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Designing Policies the Ziegler Nichols Way

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Abstract

This paper illustrates the use of two methods devised by Ziegler and Nichols to design feedback controls for System Dynamics models. The methods were originally devised through experiments with hardware systems. For this reason, they give good basic designs with very little effort.

Introduction

In 1942 Ziegler and Nichols published two methods for the design of control rules for two commonly occurring, but different, types of system. Their methods had been devised through extensive experimenting with a variety of hardware systems. The underlying purpose of their work is best illustrated by their own description (Ziegler and Nichols, 1942)

"A purely mathematical approach to the study of automatic control is certainly the most desirable from the point of view of accuracy and brevity. Unfortunately, however, the mathematics of control involves such a bewildering assortment of exponential and trignometrical functions that the average engineer cannot afford the time to plough through them to a solution of his current problem.

"The paper will thus first endeavour to answer the question, 'How can the proper controller adjustments be quickly determined on any control application?'"

The Ziegler Nichols (ZN) methods lead to one, two and three term controllers (i.e. Proportional + Integral + Derivative), a type still favoured for a wide range of hardware control applications. Though they result in linear controls, they were proven on actual systems that exhibited various nonlinear effects. They are thus well suited to System Dynamics applications, both because of their ability to cope with mild nonlinearities and because of their close connexion with classical control theory, which was the theoretical foundation of System Dynamics. In practice, because they are easy to learn and apply, they provide a systematic approach to policy design midway between often time consuming ad hoc experimentation and the difficult to acquire sophistication of rigorous classical methods such as Bode and Nichols (Thillainathan, 1978).

The Basis of Controller Design

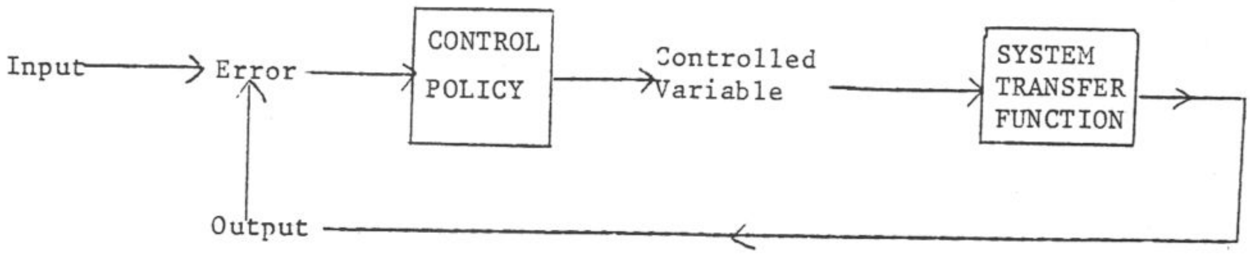


FIGURE 1.

The ZN methods apply to single feedback loop controllers of the type shown in figure 1. The controlled system is basically divided into two components, the system or Open Loop transfer function and the controller. This division is of course basic to classical feedback control design (c.f. Eveleigh, 1972). The system transfer function represents in System Dynamics terms 1) Filters to be discussed later, 2) the behavioural equations of the system, i.e. the equations describing those parts of system behaviour that for the design being carried out are considered fixed. DELAY's are perhaps the most important single component of most transfer functions, though a complex system will of course contain many other types of behavioural equation. In applying the methods, the general form of the System Transfer Function is important because it is on the basis of this form that the choice as to which of their two methods to employ is made.

Though both methods are suitable for the direct design only of single feedback loops or single input, single output controllers, it should be noticed that the System Transfer function will always contain a number (some implicit) of feedback loops. This in turn means that is is possible to use the methods sequentially to design one feedback loop, which is then considered as part of the System Transfer function for the design of the next feedback loop and so on. This sequential approach to policy design is known to work well when the feedback loops in the system operate at different speeds. It has, for example, been very successfully employed by Ratnatunga (1979) in a model of the consumption sector of an economy, where adjustments to the mix of goods purchased is rapid when the price of a particular item changes, whereas the adjustment of total expenditure across all products when income changes is a much slower process.

The rules given by Ziegler and Nicholls lead to a controller of the form

$$ZK \left\{ e(t) + \frac{1}{I} \int e(t)\,dt + D \frac{de}{dt} \right\} \qquad (1)$$

where e(t) denotes the error at time t. As well as giving rules for determining appropriate values of ZK, I and D for a full 3 term controller involving Proportional, Integral and Derivative control, they also give rules for determining ZK and I alone (proportional + integral control) or ZK alone (proportional control). Since proportional, integral and derivative action contribute to effective control in different ways, all three are often considered desirable in hardware system design. In many system dynamics applications, however, only proportional feedback control is used and the Ziegler Nicholls methods thus offer the prospect of better control than usually achieved.

Actual Controller Design

In order to make effective use of the Ziegler Nicholls methods, it is usually useful to extend the notion of the control system of Figure 1 as shown in Figure 2.

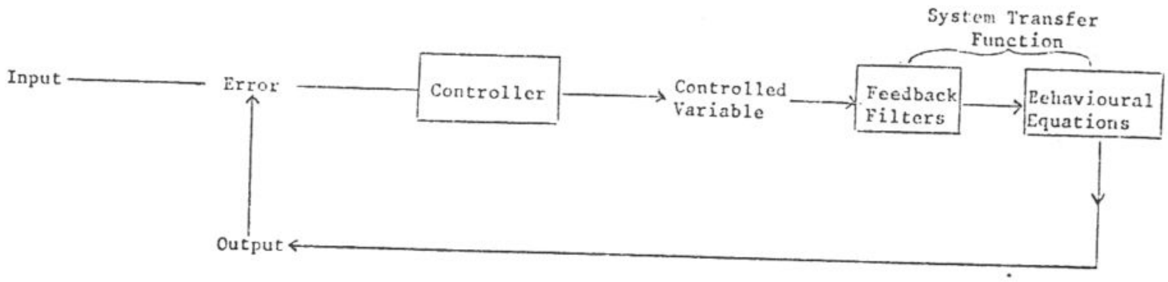


FIGURE 2.

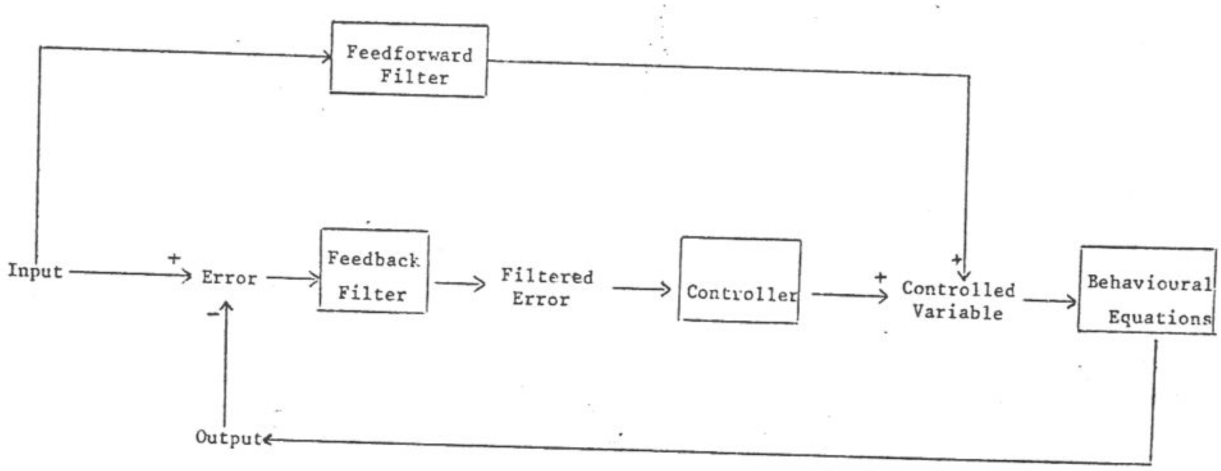


FIGURE 3.

By filters in this figure, we mean the various averaging devices used in System Dynamics models such as SMOOTH's or Moving Averages. These serve two purposes. Firstly, they reflect the fact that control policies must be based on LEVELS rather than RATES because the latter cannot be measured. Since averages of RATES are LEVELS the use of filters secures this property. Secondly, filters need to be used to shape the response of the controlled RATE to remove high frequency variations from it (Sharp, 1976).
The main change from the point of view of the Ziegler Nichols methods between Figures 1 and 2 is the splitting of the System Transfer Function into a Feedback Filter block + a Behavioural Equations block. It is the form of the behavioural equations block or alternatively the form of the output variable that determines which of the two methods should be applied. Furthermore, to apply the method to be described second, it is necessary to investigate the response of the System Transfer Function shown in Figure 2. In order to achieve a practical control set of control policies, however, it is necessary to rearrange and alter the system of Figure 2 as shown in Figure 3. Two changes will be noticed. Firstly, a feedforward element has been added to the system, i.e. the direct link between the input and the controlled variable. Such feedforward elements occur frequently in the control policies used in System Dynamics models, because of their utility in improving system performance when subjected to a ramp input (c.f. Sharp, 1976). Since the feedforward part of the control does not affect the stability and other behaviour of the feedback controller, the latter can be designed independently of Zeigler Nichols' methods. In order to implement the control rules, however, it is necessary to interchange the positions of the Feedback filter and controller in the system. For linear filters such as SMOOTH's and control rules of the type given by equation (1) the feedback elements of the systems of Figures 2 and 3 are equivalent. Equation (1), however, requires the measurement of the rate of change of error, which is not feasible. By placing the filter before the controller, it is possible to implement the control using only variables that can be measured.

A Production Planning Example

In order to demonstrate the use of the Zeigler Nichols methods, we consider a simple production planning system based loosely on a real system as described by Sharp and Coyle (1976). As will be shown, the choice of method depends on what we consider the notional input and output variables. We accordingly define the input sector of the model, the behavioural equations and the output sector. These are given in Figure 4. The control rules appropriate to each method are then discussed separately.

```
NOTE INPUT SECTOR
R  OR.KL=1000+75*SIN(6.283*TIME.K/.1667)+150*SIN(6.283*TIME.K/0.25)
X  +100*SIN(6.283*(TIME.K+1.7)/0.5)+125*SIN(6.283*TIME.K/4.5)+N.K
D  OR=(UNITS/YR) ORDER RATE
L  N.K=N.J+DT*(NORMRN(0,400)-N.J)/0.2
N  N=0
D  N=(UNITS/YR) CORRELATED NOISE IN ORDER RATE
A  DST.K=200+K*SMOOTH(OR.KL,S)
D  DST=(UNITS) DESIRED STOCK
NOTE OR AND RATE OF CHANGE OF DST ARE NOTIONAL INPUTS FOR FIRST ZN METHOD.
NOTE CUMULATIVE ORDERS AND DST FOR SECOND.
NOTE
NOTE BEHAVIOURAL EQUATIONS
R PCR.KL=DELAY3(PSR.KL,0.25)
D PCR=(UNITS/YR) PRODUCTION COMPLETION RATE
NOTE
NOTE OUTPUT SECTOR
L ST.K=ST.J+DT*(PCR.JK-OR.JK)
D ST=(UNITS) STOCK
N ST=DST
NOTE PCR IS NOTIONAL OUTPUT FOR FIRST METHOD.  CUMULATIVE PRODUCTION FOR SECOND.
NOTE
NOTE CONSTANTS
C K=0.1
C S=0.1
C DT=0.04167
```

FIGURE 4

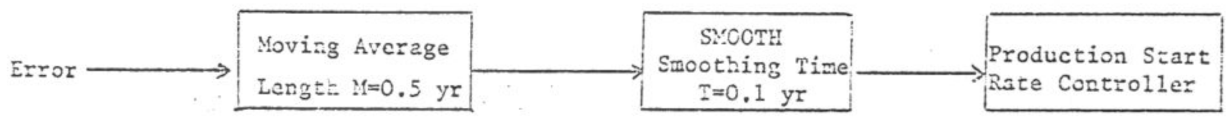


FIGURE 5

Obviously the system, though ostensibly dealing with production planning, is more general in form. Thus there is a LEVEL (ST) whose value is to be controlled to vary in line with some target (DST). A slightly more general form of a base constant plus a constant times a smoothed value than is usual has been chosen for DST. The base constant could, however, be set to zero and the same controls would still apply. Control is affected by controlling some RATE (PSR) and the system has a behavioural component that is considered fixed. In this case, and indeed quite commonly, this is a simple DELAY though more complex forms are easily coped with.

The system is driven by OR. As is the case in the real system on which the example is based, this has strong seasonal components corresponding to monthly, quarterly and six monthly seasonality. We shall assume that as is usual in such systems, the seasonal variation shall as far as possible be absent from PSR. In addition to the seasonal components, the Order Rate contains a business cycle like variation of period 4.5 years, plus correlated random noise. For a justification of the latter term see Forrester (1961).

To demonstrate the robustness of the ZN methods and their usefulness in dealing with systems with discrete time steps, we choose the unusually large value for DT of 0.04167 yr corresponding to 2 working weeks.

Designing Control Policies

The choice of which ZN method to use is determined by whether we choose as our notional output variable a RATE, in which case we use the first ZN method or a LEVEL in which case we use the second.

Applying the First Method

For this method, the notional input is OR plus the rate of change of DST and the notional output Production Completion Rate. In order to design the controller, it is first necessary to select appropriate filters. For the feedback filter, because of the strong seasonal components, we make use of a Moving Average Filter of length 0.5 years (denoted by MOVE(V.K,0.5) where V.K is the variable being averaged). This eliminates the seasonal components completely (Sharp, 1978). In addition to permit the use of derivative control, a second filter is necessary and this will be taken as a SMOOTH of length 0.1 year. The process by which Feedback error is filtered is therefore as depicted in Figure 5.

For this controller a feedforward element is needed for good response to a ramp input. For the feedforward filter to avoid seasonal effects, we again use a Moving Average of length 0.5 yr.

The equations for Production Start Rate with the first method are given in Figure 6.

```
NOTE CONTROLLER EQUATIONS FOR FIRST METHOD
R  PSR.KL=MAO.K+ZK1*(MASE.K/I1+CE.K+D1*DC.K)
D  PSR=(UNITS/YR)
A  SE.K=DST.K-ST.K
D  SE=(UNITS) STOCK ERROR
A  MASE.K=MOVE(SE.K,M)
D  MASE=(UNITS) MOVING AVERAGE STOCK ERROR
A  MAO.K=MOVE(DR.KL,M)
D  MAO=(UNITS/YR) MOVING AVERAGE ORDER RATE
A  MAC.K=MOVE(PCR.KL,M)
D  MAC=(UNITS/YR) MOVING AVERAGE COMPLETION RATE
A  CE.K=MAO.K-MAC.K
D  CE=(UNITS/YR) MOVING AVERAGE ERROR IN COMPLETION RATE
A  DC.K=A*MAO.K+B*MAOS.K+C*MAODS.K
D  DC=(UNIT/YR**2) STOCK ADJUSTMENT TERM
A  MAOS.K=SMOOTH(MAO.K,T)
D  MAOS=(UNITS/YR) SMOOTH OF MAO
A  MAODS.K=SMOOTH(MAOS.K,S)
D  MAODS=(UNITS/YR) SMOOTH OF MAOS
C  M=0.5
D  M=(YR) LENGTH OF MOVING AVERAGE
C  T=0.1
N  E=D*K/S/T
N  A=D/K+E
N  F=(K-(S+T)*E)/S
N  B=1-D/K+F
N  C=-E-F
C  ZK1=
C  I1=
C  D1=
```

FIGURE 6

The constant S in Figure 6 is as defined in Figure 4.

To use the first method, it is now only necessary to obtain the values ZK1, I1, D1. The procedure is as follows:

1) Set I1 to a very large value, e.g. 10^9 and D1 = 0.
2) Find value of ZK1 for which the system exhibits sustained oscillation for OR given by a unit STEP. Denote this value by ZKMAX.
3) Measure period of sustained oscillation (PMAX).

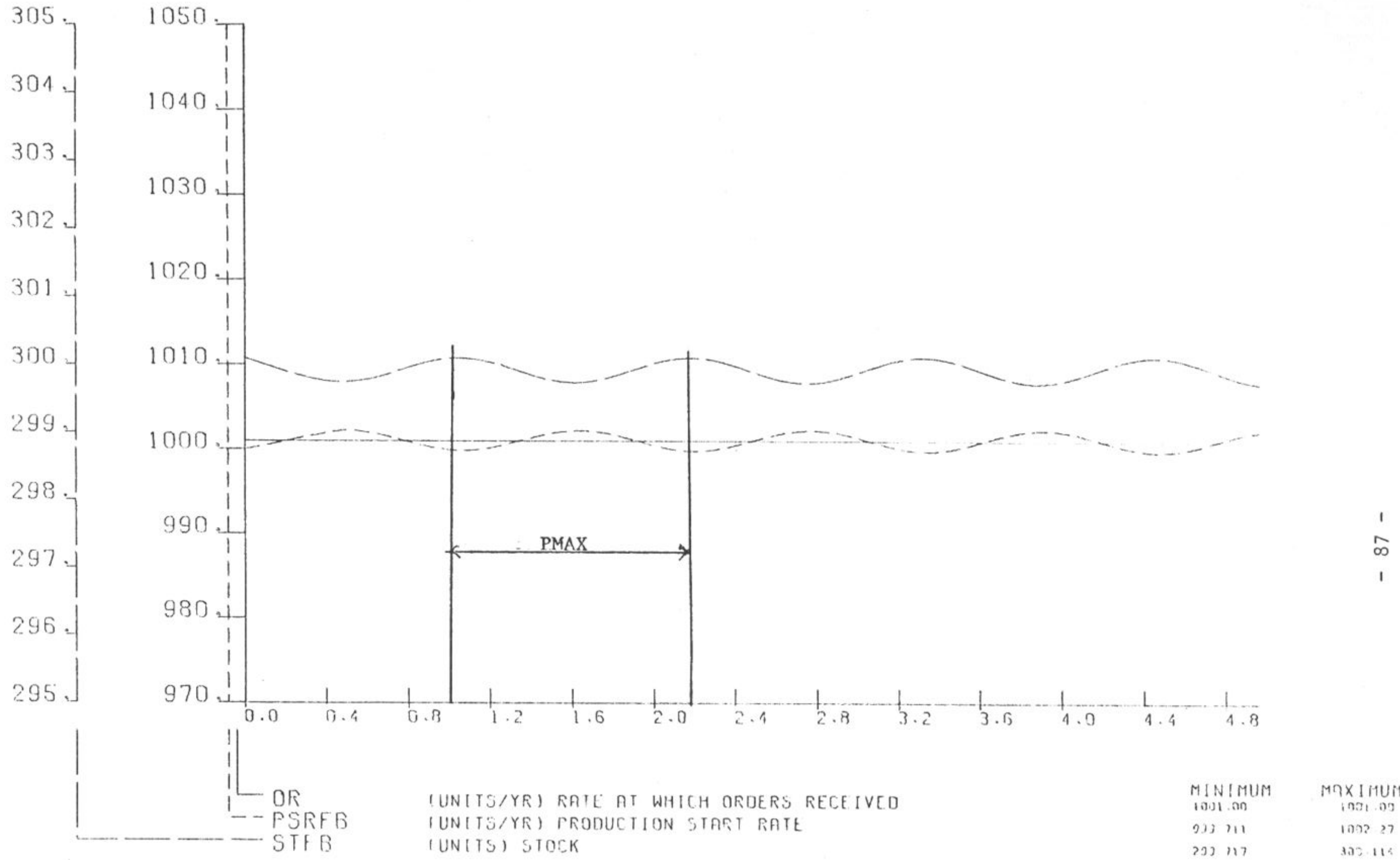


FIGURE 7

ZK=1.8
FEEDFORWARD PRODUCTION BASED CONTROL. TO FIX ZK.

The determination of the value ZKMAX is most easily carried out online. In This case, three runs of the model using the interactive DEC10 version of DYSMAP were required to establish the value ZKMAX = 1.8. A plot of the output for this value of ZK1 is given in Figure 7. The value of PMAX in this case turns out to be 1.1 years.

Once these two constants have been determined, the appropriate settings for the constants ZK1, I1 and D1 are easily determined from the rules given in Ziegler and Nichols paper. They are:

Proportional Control

$$ZK1 = 0.5ZKMAX$$
$$I1 = \infty$$
$$D1 = 0$$

Proportional + Integral Control

$$ZK1 = 0.45ZKMAX$$
$$I1 = 0.83PMAX$$
$$D1 = 0.$$

Proportional + Integral + Deriviative Control

$$ZK1 = 0.6ZKMAX$$
$$I1 = 0.5PMAX$$
$$D1 = 0.125PMAX$$

Figure 8 shows the performance of the Proportional + Integral control obtained from the above formulae (ZK1 = 0.81, I1 = 0.92) and Proportional + Integral + Derivative Control (ZK1 = 1.1, I1 = 0.55, D1=0.13).

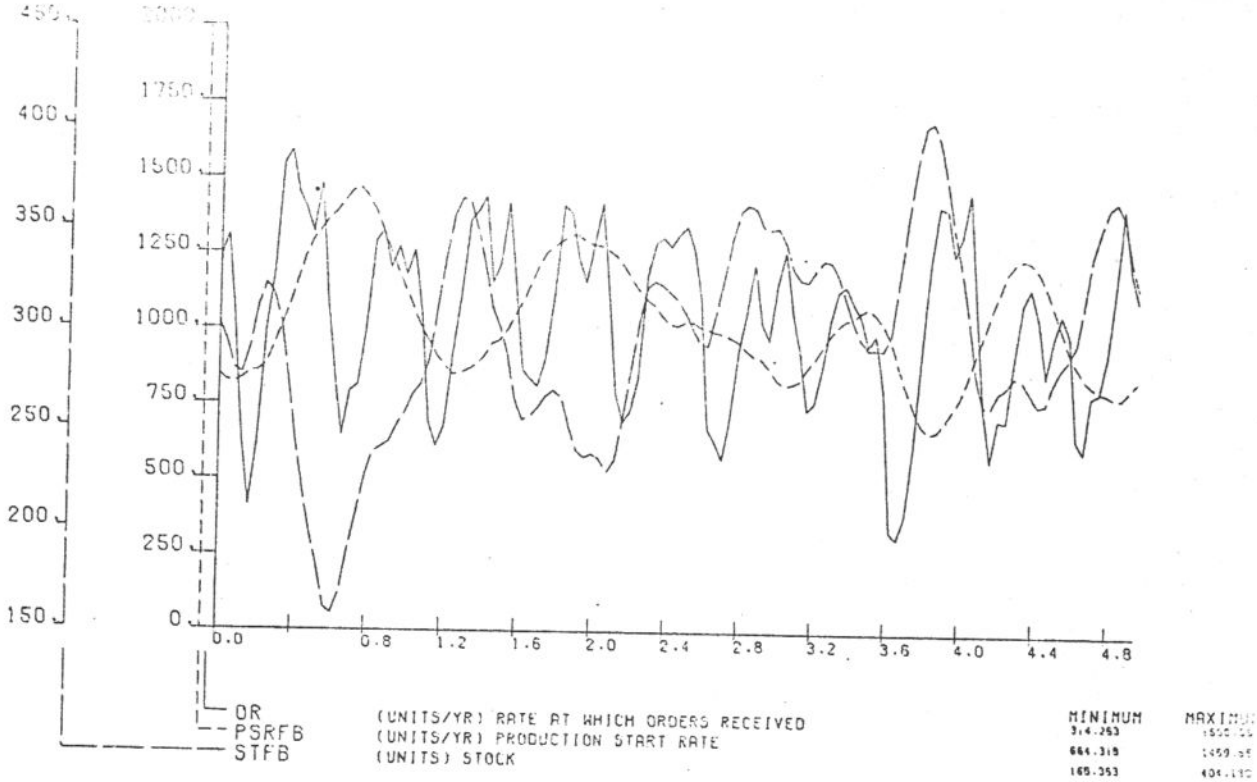


FIGURE 8

ZEIGLER NICHOLLS INTEGRAL CONTROL.
FEEDFORWARD PRODUCTION BASED CONTROL

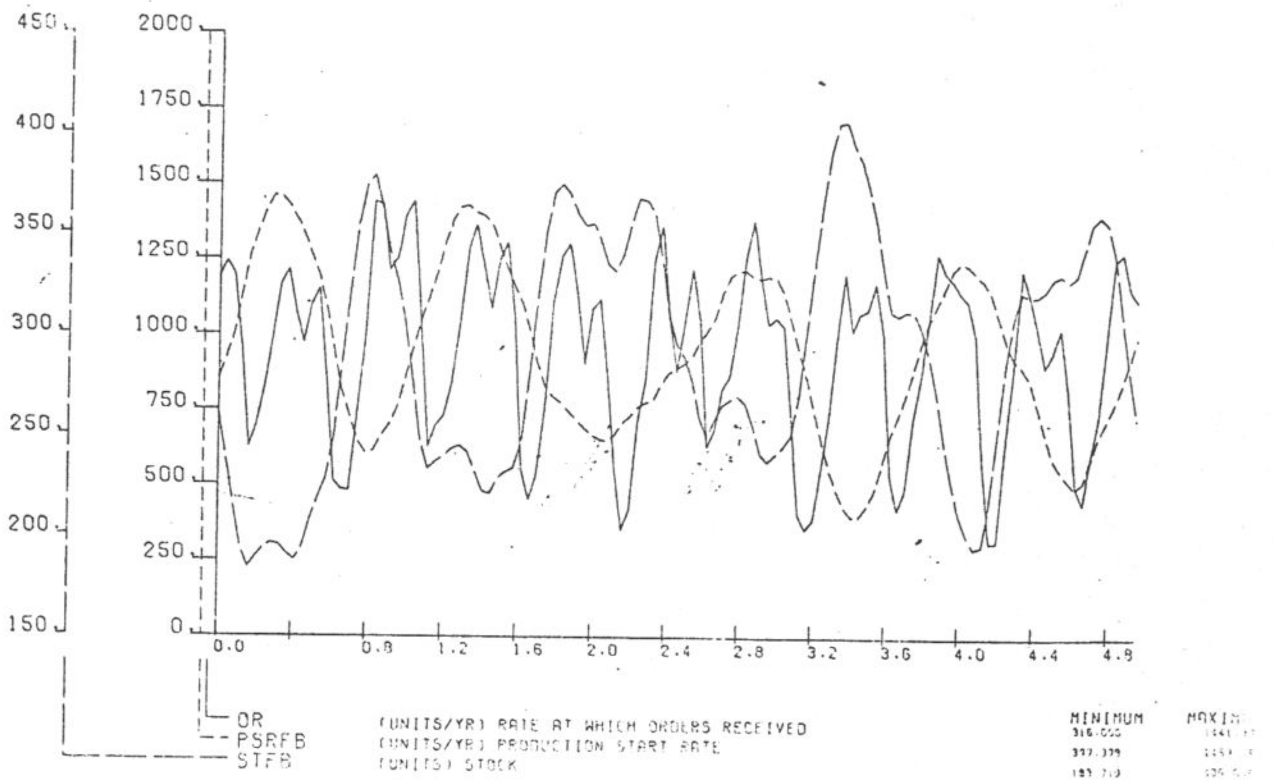


FIGURE 8

ZIEGLER NICHOLLS INTEGRAL+DERIVATIVE CONTROL
FEEDFORWARD PRODUCTION BASED CONTROL

Applying the Second Method

In this case, the notional input to the system is Cumulative Orders and the Notional Output is Cumulative Completed Production. In this case, derivative control can be attained with a simpler feedback filter. We therefore choose a six month moving average filter for both the feedforward and feedback filters. In fact, it is possible with this method to obtain satisfactory response for a RAMP input in Order Rate without a feedforward element to the controller. The equations for Production Start Rate with this method are given in Figure 9.

```
NOTE CONTROLLER EQUATIONS SECOND METHOD
A  ERROR.K=DST.K-ST.K
D  ERROR=(UNITS) STOCK ERROR
A  MAER.K=MOVE(ERROR.K,M)
D  MAER=(UNITS) MOVING AVERAGE STOCK ERROR
L  IMAER.K=IMAER.J+DT*MAER.J
N  IMAER=0
D  IMAER=(UNITS*YR) INTEGRAL MOVING AVERAGE STOCK ERROR
A  SOR.K=SMOOTH(OR.KL,S)
D  SOR=(UNITS/YR) SMOOTHED ORDER RATE
A  MSOR.K=MOVE(SOR.K,M)
D  MSOR=(UNITS/YR) SMOOTHED MOVING AVERAGE ORDER RATE
A  MCR.K=MOVE(PCR.KL,M)
D  MCR=(UNITS/YR) MOVING AVERAGE PRODUCTION COMPLETION RATE

R  PSR.KL=FF*MOR.K+ZK2*(IMAER.K/12+MAER.K
X  +D2*(G*MOR.K+H*MSOR.K-MCR.K))
D  PSR=(UNITS/YR) PRODUCTION START RATE
N  G=1+K/S
N  H=1-G
```

FIGURE 9

Note: The variable MOR and the constants K,K,S have the same definitions as in Figure 6.

To define a controller, it is now only necessary to obtain the values of the constants ZK2, I2 and D2. For the second method, this is easily done by running a simple program in batch mode to assess the characteristics of the system. Basically, the program required is that of Figure 4 with the following modifications:

a) OR is set equal to zero.

b) Adding the additional equations

```
NOTE TEST INPUT FOR SECOND METHOD
R  PSR.KL=STEP(1,0)
L  CP.K=CP.J+DT*PCR.JK
N  CP=0
D  CP=(UNITS)  CUMULATIVE PRODUCTION
```

FIGURE 10

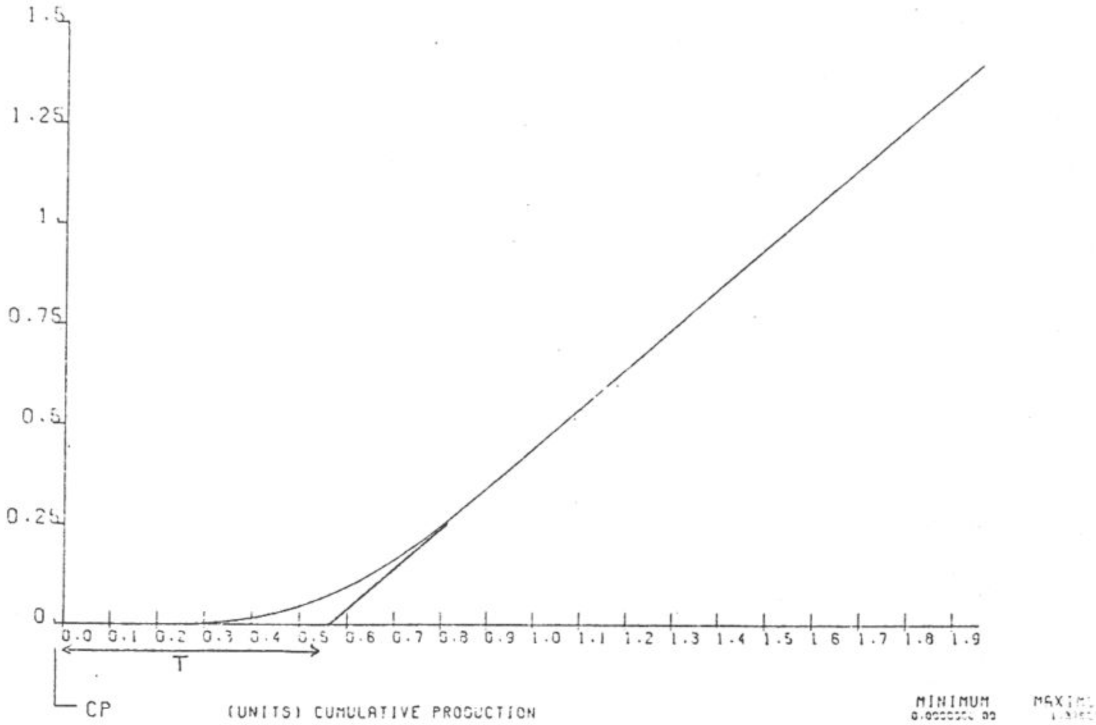
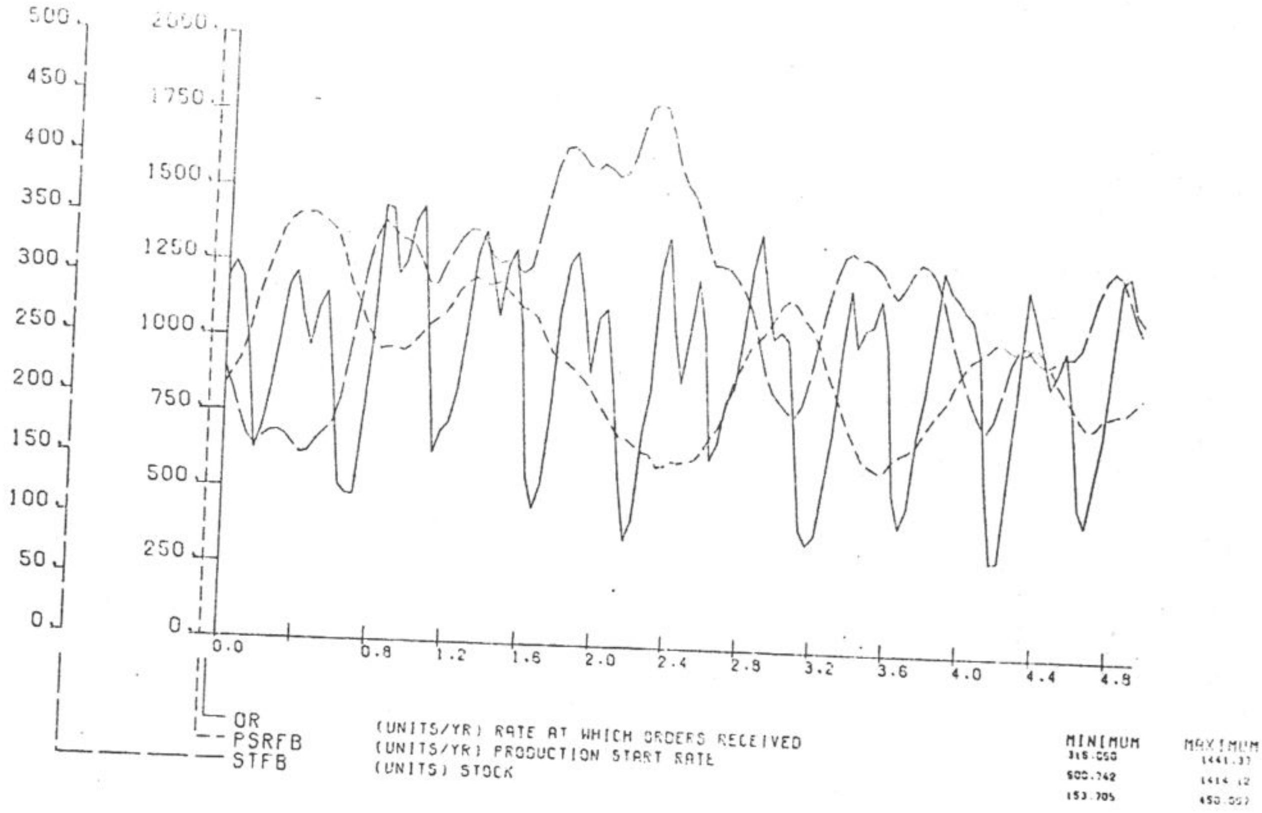


FIGURE 11

STEP TEST
CUMULATIVE PRODUCTION BASED CONTROL



FEEDFORWARD+ZIEGLER NICHOLLS PID CONTROL
STOCK ERROR BASED FEEDBACK CONTROL

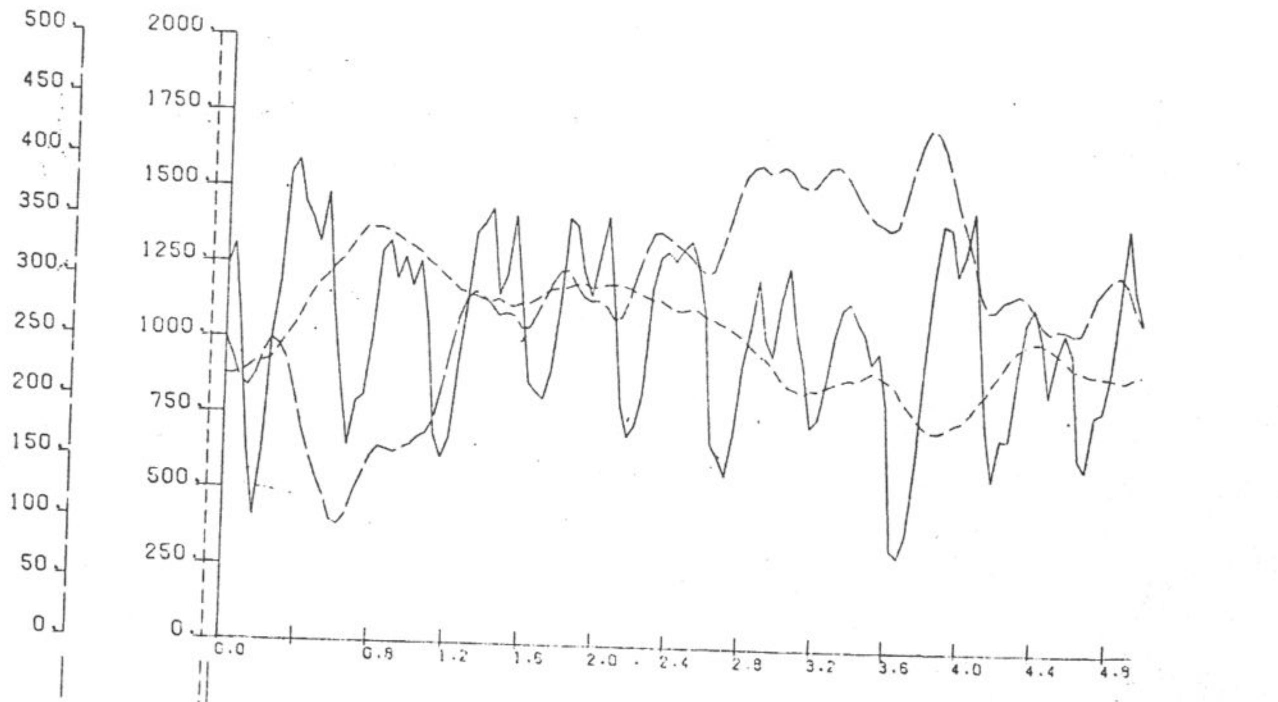


FIGURE 12

ZIEGLER NICHOLLS PID CONTROL
STOCK ERROR BASED FEEDBACK CONTROL

The values of the constants are derived from a PLOT of the variable CP. The results for this system are shown in Figure 11 In terms of the value T shown in the diagram, the necessary values are obtained as follows (Rijnsdorp, 1961):

Proportional Control

$$ZK2 = \frac{1}{T}$$

$$I2 = \infty$$

$$D2 = 0$$

Proportional + Integral Control

$$ZK2 = \frac{0.9}{T}$$

$$I2 = 3.3T$$

$$D2 = 0$$

Proportional + Integral + Derivative Control

$$ZK2 = \frac{1.2}{T}$$

$$I2 = 2T$$

$$D2 = 0.5T$$

Figure 12 shows the performance of the system with Proportional + Integral + derivative control, using the values ZK2 = 2.15, I2 = 1.1, D2 = 0.28 derived from Figure 11 and both with (FF=1) and without (FF=0), a feedforward element.

Conclusions

The ZN methods are quick to apply. Once a choice of filter has been made, the control equations are easily set up, since they take the same basic form, whether moving averages or the more usual SMOOTHs are used. The relevant constants can be quickly determined to give a reasonable basic controller that can then be refined by further experiment. The three term form of the controller is actually more sophisticated than those often used in System Dynamics. Essentially they offer the opportunity of reducing the number of experiments to be done to achieve a good control policy, since the major independent variable to be manipulated is the type of filter and the filtering time. A good guess at an appropriate value for the filtering time can usually be made on the basis of what frequencies in the input that should be as far as possible eliminated from the controlled variable. Thus in the case considered, the strong six month seasonality in Order Rate suggested the use of a 6 month Moving Average as the basic filter.

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