NONLINEARITY IN SYSTEM DYNAMICS MODELS

Part II

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Abstract

This paper classifies nonlinearities in System Dynamics Models into three types. Procedures are indicated to resolve these nonlinearities and transform the models to linear ones.

1. Introduction:

System Dynamics Models are well known for their ability to depict nonlinear relationships. These do not pose any problem for the model to be run and system behaviour generated. However, the analysis of the model becomes very difficult. Therefore, for ease of model understanding, there exists a need to generate a linear model which would approximate to the behaviour of the actual model. Another advantage of a linear model is that one can use linear Control Systems theory for a thorough analysis and design of the model system.

A cursory survey of linearization attempted in system dynamics literature indicates two types of approaches. The first is that by Ratnatunga and Sharp (1) who have proposed linearization and order reduction based upon perturbation techniques. The availability of a low-order linear model makes this approach quite attractive. But the additional computational effort and the exotic nature of the approach makes it a difficult tool to apply. The second approach is based upon local linearization by partial differentiation, and has been adopted by Cuypers (2) and Cuypers and Rademaker (3) in the analysis of World 2 model of Forrester (4). This is a relatively simple approach. Ratnatunga (5) has used this approach to generate the system matrix of an SD model in the new version of DYSMAP. Although an automatic linearization by DYSMAP would greatly facilitate model analysis, it is felt that *model understanding would be poorer without the personal involvement of the model maker.* Therefore, it is argued in this paper that model improvement in the form of *model simplification is first necessary before attempting for linearization, whether done manually or automatically.*

2. Characteristics of Linear and Nonlinear Systems:

A very naive but important aspect of model construction is to recognise the linearities and nonlinearities of the model.

Conceptually, a system is said to be linear if a state variable is related to its rate of change in a linear fashion. Fig.1 depicts a system which is linear.

The rate of change of the level variable in Fig. 1 is (MRR – SR) and not MRR alone. Therefore, a precise mathematical definition of linearity is desirable.

'A system is said to be linear if it obeys the principle of superposition.'

Proof:

Let the system with the following state equation be considered

$$\dot{\underline{x}} = A\,\underline{x} + B\underline{u} \qquad \ldots (1)$$

Let at two instances the values of state variables, exogenous input variables and the rate of change of state variables be given respectively by $\underline{x}_1$, $\underline{u}_1$, $\dot{\underline{x}}_1$ and $\underline{x}_2$, $\underline{u}_2$, $\dot{\underline{x}}_2$ such that the following holds

$$\dot{\underline{x}}_1 = A\,\underline{x}_1 + B\underline{u}_1 \qquad \ldots (2)$$

$$\dot{\underline{x}}_2 = A\,\underline{x}_2 + B\underline{u}_2 \qquad \ldots (3)$$

Principles of superposition demands that the rate of change of state variables corresponding to the values of state variables and control variables at

$(\underline{x}_1 + \underline{x}_2)$ and $(\underline{u}_1 + \underline{u}_2)$ respectively be

$$\dot{\underline{x}}_1 + \dot{\underline{x}}_2 = \frac{d}{dt}(\underline{x}_1 + \underline{x}_2).$$

In fact adding the left hand sides and the right hand sides of Eqn. (2) and Eqn. (3) one gets the same result,

viz.

$$\dot{\underline{x}}_1 + \dot{\underline{x}}_2 = A\underline{x}_1 + A\underline{x}_2 + B\underline{u}_1 + B\underline{u}_2$$

or

$$\frac{d}{dt}[\underline{x}_1 + \underline{x}_2] = A[\underline{x}_1 + \underline{x}_2] + B[\underline{u}_1 + \underline{u}_2] \qquad \ldots\ldots (4)$$

Thus the principle of superposition holds and the system modelled by Eqn. (1) is linear.

The above proof holds true even when the elements of A and B are time-varying, or are functions of exogenously defined input variables, because superposition applies, although as Gibson (6) points out the response of a given system to a given input very definitely depends upon the time at which the input is initially applied.

Unfortunately, there is no accepted definition of nonlinearity. In fact Gibson (6) is of the opinion that *nonlinear systems are simply all those systems that are not linear.* Therefore, although the class of systems whose parameters are state-dependent (i.e. if the elements of A are functions of state variables) are nonlinear, they do not provide sufficient conditions for nonlinearity.

Although the mathematical definition of linearity is precise, the conceptual definition given earlier can very well serve as

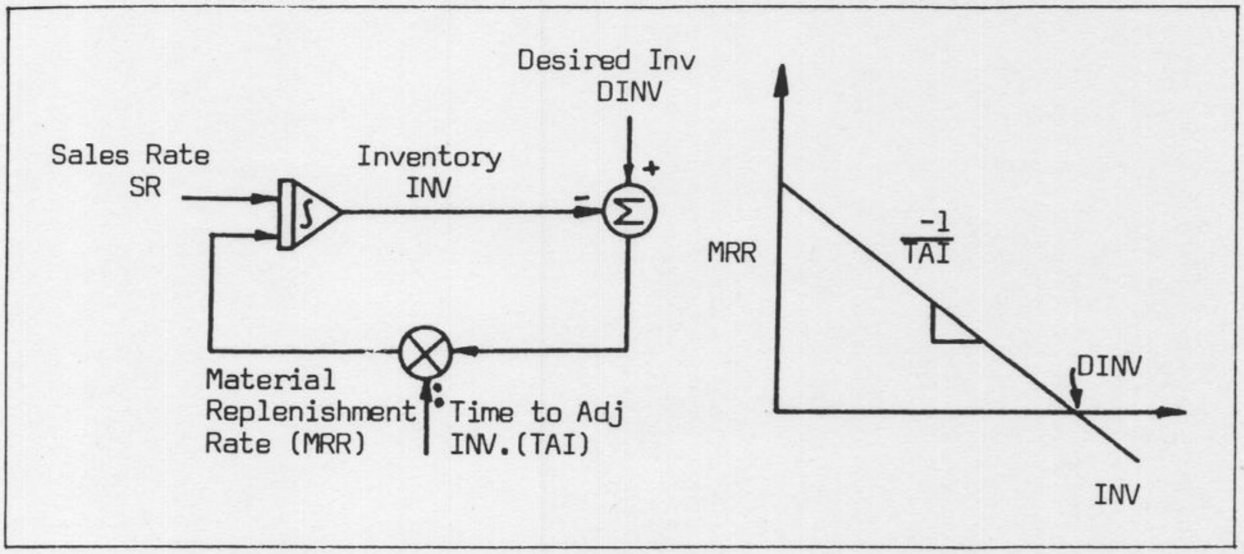

Figure 1: A linear system

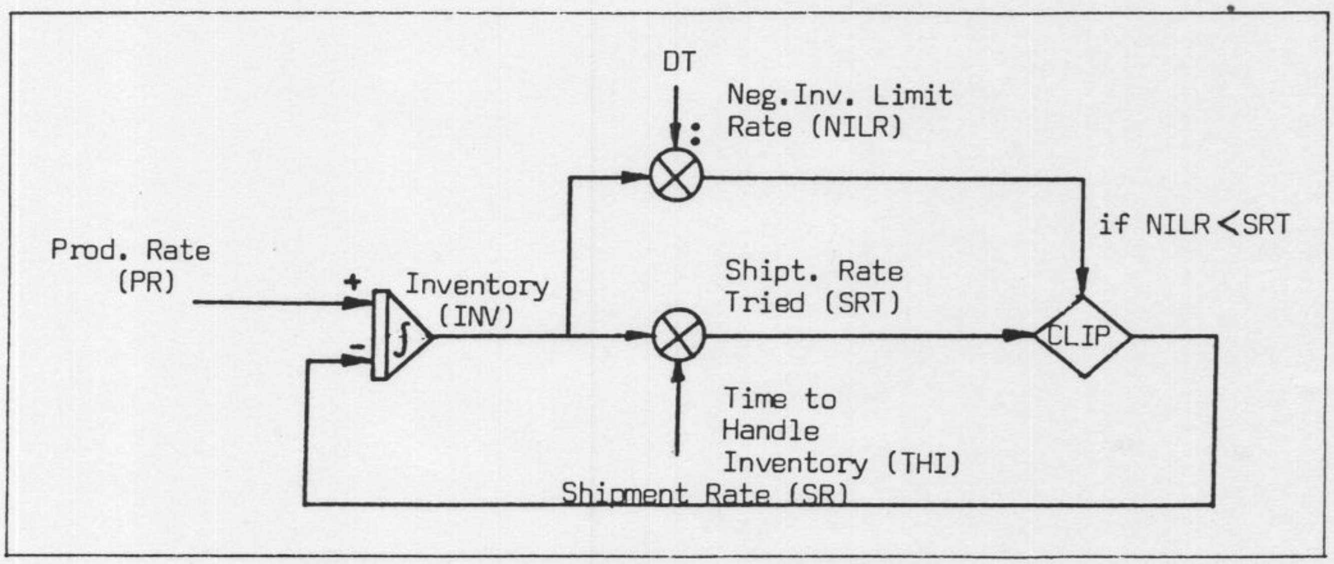

Figure 2: (Mis)Use of CLIP to constrain Inventory from becoming Negative

a working guide to identify linearity or otherwise. In fact it is contended here that the analogue scheme of representation as suggested in an earlier paper (Mohapatra (7)) is capable of indicating nonlinearities, when present.

3. Broad Classifications of Nonlinearities:

It is possible to express a linear SD model in the differential equation form and the elements of the plant matrix can be derived from the constants associated with ⊗ – operators along the path between two state variables (Mohapatra (7)). One many encounter two types of difficulties in the process of computing path transmittance. These are (i) presence of functions like MAX, MIN, CLIP, TABLE and TABHL etc., and (ii) association of ⊗ – operators with other endogenous variables rather than constants. These are indicative of presence of nonlinearities.

Nonlinearities can conveniently be classified into three types:

a) nonlinearities due to use of limiting functions such as MAX, MIN or CLIP.

b) nonlinearities due to the presence of table functions along a path.

c) nonlinearities due to the association of some of the ⊗ – operators along a path with variables of other paths.

All possible combinations of these three types of nonlinearities may actually occur in an SD model.

4. Nonlinearities due to use of Limiting Functions:

This type of nonlinearities usually occurs when the model maker wishes to constrain a variable so that it does not go beyond a limit value. MAX, MIN and CLIP functions usually fall in this category. These functions usually create sharp discontinuities *which cause nonlinearity only at the points when these functions become active.*

Two ways may be proposed to tackle the situation;

a) The functions may just be deleted from the final model because it may be redundant. The redundancy might be due to an unnecessary inclusion in the first place or because it might no longer be required since it was first included to overcome some unwanted model behaviour in the initial phase of model building but eventually an intelligent fine tuning of the model makes it redundant.

b) The limiting functions may be substituted by table functions. Though the latter is a nonlinearity, it is relatively easy to linearize it and would be discussed later. In many aggregative situations table functions may be more realistic than limiting functions.

Certain examples, drawn from students' theses, are given below for elucidating the points.

Example I:

Fig. 2 illustrates an example of misuse of CLIP function. In this particular example the use of CLIP function is redundant since SRT – INV – SRT is a first-order negative loop whose property is asymptotic growth or decay meaning that INV is guaranteed to be positive. Therefore use of CLIP is definitely unwise.

Example II:

Fig.3 is a model with two MAX functions. Fig. 4 & 5 are its simplifications.

Fig. 5 is correct only when printouts of CFLOW for various runs would show that it is greater than or equal to zero for all cases.

Example III:

Fig. 6 depicts a situation in which the cash discrepancy is met immediately by credit from bank provided current ratio is low, but is paid back as soon as possible due to high interest rate, subject to availability of funds. The liberal use of DT, MAX –, and CLIP – functions may not be worth the trouble because this may form a very trivial part of the original model. Also, immediately borrowing the positive cash discrepancy may defeat the very purpose of minimum cash level. The sharp discontinuity at current ratio = 1 may not also be a practical proposition. Similarly, repaying after DT – time does not seem to be a very sound practice.

Fig. 7 shows how TABHL functions may be used instead of MAX and/or CLIP functions. The remaining MAX function may also similarly be substituted by a TABHL function as shown in Fig. 8.

The wisdom for using so many TABHL functions may of course be challenged. But at least the latter does not provide sharp discontinuities.

Example IV:

Fig. 9 shows a redundant use of MAX function and it is also possible to eliminate the MIN function by means of a table function which discourages shipment rate if clinker stock falls short of a desired value.

It may be mentioned that though it is not possible to get rid of such limiting functions in all cases, in most of the cases, especially in aggregative situations, elimination of such functions may both be feasible and desirable.

5. Nonlinearity due to Nonlinear Relationships (Defined through Table Functions) between Consecutive Variables:

This class of nonlinearity is relatively easy to resolve. Usually a table function reflects the assumptions of the analyst regarding causal relationships between two variables. Therefore, any variation from the set relationships should not cause great concern, particularly when the operating zone is approximately linear.

The standard practice in SD modelling is to consider very high/low conceivably feasible values for the causal variable and draw the relationships. But in most cases the operating zone is quite narrow. Therefore, in such cases an assumption of linearity in the active operating zone may be very appropriate. However, in cases where sharp changes in slopes occur, only piecewise linearity would be a valid assumption. The actual operating zone has, of course, to be ascertained by running and rerunning the model.

Some examples are given below to clarify the points.

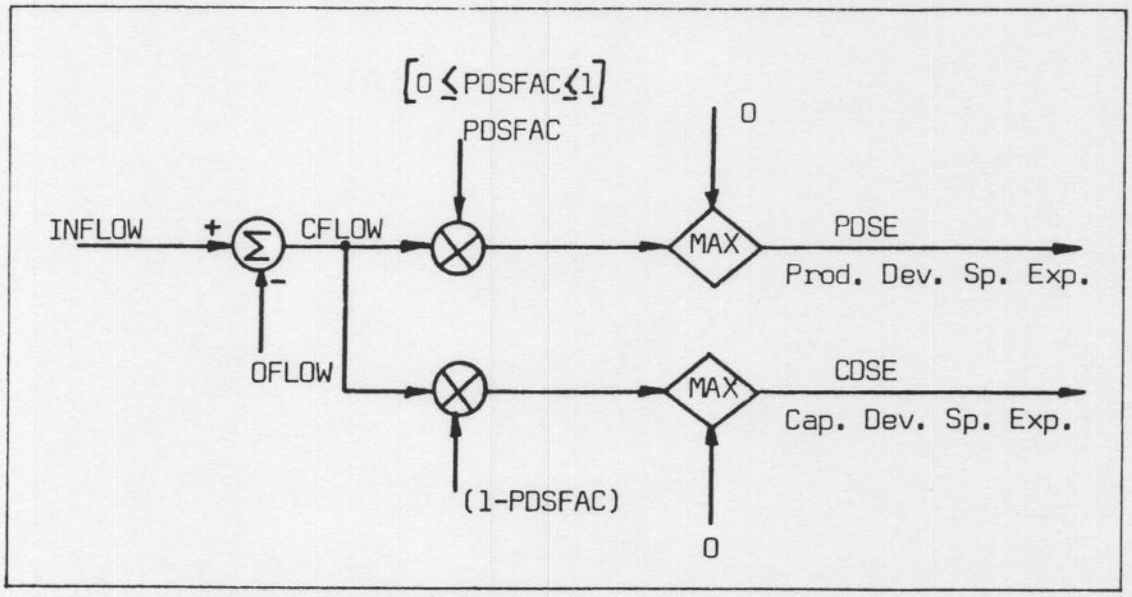

Figure 3: Use of MAX Functions

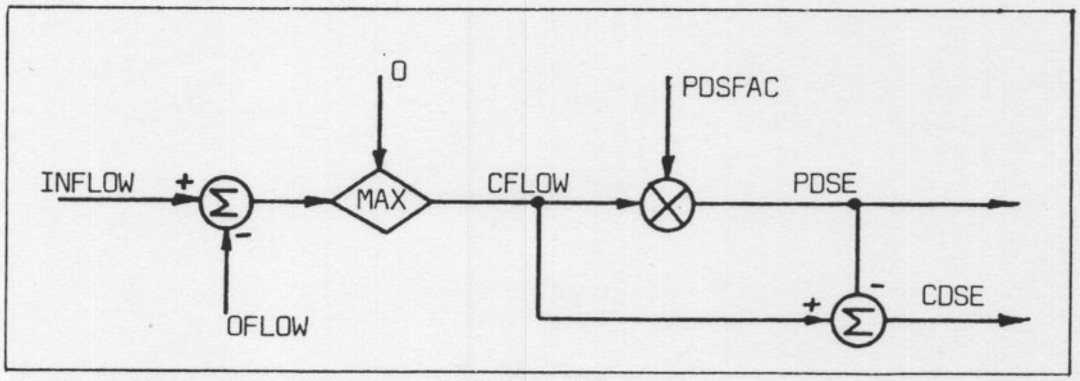

Figure 4: Reducing number of MAX Functions

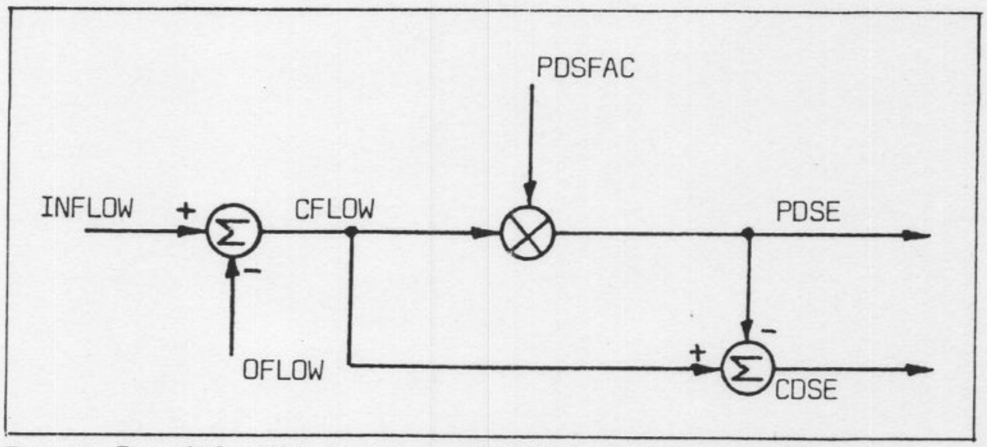

Figure 5: Possible Elimination of MAX Function provided

CFLOW ≥ 0

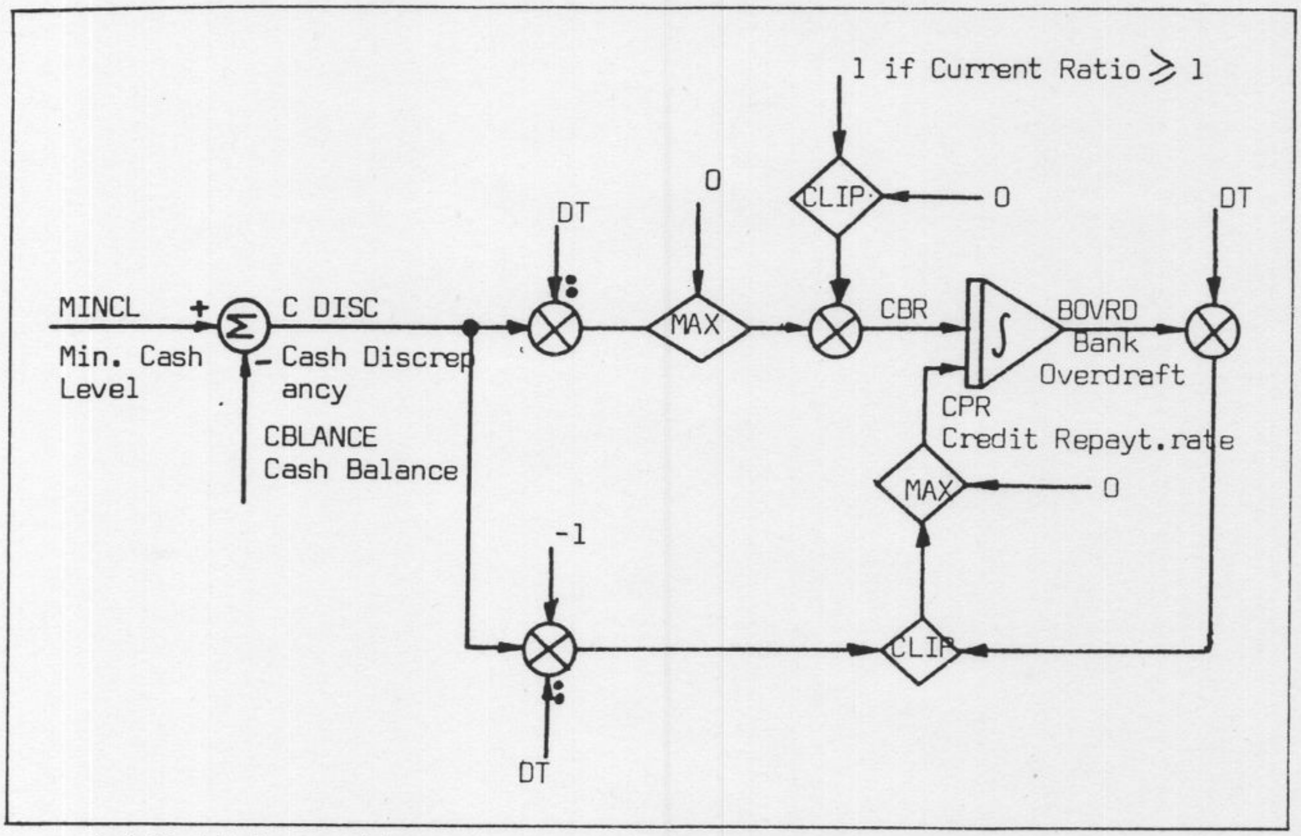

Figure 6: A Model Apparently made Complex with MAX and CLIP Functions

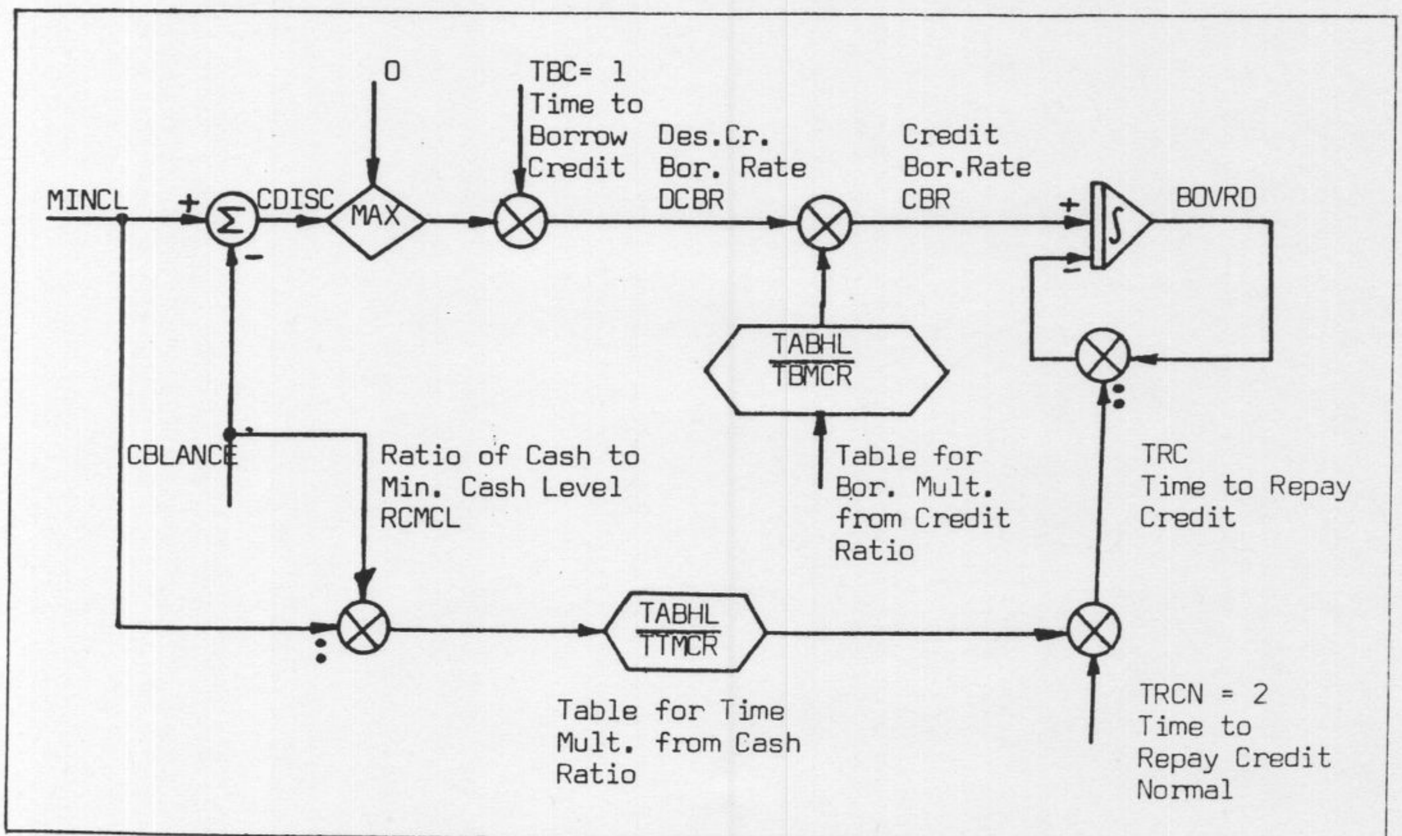

Figure 7: Alternative Form of the Model with the Use of TABHL functions

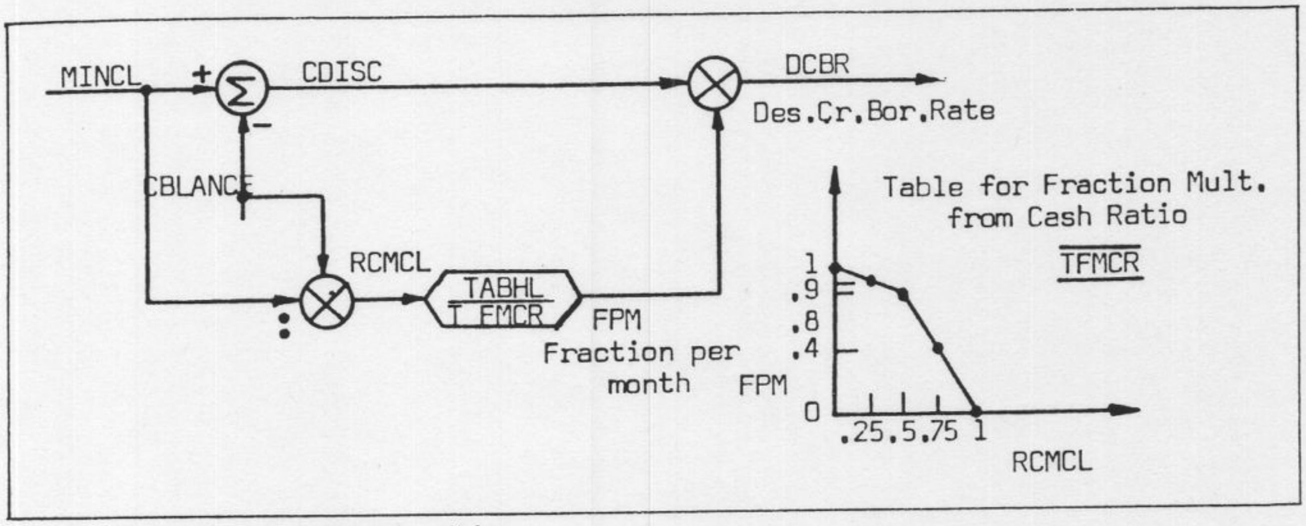

Figure 8: Eliminating MAX function

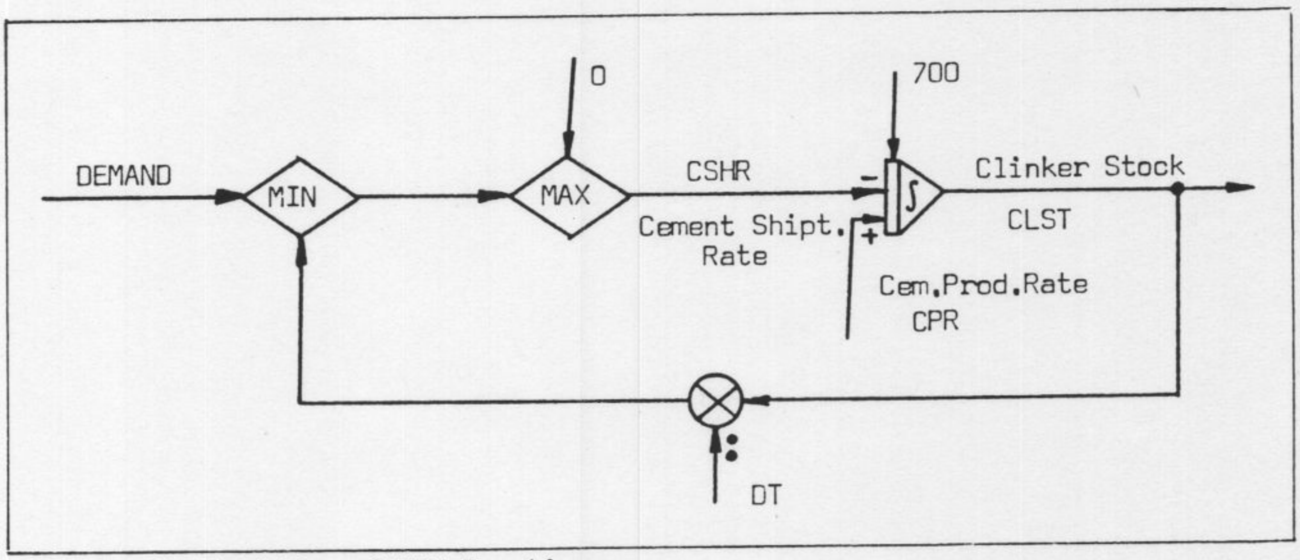

Figure 9: Redundant Use of MAX Function

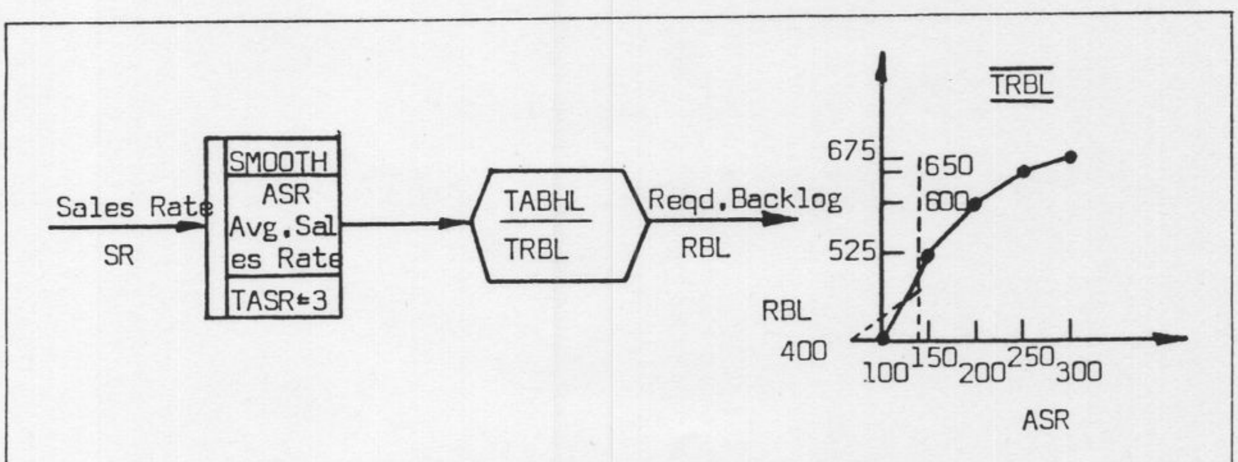

Figure 10: TABHL - Function Relating ASR with RBL

Example A:

Sales rate, defined in Fig. 10, is exogenous. Hence, if it is assumed to undergo a 40% step increase, then naturally the maximum value of ASR can only be 140, if the original steady state value is 100. Therefore, one may approximate the above drawn model by that shown in Fig. 11 where M equals the slope and C is the intercept of the straight line, approximating the active part of the table function. If, however, the sales rate is contaminated with a sinusoidal fluctuation with amplitude equal to 40% of the steady state value, then the minimum value of ASR will be lower than 100. In such a case three procedures may be followed.

(i) The range of ASR in the Table function may be increased to include this value of ASR because after all the present TABHL function is only a hypothesis and is not exact. For example, a value of 350 for RBL may be taken for ASR equal to 50 and be incorporated in the TABHL function, and thus a suitable straight line may be fitted. Such inclusion of additional points may be defended on the grounds that, after all, table functions are based upon conjectures and that the limit values of the causal variable should, after all, be included.

(ii) A straight line may still be fitted (probably with a gross simplification) if the achieved limit value is close to the defined limit value. For example, when minimum value of ASR is less than 100, but close to it then the dotted line shown in Fig. 10 may be assumed to fit the function.

(iii) The last resort is, of course, to have piecewise linearity fits.

For example,

$$\text{RBLK} = \begin{cases} (\text{ASR.K})(\text{M}) + \text{C} & \text{if ASR.K} \geq 100 \\ 400 & \text{if ASR.K} < 100 \end{cases}$$

Example B:

Fig. 12 illustrates an example where the analyst presupposes an apparently non-linear form of causal relationship between PDSFAC and ADCR. Therefore, a quick glance reflects the view that the function can be broken down conveniently to three piecewise linear forms

a) ADCR $\leqslant$ 0.9 b) 0.9 $<$ ADCR $\leqslant$ 1.2

c) ADCR $>$ 1.2

However, all the runs for the model of which the above diagram is only a part, reveals the fact that all the values of ADCR are between 1 and 1.3496. Hence, only two piecewise linearities need be defined. Equation defining PDSFAC may be written as

A PDSFAC.K = MAX (PDSFCT.K,O)

A PDSFCT.K = M * ADCR.K + C . . . (5)

NOTE M IS SLOPE AND C IS INTERCEPT

However, only one straight line may also be fitted and the results may also be tested. If the results are acceptable then such a fit may also be accepted.

6. **Nonlinearities due to Multiplying Effect of Multiple Causal Variables upon another Variable**:

This type of nonlinearity is identified as soon as one confronts a ⊗ – parameter along a path being associated with a variable of another path, rather than a parameter. Such cases are quite common in SD and are quite hard to tackle. Two approaches are discussed below for linearization. The first takes advantage of the particular structure of nonlinearity and the second, which is based on partial differentiation, assumes that although a sub-system is nonlinear, small perturbations of the variables are related linearily.

6.1 **Linearization made feasible by Particular Model Structure**

Certain types of model structure help resolving nonlinearities quite easily. Certain examples follow.

Example a:

Figure 13 depicts a case in which RCOL is a function of AOL and PPDSE, the latter being a model variable. In computing the transmittance of the path between AOL and OL, one comes across one ⊗ –symbol which is not associated with a parameter, but which is a function of other variable such as PPDSE. Therefore, this implies a nonlinearity situation.

Various runs of the original model of which Fig. 13 is a part, confirm that the range of X is 1.0912 $\leqslant$ X $\leqslant$ 1.7560. A straight line when fitted to TPDSE in this zone band of X provides one with slope = M = 0.86, and intercept = C = –0.86. Therefore, one may write the following:

R RCOL.KL = (AOL.K) (X.K*M + C) . . . (6)

But X.K = PPDSE.K/AOL.K . . .(7)

Using Eqn. (7) in (6), one obtains the following:

R RCOL.KL = (M) (PPDSE.K) + (C) (AOL.K)

C M = 0.86 . . . (8)

C C = -0.86

The corresponding visual representation is shown in Fig. 14. C is taken as 0.86 and not–0.86 whereas the arrow inputted to ⊗ – operator has now a – sign, so that the actual direction of causation is retained.

Example b:

Fig. 15 represents another case where a seemingly complex nonlinearity can be resolved fairly easily. Various computer runs indicate the range of DCR as 1 $\leqslant$ DCR $\leqslant$ 1.3580. Then the table TDCR is defined in a different way and is shown in Fig. 16, where the x-axis is taken as the reciprocal of DCR. The operating range of $\frac{1}{\text{DCR}}$ is now $0.74 \leqslant \frac{1}{\text{DCR}} \leqslant 1$ where $0.74 = \frac{1}{1.3580}$. A straight line which fits this operating zone of 1/DCR has slope M = 72.7 and intercept C = -6. Therefore, the equation for OV is given by the following :

A OV.K = (CAPAC.K/OL.K)(M) + C . . . (9)

Therefore, from Fig.15 the following equation may be written:

INFLOW.K = (OL.K) (OV.K)

$$= (\text{OL.K}) \left[\left(\frac{\text{CAPAC.K}}{\text{OL.K}} * \text{m} \right) + \text{C} \right]$$

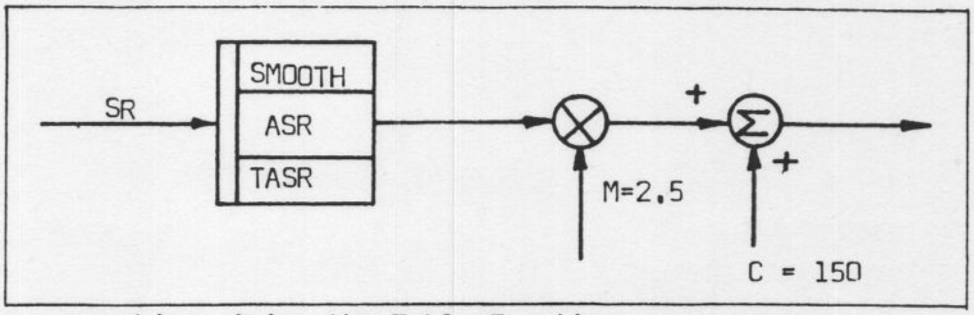

Figure 11: Linearizing the Table Function

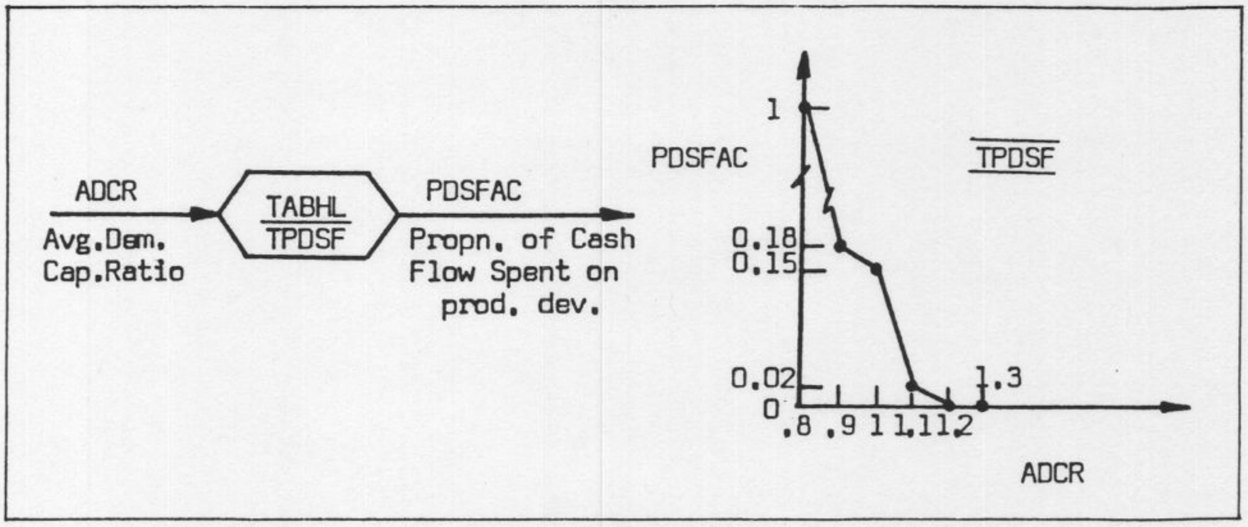

Figure 12: Apparrently very Nonlinear Table Function

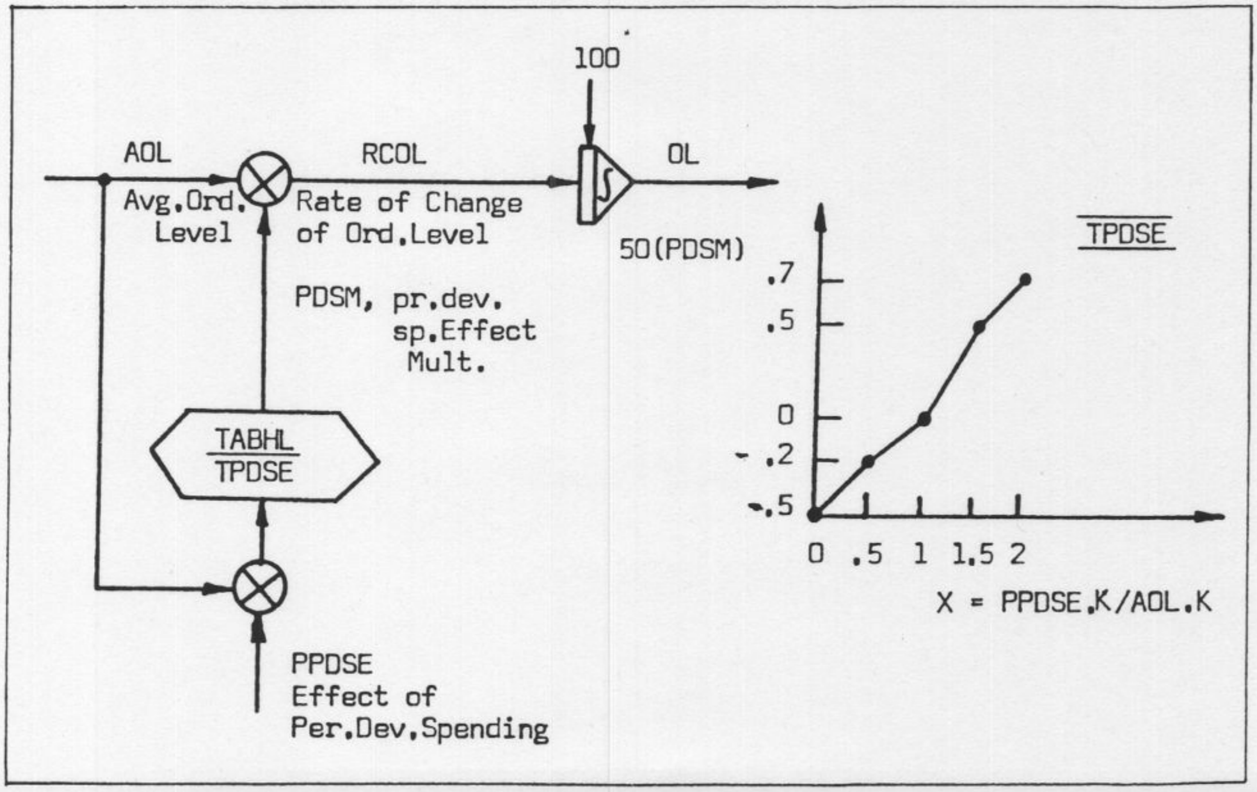

Figure 13: Parameters being Function of a Dependent Variable

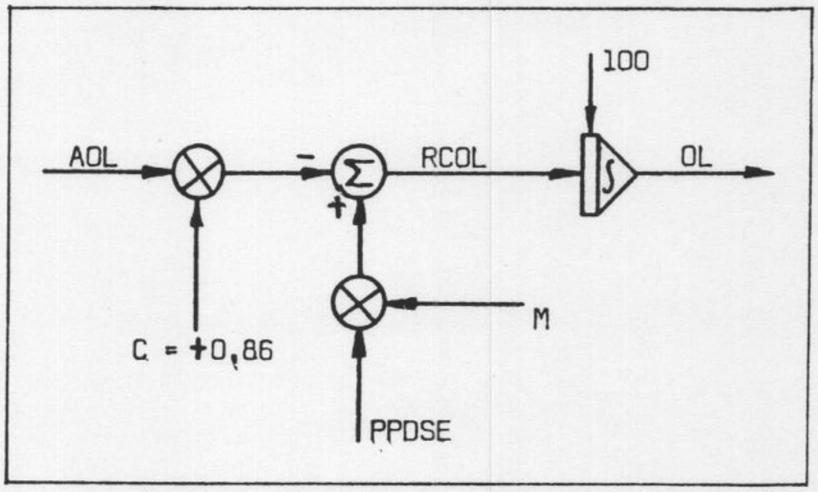

Figure 14: An Equivalent Linear Representation of Fig.13

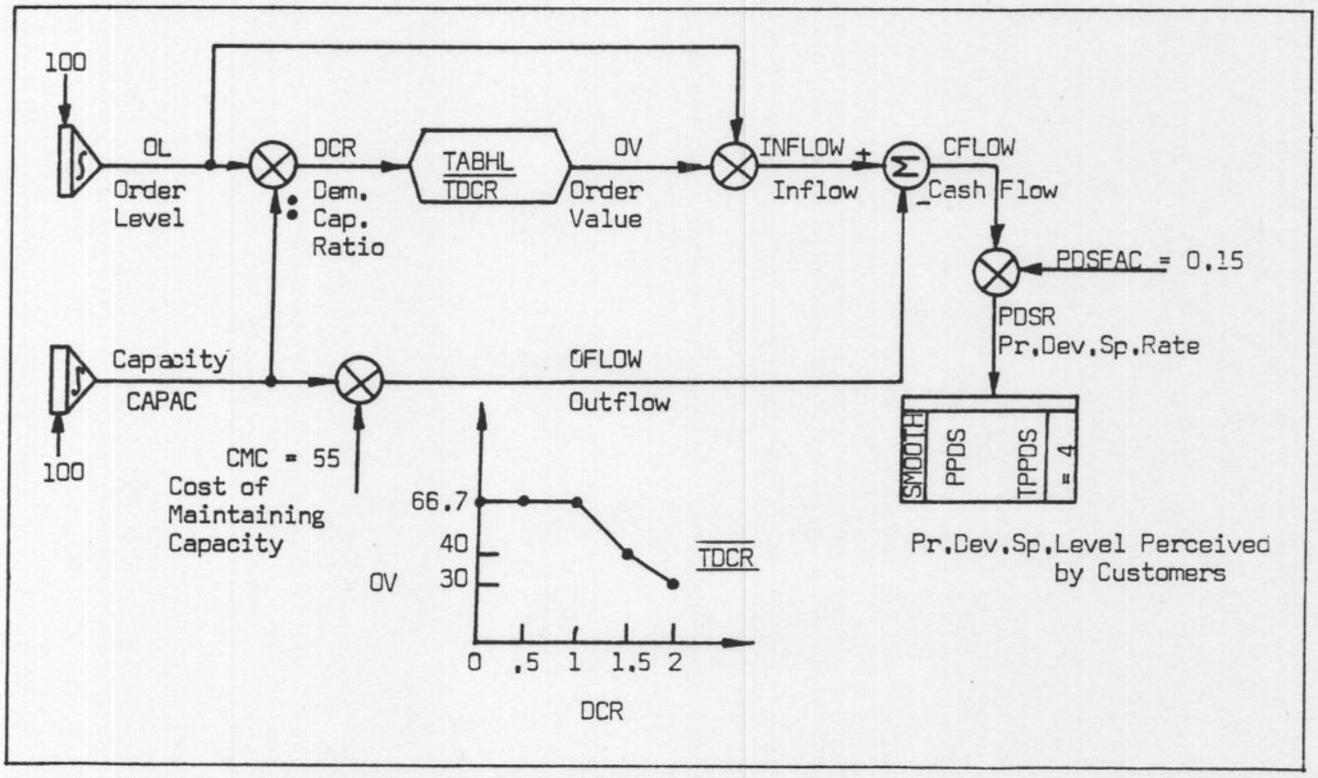

Figure 15: Nonlinearity in the Path to PPDS from OL and CAPAC

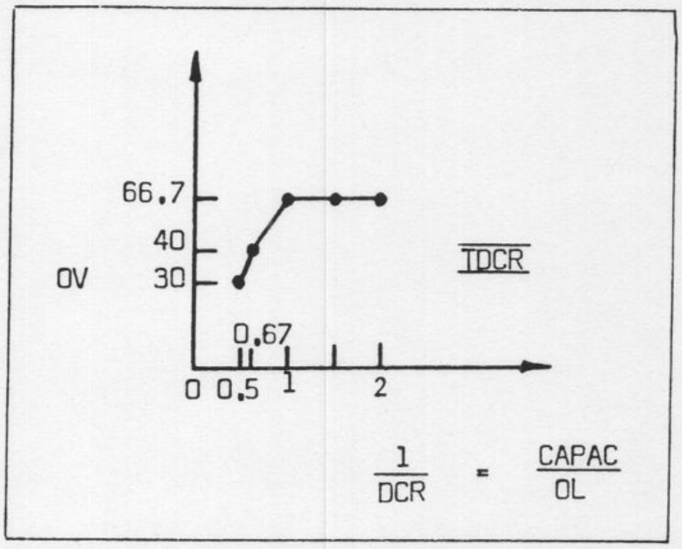

Figure 16: A Different Representation of Table TDCR

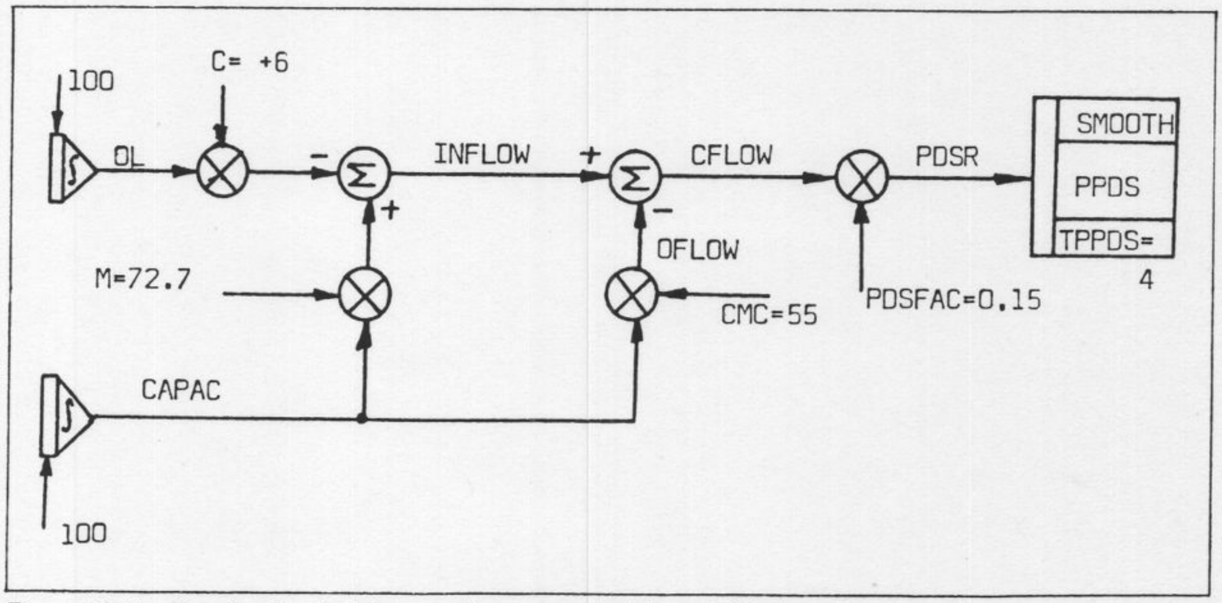

Figure 17: An Equivalent Linear Representation of Fig. 15

or,

A INFLOW.K = (CAPAC.K) (M) + (OL.K) (C)

C M = 72.7 . . . (10)

C C = -6

The equivalent representation for Fig. 15 is then given by Fig. 17.

It may be noticed in Fig. 17 that C = 6, and not -6, whereas ⊗- operator is now having an arrow with – sign. This way of representation keeps intact the direction of causation and, therefore, the ultimate polarity of loops.

As is said earlier, this way of linearizing is feasible only if the model structure permits it and this can never be generalized. In the next section a general method of linearization is discussed.

6.2 **Local Linearization with First-Order Accuracy**:

The earlier approaches should first be used wherever possible to free the paths between state variables from containing ⊗ – operators associated with variables and not parameters. As long as ⊗ – operators are associated with variables, non-linearity persists and the usual approach for the computation of path transmittance fails. In such a situation it is possible to approximate such that the small perturbations from the reference points may be assumed to be linearly related although the actual relationships among the variables are nonlinear.

It may be shown that if the ith state variable is related with other state variables by the following state differential equation:

$$\dot{x}_i(t) = f_i\,(x_1, \ldots, x_n) \quad \forall i = 1, \ldots, n \qquad \ldots(11)$$

then a small deviation from the reference trajectory is given approximately by

$$\delta\dot{x}_i = \frac{\partial f_i}{\partial x_1}\,\delta x_1 + \ldots\ldots + \frac{\partial f_i}{\partial x_n}\,\delta x_n \quad \forall i = 1, \ldots, n \qquad \ldots(12)$$

where $\frac{\partial f_i}{\partial x_j}$ indicates the partial derivatives w.r.t. the jth state variable, x_j; and δx_j is the small deviation from the nominal value of x_j.

Therefore, the Eqn. (12) may be written as

$$\delta\underline{\dot{x}}\,(t) = A\,\delta\underline{x}\,(t) \qquad \ldots(13)$$

where, A is the Jacobian Matrix of partial derivatives given by

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \ldots\ldots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_n}{\partial x_1} & \ldots\ldots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

The partial derivatives are to be computed at some reference values and the assumption of continuity is quite evident. This is the reason why the sharp discontinuities like MAX, MIN or CLIP functions are first to be converted to continuities (as discussed in § 4). This type of linearization provides accuracy upto first-order. This is illustrated by the following examples.

Example 1:

Paths to ADCR from OL and CAPAC in Fig. 18 pass through ⊗ – operators containing variables rather than parameters, thereby presenting cases of nonlinearities which can only be resolved by local linearization. From Fig. 18 one obtains the following:

$$\frac{d}{dt}\,ADCR(t) = \frac{1}{TADCR}\,\left[DCR(t) - ADCR(t)\right]$$

$$= \frac{OL(t)}{CAPAC(t)} \times \frac{1}{TADCR} - \frac{ADCR(t)}{TADCR} \qquad \ldots(14)$$

The small perturbation is then given by

$$\partial\left[\frac{d}{dt}\,ADCR(t)\right] = \frac{1}{CAPAC(t)} \times \frac{1}{TADCR} \times \partial\left[OL(t)\right]$$

$$- \frac{OL(t)}{CAPAC^2(t)} \times \frac{1}{TADCR} \times \partial\left[CAPAC(t)\right]$$

$$- \frac{1}{TADCR}\;\partial\left[ADCR(t)\right] \qquad \ldots(15)$$

Eqn. (12) may be written in the following form:

$$\frac{d}{dt}\left[\partial\,ADCR(t)\right] = \begin{bmatrix} \frac{1}{(100)(4)} & \frac{-100}{(100)^2(4)} & \frac{-1}{4} \end{bmatrix} \begin{bmatrix} \partial OL(t) \\ \partial CAPAC(t) \\ \partial ADCR(t) \end{bmatrix} \qquad \ldots(16)$$

It may be noticed here that the initial values of OL and CAPAC have been used in the computation of the partial derivatives. Eqn. (15) indicates that path transmittances between small deviations can be computed from a different type of figures, as shown in Fig. 19 and 20.

Example 2:

Suppose that the table TPDSF can be linearized such that

A PDSFAC.K = (ADCR.K) (M3) + C3 . . . (17)

then, since ⊗ – operator in Fig. 21 is not associated with a

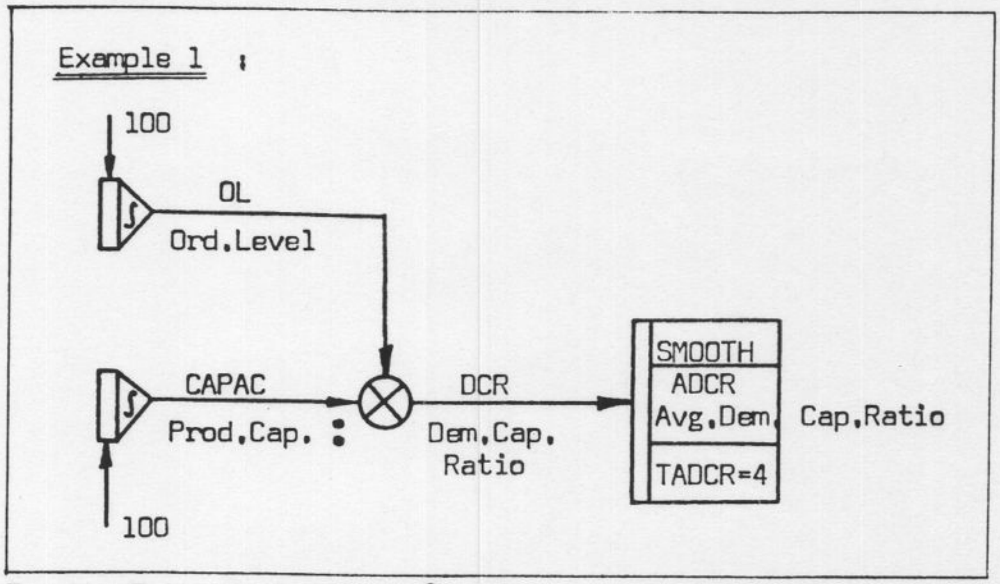

Figure 18: Paths Containing ⊗- Operators Association with Variables

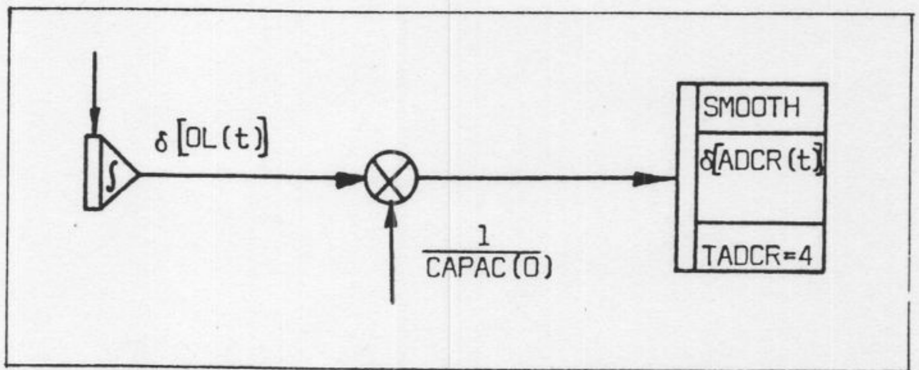

Figure 19: Path between δ[OL(t)] and δ [ADCR(t)]

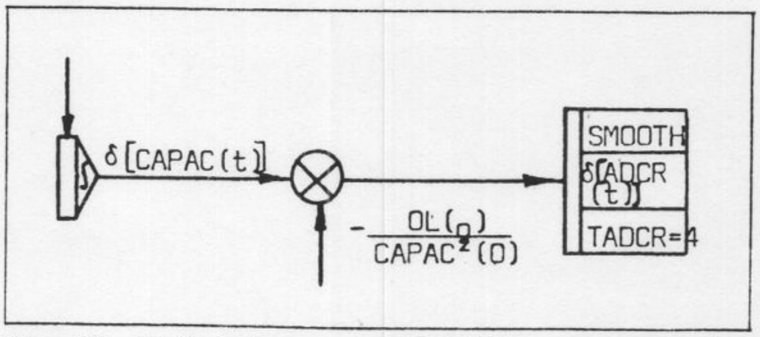

Figure 20: Path between δ CAPAC(t) and δ ADCR(t)

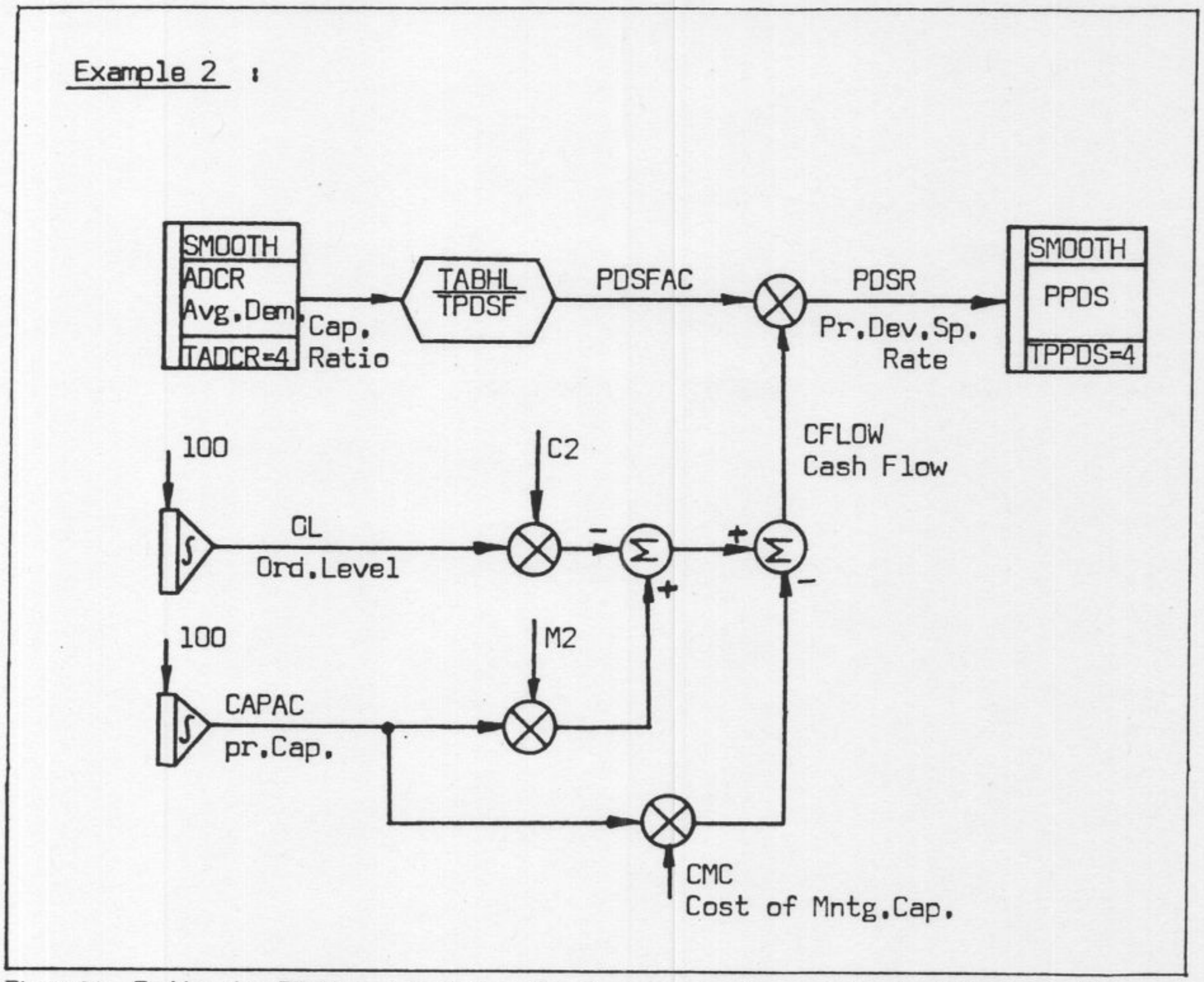

Figure 21: Paths to PPDS containing ⊗- Operators Associated with Variables

dividing variable the following equation may be directly written from Fig. 21.

$$\frac{d}{dt}\left[\delta PPDS(t)\right] = \left[a_1\ a_2\ a_3\ a_4\right] \begin{bmatrix} \delta CAPAC(t) \\ \delta OL(t) \\ \delta ADCR(t) \\ \delta PPDS(t) \end{bmatrix}$$

where,

a_1 = (M2 – CMC) * [(M3) * ADCR (O) + C3] /TPPDS

a_2 = – C2 * [M3 * ADCR (O) + C3] /TPPDS

a_3 = M3 * [(M2-CMC) * CAPAC (O) – C2*OL(O)] /TPPDS

a_4 = –1/TPPDS

It may be noted that

M3 * ADCR (O) + C3 = PDSFAC (O)

Thus from the above two examples it is understood that as long as there is no variable along a path which is used to divide another variable, one can use the same method of computation of path transmittance, but now it is to be defined between the first order deviations of the state variables. However, if a variable is dividing along a path (like that in Fig. 18 defined between CAPAC and ADCR), it may be supposed to be associated with (– 1/variables^2) (as shown in Fig. 20).

7. Chapter 9 Problem (Coyle(8))

Fig.22 depicts the analogue representation of the Chapter 9 Problem by Coyle(8). Apparently it is a highly nonlinear system since it is beset with all types of nonlinearities discussed earlier, viz., (i) sharp discontinuities like two MAX functions, (ii) TABHL functions like TPDSE, TPDSF and TDCR, and (iii) variables are defined by multiplication and division of other variables like six such operations are present.

The table functions, TPDSF, TPDSE and TDCR, are already shown in Fig. 12, 13 and 15 respectively.

Various runs and print outs of the variables indicate the following ranges of some of the key variables:

$$\begin{aligned} &1.0000 \le DCR \le 1.3580 \\ &1.0000 \le ADCR \le 1.3496 \\ &1.0912 \le X = \frac{PPDSE}{AOL} \le 1.7560 \\ &1170 \le CFLOW \end{aligned} \qquad \ldots(19)$$

Eqn. (19) and table function TPDSF indicate that

$$0 \le PDSFAC \le 0.15 \qquad \ldots(20)$$

Eqn. (20) and (19) obviate the need for the two MAX functions (see also Fig. 3, 4 and 5).

It is obvious now that the operating parts of TDCR and TPDSE can be conveniently linearized. TPDSE may be approximate by

PDSM.K = (0.0172) (PPDSE.K/AOL.K) – 0.0172 ...(21)

Defining the x-axis of TDCR as $(\frac{1}{DCR})$ and linearizing the active part of the function, the following may be written:

OV.K = (72.7) (1/DCR.K) – 6 ...(22)

Taking advantage of the special structure of the model (see Fig. 13, 14, 15, 16 and 17) one would be able to get rid of two TABHL functions and four ⊗ – operators.

TPDSF cannot be substituted by one straight line but piecewise linearization of the following type seems feasible.

PDSFAC.K = MAX(0, -1.3*ADCR.K+1.45) ...(23)

This assumes that PDSFAC = 0 when ADCR ⩾ 1.11. Thus while the table function is eliminated, a sharp discontinuity has been created, therefore, the model is not fully linearized.

Running the model with these modifications gives quite similar results, therefore it must be acceptable. Values of PDSFAC, when tabulated, show that it is never zero. Hence the MAX function can also be eliminated. Eqn. (23) may be written as the following:

PDSFAC.K = - 1.3*ADCR.K + 1.45 ...(24)

Fig. 23 shows the simplified analogue representation of Chapter 9 problem. It may be noticed that only one ⊗ – operator associated with variables remains necessitating use of first-order deviations for linearization. If one ignores the smoothing level variable ADCR, then it is also feasible to get rid of this nonlinearity, since one can take advantage of the special structure of the model.

8. Conclusion:

This note makes an attempt to classify various types of nonlinearities encountered usually in SD models. Techniques are discussed which make possible the elimination of nonlinearities in most of the cases.

An outgrowth of the present discussion could be a desire on the part of the analyst to have DYSMAP facilities extended to cover the following cases:

i) to automatically indicate if the limiting functions are inactive (because printouts are not obtained usually after each DT)

and ii) to automatically indicate the active portion of the table functions, and the slopes and intercepts of the best linear fit over this range.

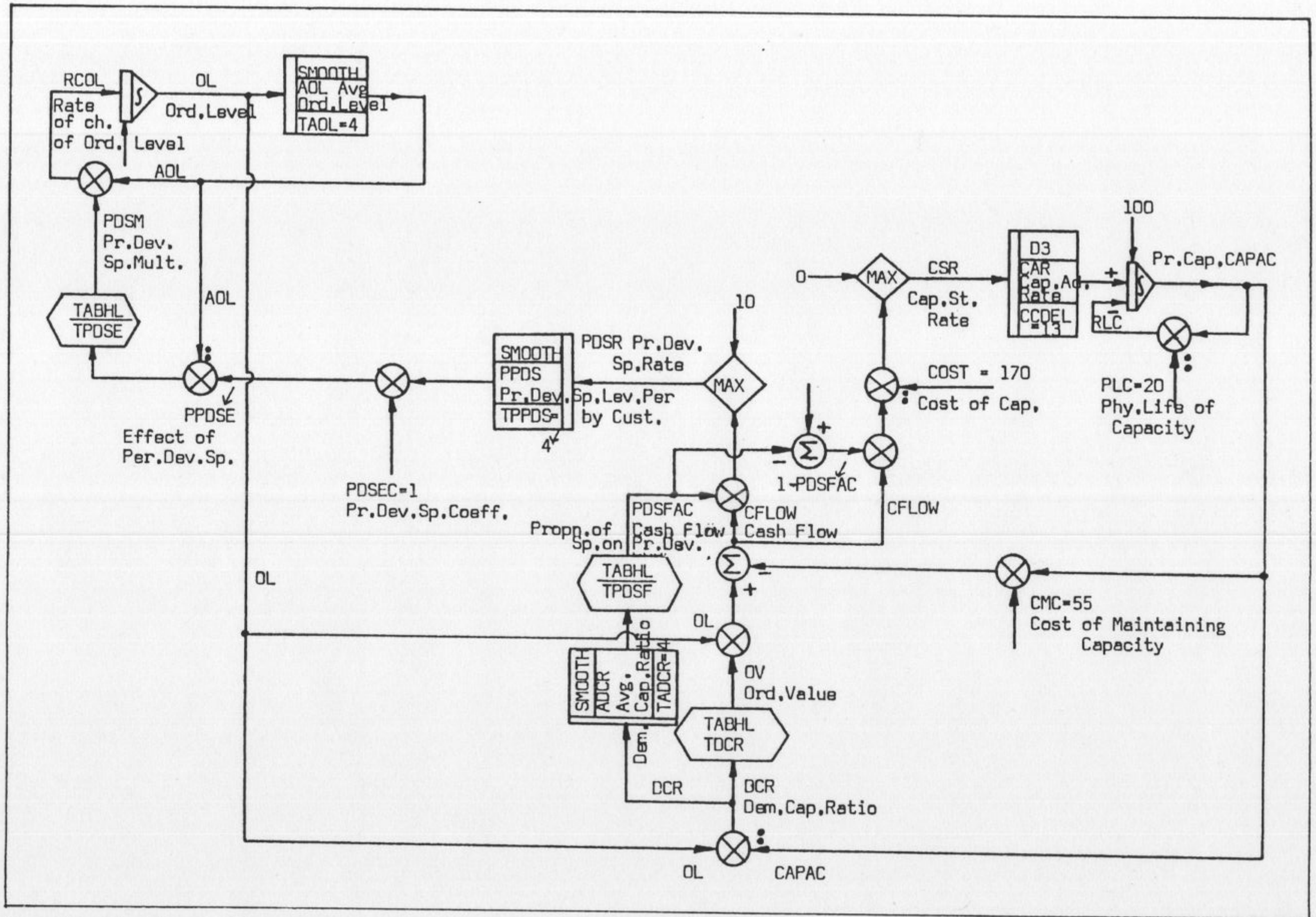

Figure 22: Chapter 9 Problem

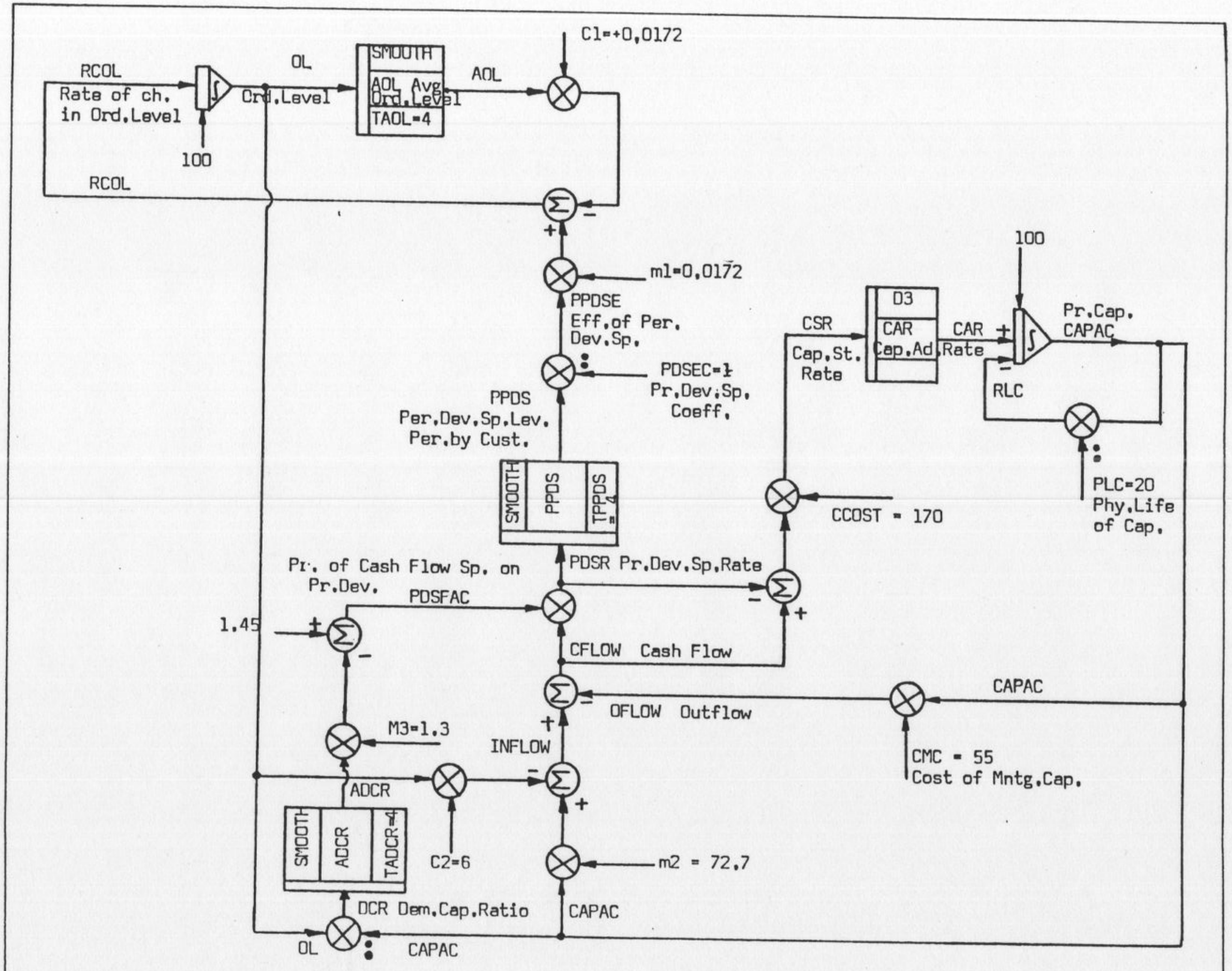

Figure 23: Simplified Ch.9 Problem

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