J} - \text{WDV.J}) / \text{DT}, \text{RVI}, \text{RUI}) \quad (23)$$

REVA=( $\$$ ) Cumulative change in Capital Value due to Plant Revaluation.

Note that future depreciation of the revalued plant merely transfers cash on the Right Hand Side of the Capital Balance Sheet from Written-down Value to Cash and has no effect on the equality between the two sides.

#### Caution

The treatment we have provided here has ignored the possibility of the plant scrapped in equation 8 having any scrap value, or the prospect that revaluation of plant may be taxable. There is no great difficulty in writing appropriate equations for these cases as long as the modeller remembers to include CHECK equations, to ensure that the equations used have the same impact on both sides of the balance sheet.

#### References

Batty, J.

Management Accountancy,  
McDonald and Evans, 1963.

Coyle, R.G.

The Calculation of Average of a  
Group of Items,  
Dynamica, 3, 1, 1976.