

NON-LINEAR SYSTEM DYNAMICS: SOME THEORETICAL AND APPLIED ISSUES

Mr. J.R. Beaumont. University of Keele

Mr Beaumont is currently a Lecturer in Geography at the University of Keele, his research interests lie in Dynamical Systems Analysis, Urban Geography of Energy and Public and Private Facility Location Problems

INTRODUCTION

Broadly speaking, the essence of a system is its complex connective structure and its dynamical behaviour. Many conventional analyses, however, consider these fundamental features in a restricted way, and, in so doing, they undermine our ability to fully comprehend the system of interest, and, perhaps more significantly, they reduce our ability to control it in a meaningful way. Given particular interest in the complex interconnections between a system's sub-systems, for example, it is somewhat surprising that techniques, such as conventional multivariate statistical approaches founded on constrained (often linear) functions (which are a special type of relation), are frequently employed to analyse this characteristic. Unfortunately, by their very nature, these taxonomic procedures destroy much of the structure of interest. The basic argument is that theoretical deductions may be as much a result of the specific approach and representation applied as of the system's features. Specifically, attention is drawn to the fact the employment of linear functions is inhibiting.

A related issue is the balance between mathematical tractability and realism. Non-linearities in dynamical systems engender analytical complications, and, therefore, it is often deemed desirable to use linear approximations (see, for example, Mohapatra (1980)). Whilst model simplification is obviously an acceptable aim, it must be appreciated that the presence of non-linearities, representing scale (dis-) economies, realistic spatial interaction, and so on, is often the reason for the complex behaviour of many systems. Thus, by restricting analyses to linear relationships, much interesting complex dynamical behaviour is no longer tenable.

This paper is primarily conceptual, introductory and speculative. Recent developments in bifurcation and catastrophe theory are mentioned, and particular attention is given to the complementary nature of deterministic and stochastic modelling. Following the work of Prigogine and his colleagues in the field of physical chemistry (Nicolis and Prigogine, 1977), it is argued that stochastic fluctuations play a fundamental role in the self-organisation process at bifurcation (or critical) points by determining the specific state to which a system evolves — 'order through fluctuation'; away from criticality, system dynamics are determined by deterministic, differential equations and stochastic fluctuations have little effect. General implications for applied and theoretical work are noted. In section two, it is shown that mathematics is both a tool of analysis and a language of conceptualisation. The discussion of multiple, steady-states found in non-linear systems provides the foundation for a detailed consideration, in section three, of the distinction between 'classical' stability and structural stability. It is argued that stochastic elements

are of paramount importance in the modelling of dynamical systems, because they actually determine the particularly trajectory that a system follows. A number of conceptual issues are raised with regard to the modelling of dynamical systems, and, in the final section, some general implications for man's role in system management are outlined.

(2) MATHEMATICAL REPRESENTATION

A commonly adopted convention is that a system's state is defined by a set of state variables, (\underline{x}), with specified values. These represent individual elements of a system, and their relationship with each other are often given by suitable (linear or non-linear) functions. The magnitude and nature of a function's impact is determined by their associated set of parameters. More specifically, investigation of a dynamical system involves an examination of the way a system evolves over time, and, therefore, a description of system dynamics must consider system states at various times, especially the sequence of states. Formally, this can be represented by a set of simultaneous differential equations,

$$\frac{dx}{dt} = f(\underline{x}, \underline{a}) \quad (1)$$

where \underline{x} is the set of state variables and \underline{a} is the set of parameters. Analyses focus on the effect of changes in parameter values over time on the system's qualitative dynamical behaviour.

Particular attention is given to a system's steady-state, that is, its time independent state. A steady-state is when

$$f(\underline{x}, \underline{a}) = 0 \quad (2)$$

For non-linear systems, determination of such a state is usually analytically intractable, although numerical integration has been facilitated by computer developments. It now becomes clear why to disregard or approximate non-linearities can destroy the recognition of inherent system dynamics. Multiple steady-states are only found in non-linear models, and their existence, indicating the possibility of more than one future, is of fundamental significance in the conceptualisation of system dynamics; linear systems have one unique steady-state.

A simple model which illustrates the existence of multiple steady-states is the logistic growth model, which is widely used in demographic studies and ecosystem management. As figure one illustrates, there is a saturation level (or carrying capacity). For example, for a population x , the rate of growth

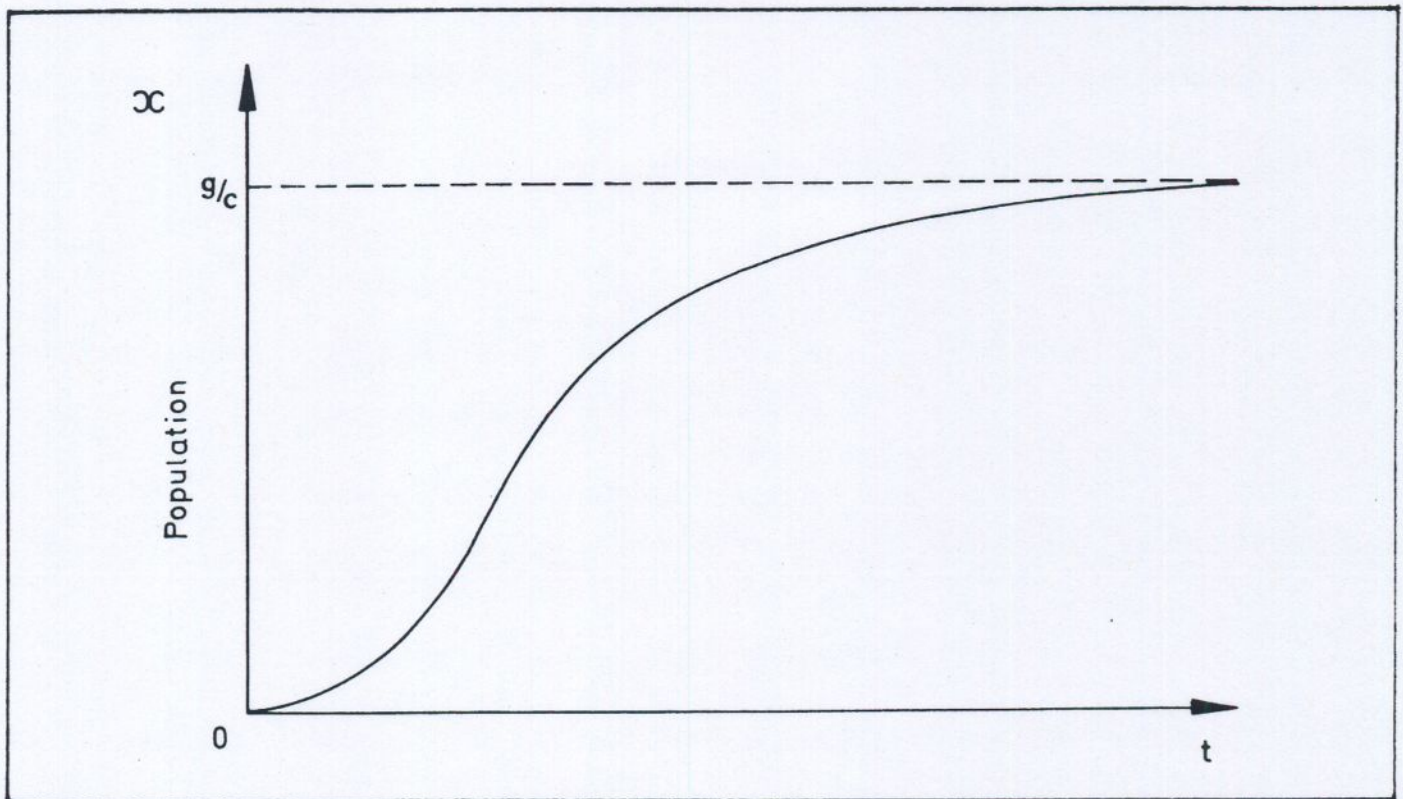


Figure 1. Logistic growth

can be written as a non-linear, differential equation

$$\frac{dx}{dt} = (g - cx)x \quad (3)$$

where g is the intrinsic growth rate (excluding carrying capacity) and c relates to the effect of the carrying capacity; specifically, it is the rate at which the addition of population reduces its intrinsic growth rate. For a steady-state to occur, that is when the population stop growing (or decreasing)

$$gx = cx^2 \quad (4)$$

because

$$\frac{dx}{dt} = 0 \quad (5)$$

This situation arises for two population values, x equal to zero (extinction) and x equal to g/c (the saturation level).

(3) 'CLASSICAL' STABILITY AND STRUCTURAL STABILITY

Given the existence of multiple steady-states, which state occurs when? In the present context, it is important to distinguish between 'classical' stability and structural stability. Classically, stability analysis was concerned with perturbations to a system's initial conditions or its external environment, whereas structural stability is concerned with perturbations to a system's structure itself. In both cases, analysis focuses on whether system behaviour is altered by the dis-

turbances, although 'classical' stability analysis involves an examination of trajectories in the neighbourhood of a steady-state for one particular system and structural stability involves an examination of trajectories for a set of systems which are only slightly different. Attention here focuses on the concept of structural stability, which is of central significance in the recently developed and much discussed catastrophe theory. Thom's (1975) catastrophe theory demonstrates the important relationship between structural stability and morphogenesis (although bifurcation theory, which is not restricted to gradient systems described by a potential function, is likely to have wider applicability). It should be noted that catastrophe theory is restricted in its description, because it only represents switches between steady-states of different dynamical regimes, (although it is possible to incorporate 'slow' equations (Zeeman, 1977)). Such instantaneous changes are termed 'catastrophes' (although no suggestion of disaster is necessarily meant). In contrast, the principle of 'order through fluctuation', which is described in the next section, provides a mechanism by which particular state transitions can be explained (and, in so doing, overcomes a major problem of catastrophe theory). Stochastic fluctuations are the mechanism for change.

A phase space representation of system behaviour is widely used; a trajectory in the phase space plots the change over time of a system from an arbitrary initial state. Stable steady-states are termed 'attractors', and they have trajectories leading to them; if a steady-state is unstable, trajectories leave them and they are termed 'repellers'. The area in phase space which defines the set of possible initial states which are attracted to a particular stable, steady-state is called the 'domain of attraction'. Basically, structural stability involves an analysis

of how domains of attraction, particularly their boundaries, are modified by alterations in the value of a system's parameters. Such alterations take place at so-called 'critical' parameter values. Elementary catastrophe theory involves a classification of the types of transition from being in the domain of one steady-state to being in the domain of another.

Following Casti (1979, p.52), this is portrayed in figure two,

'The point x initially lies in the domain of the attractor P . Because of changes in the system dynamics, the domain of attraction of P shrinks from I to II , while that of Q expands from 1 to 2 . The point x is now drawn toward Q rather than P . Of course, the locations of P and Q themselves depend upon the system structure, so the points in the figure are actually regions containing P and Q . What is

important is that the regions of P and Q are disconnected'.

It is noted that such a phase space representation is not used directly in catastrophe theory; instead, it gives (critical) parameter values which cause a change in the domain of attraction. Such alterations of the boundaries of the specific domains relate to the idea of system persistence, and this is discussed in the section on planning implications.

(4) INCORPORATION OF STOCHASTIC ELEMENTS INTO A DETERMINISTIC FRAMEWORK

To date, the representation of system dynamics has been within a wholly deterministic framework. However, as it has already been demonstrated, no single, unique trajectory of behaviour exists as a parameter value changes over time. Bifurcating steady-state solutions for alterations in the value

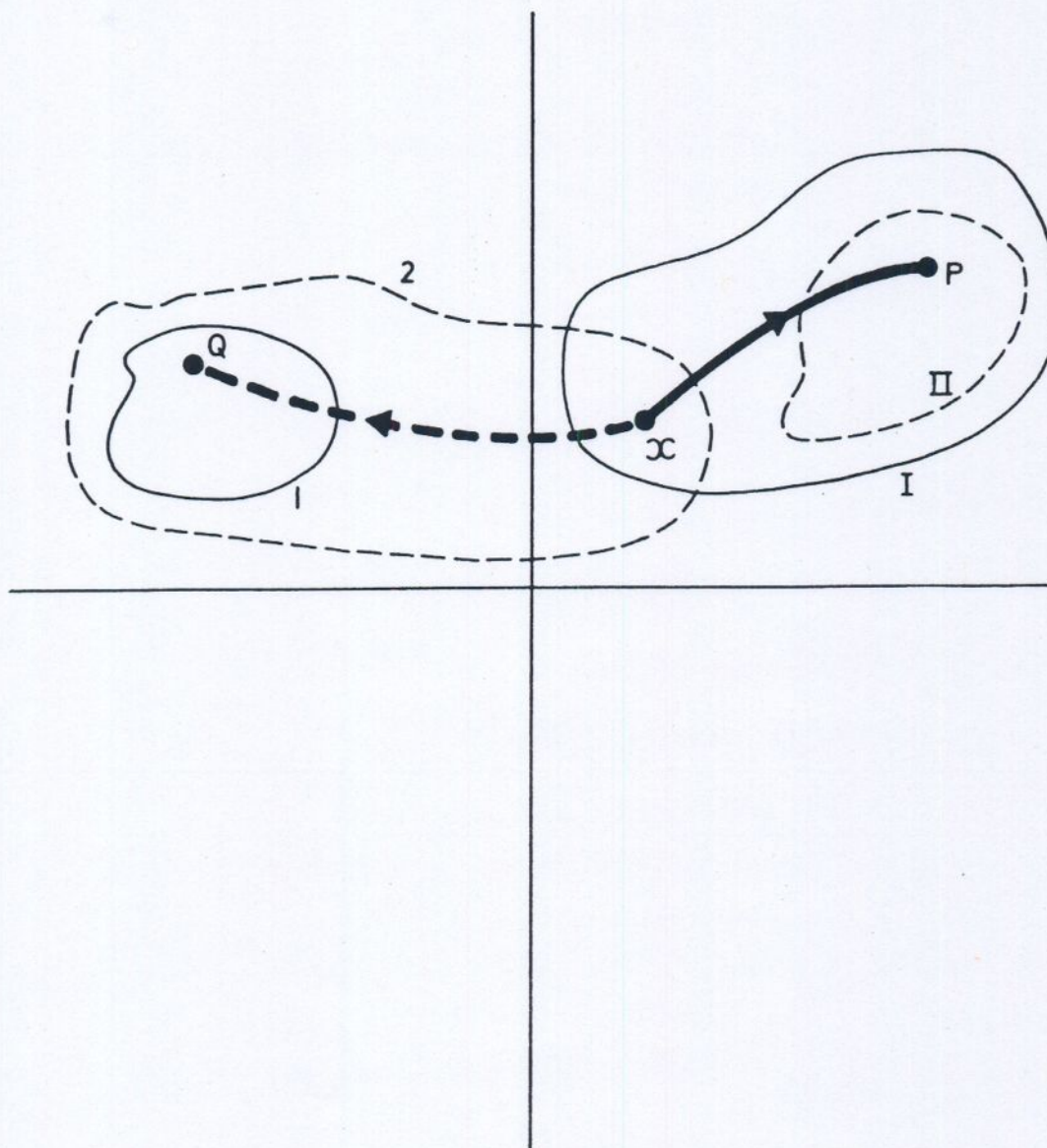


Figure 2. Shifting domains of attraction

(Source: Casti, 1979)

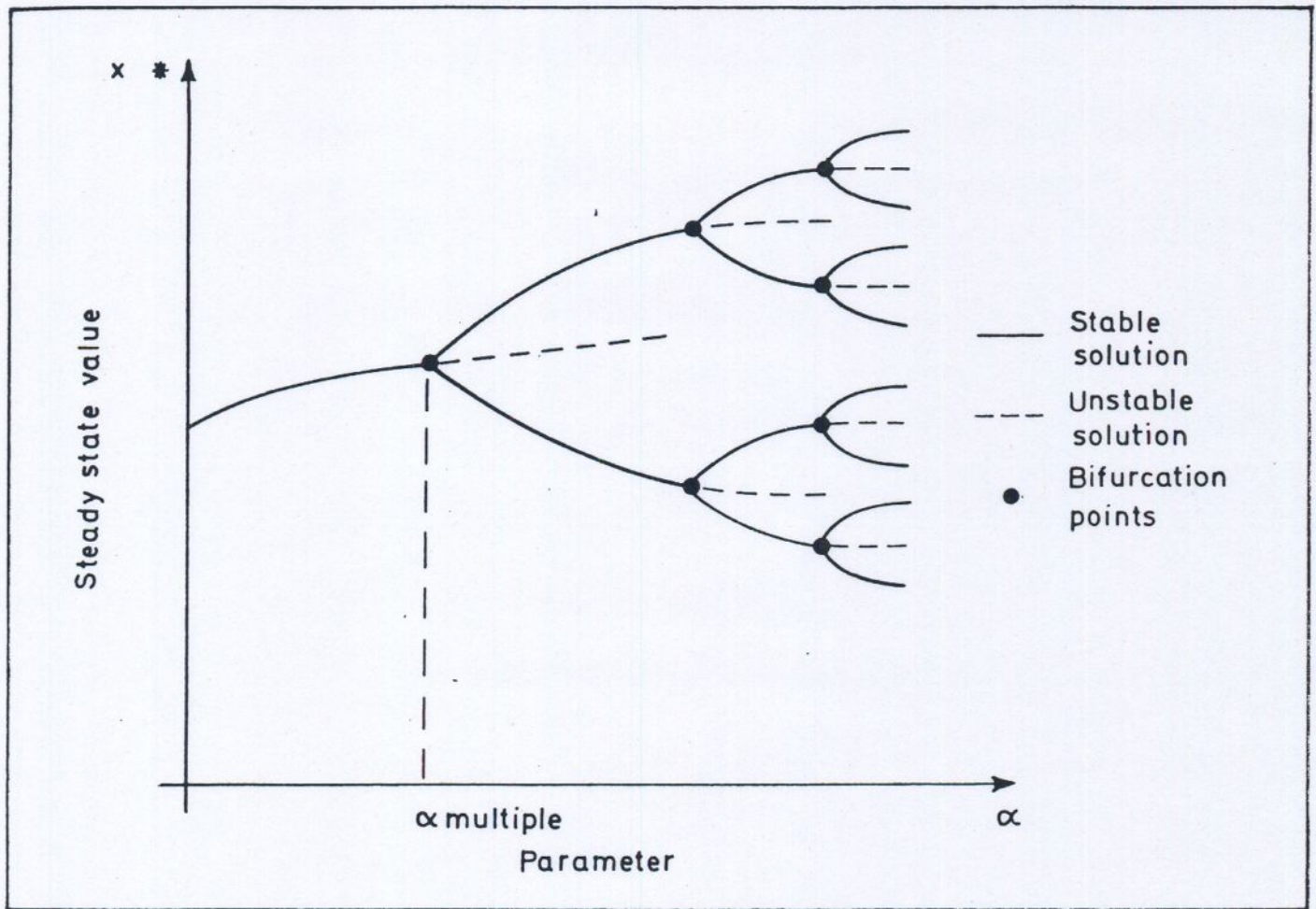
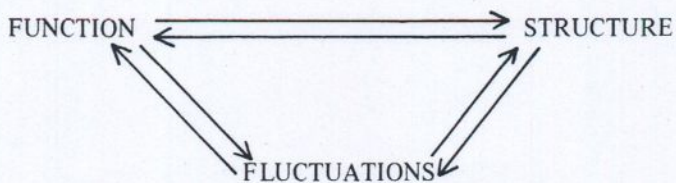


Figure 3. Bifurcating steady-state solutions as a parameter changes

of a parameter are shown in figure three; for α greater than α multiple, multiple, steady-states exist. At present, however, one is unable to specify the particular trajectory followed by the system.

It is the stochastic fluctuations which determine the system's specific evolutionary path. Nicolis and Prigogine (1977) schematically portray this complementary role of determinism and fluctuations in system evolution as



For example, a system's function is described by differential equations, its different behavioural regimes that are exhibited arise from the structural instabilities inherent in the system, and the fluctuations actually trigger the transitions between states at critical points.

It can be suggested that the deterministic equations relate to the mean value of 'macro' variables, and the stochastic ('micro') elements are only of importance in the neighbourhood of a critical point. Indeed, given inherent variance in the

macroscopic description of the system, the fluctuations can be conceptualised as endogenous local characteristics, and not as exogenous inputs as in time-series analysis.

In summary, it is desirable to incorporate both the micro- and macro-characteristics in modelling system dynamics. System description is at a macro level, and, therefore, it possesses a mean (average) perspective, but the micro-level variations are important forces determining system behaviour. This conceptual framework presents an opportunity to generate new theoretical insights from the modelling of dynamical systems. Moreover, it has significant implications for practical work, particularly planning and man's control.

(5) MAN'S ROLE IN SYSTEM CONTROL

For control to have meaning, there must be explicitly stated goals and the current preoccupation with efficiency and optimal solutions commonly involves active control to either preserve a given structure or change a system into a desirable structure. However, with regard to the modifications of domains of attraction by changing parameter values (specifically, the ideas of a system's persistence at a particular state), it can be suggested that such actions may produce transient benefits at the cost of a contraction of the specific state's domain of stability. Since the mid-1960's, for instance, a number of socio-economic indicators such as crime, 'poverty', unemployment and inflation levels could arguably be inter-

puted as indication that a temporary improvement in the quality of life has been achieved at the expense of an associated reduction in the stability of the regime within which such behaviour can persist. That is, a system's (long-term) persistence capabilities can actually be undermined. Consequently, there is an increased possibility of sudden, potentially disastrous, irreversible transitions resulting from apparently insignificant decisions. There is, therefore, an indication of a need to consider alternative courses of action.

At worst, planning strategies must not reduce a system's ability to counter perturbations. Otherwise, at some future time, some fluctuations, which could formerly be absorbed without resulting in a sudden change in behaviour, will actually trigger a transition between regimes. It is suggested that any system management should adopt a perspective founded on persistence rather than on stability; that is, there should be a shift away from the current focus on steady-states to the broader features of attractor domains (see also Holling (1976)).

Thus, there is an implicit suggestion that planning cannot indefinitely constrain development processes within a given structure. It has already been postulated that change is inevitable (even if man desires otherwise), and a further corollary is that such evolutionary processes should be identified and appreciated. Planning in such an evolutionary context will result in greater uncertainty because of an increase in potential futures. Perhaps man's role should be thought of as one of assisting desirable evolutionary processes. Obviously, transitions to new regimes will eventually occur, and, although it can be argued that a laissez-faire approach relying on invisible hands is more appropriate than one founded on stability, planners could act as catalysts in the evolution of different dynamical behaviour. Such notions have many parallels with the Baconian philosophy that man can master nature only by obeying her. Accordingly, the common thesis that man's relationship with nature is an ever-increasing one of control and ascendancy must be carefully interpreted. No proposal, however, is made to reduce man's intervention in a system's evolution — far from it, the approach may require increased control. Fundamentally, the implications of an emphasis on persistence must be appreciated — ultimately, man's power is restricted, although it is potentially crucial.

An adoption of such an approach does not mean that present-day features of the planning process will not remain of significance. For example, clearly, planning must be continuous and not exhibit discontinuities associated with the system's be-

haviour. The recognition of sudden transitions between states raises a fundamental problem with respect to current forecasting methods, which are usually trend extrapolations that do not take account of the possibility of discontinuous behaviour and evolution to new structures. The potentiality of alternative futures presents both a philosophical and methodological challenge for researchers interested in system dynamics.

In conclusion, the role of man is conceptualised in conjunction with a system's inherent dynamics, assisting, rather than impairing, endogenous organising mechanisms. Moreover, uncertainty is explicitly appreciated, and future options are left open. In the end, the success of this open-ended planning process is dependent upon the political environment in which it functions and the theory employed.

(6) CONCLUSION

The basic argument is that recent work on structural stability raises a number of issues related to the modelling of system dynamics. It is no longer necessary nor desirable to be constrained by linear functions; complex, dynamical behaviour resulting from more realistic, non-linear functions is now comprehended much more. Whilst initial advance is more likely to be in conceptual, than in operational, terms (as with system dynamics in general), work also needs to be undertaken to operationalise the mathematical models. In the immediate future, one avenue to pursue is a re-examination of existing simulation models, because it is likely that enhanced interpretation and even new insights will be forthcoming if couched in terms of structural stability and critical points. Moreover, a fruitful and related research topic would be to examine the variety of results derived for the same model with different sets of random fluctuations.

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