

Evaluating Objective Function Trajectories:

What is in the Eye of the Beholder

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Introduction

Although the system dynamics literature covers issues of how to construct, analyze, test, validate, and implement dynamic models, surprisingly little attention has been paid to how managers react to and interpret the output from system dynamics models (see Gardiner and Ford, 1980; Rohrbaugh and Andersen, 1979). That is, system dynamicists construct feedback models that are simplifications of a complex reality and then conduct policy tests on these abridged representations. However, decision makers not trained in system dynamics may find that even these allegedly simplified models may be quite complex and difficult to evaluate, since model output typically consists of scores of variables interwoven over time.

From a psychological point of view, evaluating the output from a system dynamics model raises several interesting questions. First, how do decision makers integrate information about numerous variables that change over time and how do managers evaluate changing patterns of system states over time (some such patterns have even been nicknamed as "worse before better," "better before worse," and "trade-off" patterns of behavior)?

A second question is whether or not important differences exist between individuals with respect to how they evaluate a system's output. If so, policies preferred by one decision maker may not be preferred by another. This question quite obviously has important implications for implementing

model results. For example, do some decision makers emphasize steady state responses to policy innovations while other decision makers emphasize the transient response? If so, these two classes of decision makers would tend to evaluate "worse before better" policies in consistently different ways.

This paper summarizes some recent work on these two questions concerning how managers evaluate system trajectories over time and how researchers can attempt to measure individual differences in decision maker's preference structures. In particular, the paper presents an empirically derived taxonomy for classifying patterns of individual differences in decision makers' evaluation of the output from system dynamics models. Before delving into the research methods and conclusions, a brief overview of some of the existing problems with evaluating objective functions is presented along with a sketch of the psychological theory that is used to underpin much of the research presented below.

Existing Patterns with Traditional Objective Functions

The creation of workable and reliable objective functions has been an elusive goal of economists, policy analysts, and dynamic modelers since the beginning of quantitative analyses of social policy. The development of such functions would produce several dramatic benefits for the formation of social policy. Not only would objective functions provide a precise index of system performance, but they would also clarify and explicate the criteria being used in the process of policy formation. Perhaps more importantly, objective functions would allow analysts to rank order preferred sets of policy alternatives.

From a technical perspective, as well, the creation of workable and reliable objective functions would be extremely valuable in dealing with

questions related to parameter sensitivity. The testing of sensitivity would be greatly simplified if analysts could evaluate the reaction of the overall system performance to changes in parameters, rather than focusing on trajectories, characteristic modes, or recommended policies. Furthermore, the field of automatic control is immediately available with a host of optimization techniques which would be applicable to dynamic models upon the development of workable and reliable objective functions. Unfortunately, severe conceptual and technical problems currently appear to inhibit their possible use.

Typically, engineers and economists have been able to side-step many of the difficult problems involved in the construction of dynamic objective functions by evoking notions of minimum energy (or cost), maximum efficiency, or minimum total energy. In general, the problem has been solved by specifying a quadratic objective of the form

$$O(t) = X(t) Q X(t)^T$$

where $O(t)$ is the objective function at some specified time t , $X(t)$ is the vector of system states at time t , and Q is a matrix of weights applied to the various quadratic terms. The evaluation of dynamic trajectories typically has been handled by taking the integral of the quadratic objective function defined above with an added term for the evaluation of the system's end-state with the form

$$O(t_f) = \int_{t_0}^{t_f} X(s) Q X(s) ds + f(X(t_f))$$

where $O(t_f)$ is the overall evaluation of the objective function at the final time t_f and $f(X(t_f))$ is the relative weight given to the system's final state (assuming that the system is stable). The justification for the use of such a simplified dynamic objective function has typically rested on a priori

deductions concerning what the proper objective should be rather than upon detailed empirical investigation of what is the actual preference structure of an individual decision maker.

For many engineering problems and some economic problems, a priori specification appears to be justifiable. Most policy problems, however, seem to demand a more complex integration of the variety of social and political variables contained in a dynamic model. Early developments in the field of Gestalt psychology coupled with several recent advances in psychologist's ability to measure and quantify decision makers' preference structures provide the tools needed to develop more workable, empirically derived objective functions.

An Organizing Psychological Framework

The framework needed to integrate and evaluate complex patterns of many variables changing through time is contained in the arguments of Max Wertheimer and other Gestalt psychologists in the second decade of this century, arguments that ultimately redirected structural and behavioristic psychology. They insisted that perception, for example, is more than the sum of individual sensations or that behavior is more than a "bundle of reflexes." These radical configurationists rejected molecular psychological models and proposed that one should begin with the complex, holistic system as the basis for scientific progress. As Wolfgang Kohler suggested in his definitive statement of the Gestalt theory:

The stimulus-response formula, which sounds at first so attractive, is actually quite misleading When the term is taken in its strict sense, it is not generally "a stimulus" which elicits a response. In vision, for instance, the organism tends to respond to millions of stimuli at once; and the first stage of this response is organization within a correspondingly large field The right psychological formula is therefore: pattern of stimulation--organization--response to products of organization. (Kohler, 1947, pp. 164-165)

Kohler proceeded to outline a type of "system dynamics" theory of psychological functioning as a keystone for the Gestalt approach:

Everything in this field [sensory experience] points toward a theory in which the main emphasis lies on dynamic factors rather than on anatomically prescribed conditions. Moreover, in many observations field dynamics is almost directly revealed to the subject. This is the case for instance, when sudden stimulation, or a change of stimulation, is followed by sensory events rather than states Without the great historical prestige which machine theory still enjoys, nobody would hesitate to take these observations as evidence of dynamic interaction There is no question that so long as dynamics remains undisturbed by accidental impacts from without, it tends to establish orderly distributions Dynamic self-distribution in this sense is the kind of function which Gestalt Psychology believes to be essential in neurological and psychological theory. (Kohler, 1947, pp. 122-32)

Due to Kohler's belief that the dynamic distributions of sensory organization and sensory fields are functional wholes, he stressed the need for psychologists to investigate the overarching forms of environmental stimulation, as well as the systematic integrations that underlie the organism's responses. This molar rather than molecular approach is fundamental to our present interest in evaluating objective function trajectories.

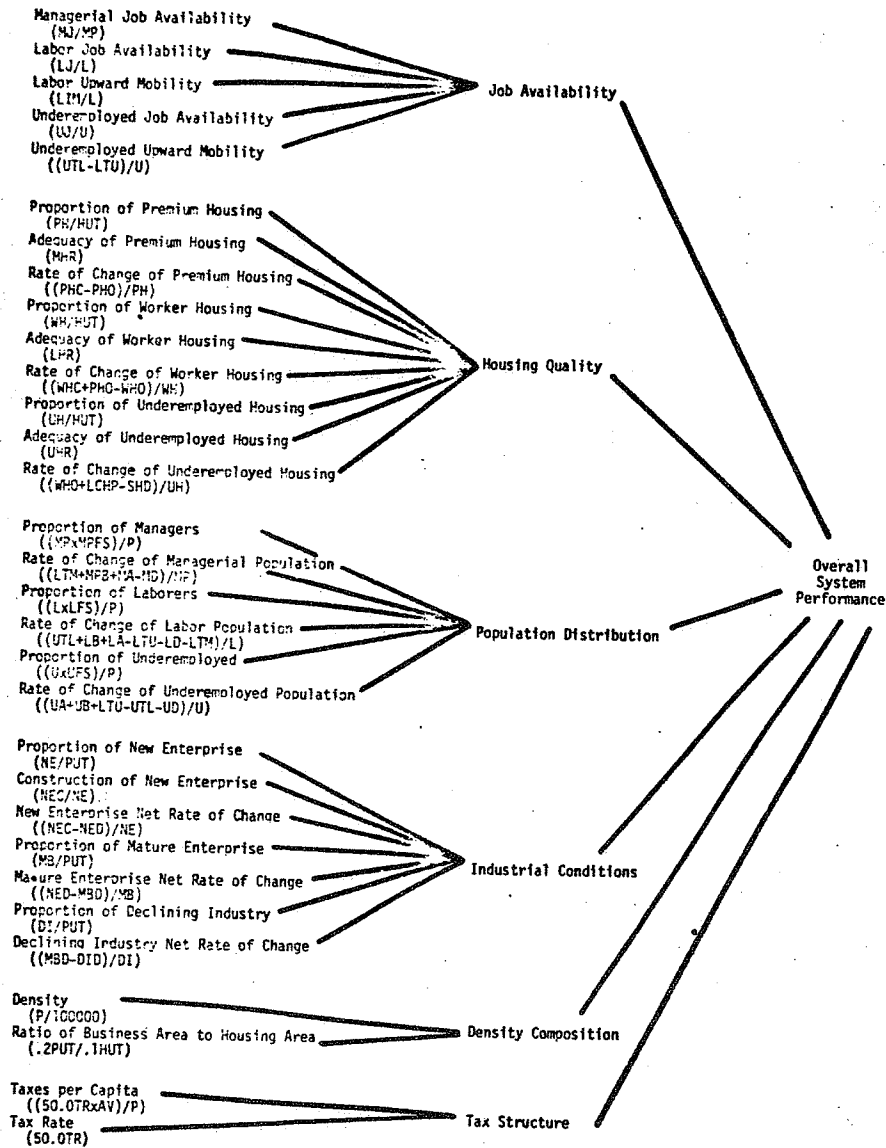
In the work reported below, social judgment analysis, a set of empirical techniques grounded in the theory of experimental cognitive psychology is proposed as a basis for the development of objective functions that could be used to summarize the multivariate performance of a variety of social systems (see Hammond, McClelland, and Mumpower, 1980; Hammond, Mumpower, and Smith, 1977). To illustrate the use of social judgment analysis in the development of workable and reliable objective functions, examples have been drawn from Forrester's (1969) Urban Dynamics, a complex, nonlinear, dynamic, feedback model designed to capture many of the interactions in a generic urban area.

The focus of the research reviewed in this section is on the construction of such dynamic objective functions that summarize the patterns of stimulation provided by the output of the Urban Dynamics model in response to a variety of policy tests.

Cross-sectional Summary of the Sensory Field.

In order to form an objective function, each n-tuple in an n-state model should map into a single value. The resulting unidimensional measure allows for the preferential ranking of all possible states of the system. Of the 124 system variables that Forrester tabled in Urban Dynamics, 36 were selected as potentially useful to the summary of the urban system sensory field. These 36 key system variables were combined and organized to form the hierarchical judgment model shown in Figure 1. More specifically, the 36 key system variables were variously combined to derive 31 explicit criteria at the bottom of the hierarchy which were subsequently clustered to form 6 separate system goals: job availability, housing quality, population distribution, industrial conditions, density composition, and tax structure. These 6 system goals, when integrated by a decision maker, provide the basis for constructing an objective function of overall system performance.

In order to develop a data base for the current research, the effects of 11 different urban policies on the 36 key system variables were extracted from the tabled values in Urban Dynamics for the two time periods cited: 10 years following implementation of a policy and 50 years following implementation. The 10-year cross section captured the short-term effects of the policy being implemented and the 50-year cross section reflected the long-run equilibrium effects of the policy. By including one more set of conditions defined at initial equilibrium, a total of 23 alternative observations of the 36 system variables and, therefore, 23 alternative profiles of the 31 derived criteria,



were constructed. Thus the 23 profiles included one base equilibrium run plus one short-run (10-year) and one long-run (50-year) set of effects for each of the 11 policies investigated by Forrester.

Each complete profile was split into sections corresponding to the 6 separate system goals. For example, Figure 2 illustrates two of the 23 profile sections pertaining to job availability, as well as two of the 23 profile sections pertaining to population distribution. It should be noted that, although the profile sections for each system goal contain the same criteria, the criteria take on different values in each profile section.

The evaluation of system states on the basis of the 31 criteria is a complex cognitive problem that can only be understood in the context of individual judgment. According to social judgment theory (Hammond, Brehmer, Stewart, and Steinmann, 1975), such an evaluation process demands the integration of the sensory field containing any or all of the 31 criteria. Social judgment theory proposes that the summary of such multiple stimuli in the judgment process can be represented by (a) the particular degree of importance placed on each criterion--referred to as weight; (b) the specific form of the functional relation between each criterion and the final judgment--referred to as function form; and (c) the particular method for integrating all of the criteria--referred to as the organizing principle. If repeated judgments are made about a variety of system states, the covert cognitive process of an individual's judgments can be mathematically modeled using multiple regression statistics, as well as converted to pictorial representation by means of interactive computer graphics (Hammond, Rohrbaugh, Mumpower, and Adelman, 1977).

Four students of system dynamics in the Graduate School of Public Affairs at the State University of New York at Albany evaluated the 23 alternative

FIGURE 1. Hierarchical Judgment Model of the Urban System.

PROFILES OF JOB AVAILABILITY

Managerial Job Availability (managerial jobs/manager)	.72
Labor Job Availability (labor jobs/laborer)	1.03
Labor Upward Mobility (% laborers to managers/year)	.9%
Underemployed Job Availability (underemployed jobs/underemployed)	.55
Underemployed Upward Mobility (net % underemployed to laborers/year)	1.5%

Managerial Availability (managerial jobs/manager)	.78
Labor Job Availability (labor jobs/laborer)	1.10
Labor Upward Mobility (% laborers to managers/year)	1.1%
Underemployed Job Availability (underemployed jobs/underemployed)	.75
Underemployed Upward Mobility (net % underemployed to laborers/year)	3.0%

PROFILES OF POPULATION DISTRIBUTION

Proportion of Managers (managerial persons/total population)	.06
Rate of Change of Managerial Population (% growth of managerial population/year)	0.0%
Proportion of Laborers (labor persons/total population)	.41
Rate of Change of Labor Population (% growth of labor population/year)	0.0%
Proportion of Underemployed (underemployed persons/total population)	.53
Rate of Change of Underemployed Population (% growth of underemployed population/year)	0.0%

Proportion of Managers (managerial persons/total population)	.07
Rate of Change of Managerial Population (% growth of managerial population/year)	2.2%
Proportion of Laborers (labor persons/total population)	.44
Rate of Change of Labor Population (% growth of labor population/year)	1.6%
Proportion of Underemployed (underemployed persons/total population)	.49
Rate of Change of Underemployed Population (% growth of underemployed population/year)	-1.3%

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profiles of system states (divided into the 6 separate system goals) described above. Each individual's 6 sets of judgments were made on a 20-point rating scale from 1 (a completely unacceptable system state) to 20 (a very acceptable system state). Figure 2 illustrates the exact nature of the judgment tasks that were given to the four participants. The participants would consider the profile sections (such as those shown in Figure 2) and express their relative preferences for the profile sections by assigning to them ratings on the judgment scale (1-20).

Once judgments had been made concerning each of the 6 profile sections at the more detailed side of the hierarchical model shown in Figure 1, participants were then asked to specify how to combine information from all 6 system goals into a single, overall evaluation of system performance. Figure 3 depicts two of the possible 23 alternative profiles that participants evaluated in assessing overall system performance. As in Figure 2, each participant responded to the profiles using a 20-point rating scale. It should be noted that these profiles (as shown in Figure 3) were constructed by using the prior judgment scales that ranged from 1 to 20. Thus, the 6 sets of judgments for each individual became a final set of profiles themselves, about which a last set of judgments were made concerning the overall acceptability of the 23 system states.

Stepwise multiple regression analyses were used to develop models of the judgment process of the four participants. Criteria were entered into the regression equations only if they were found to be statistically significant predictors of the participants' judgments ($p < .05$). The resulting multiple Rs ranging from .74 to .99 (an average of .94) indicated that a major proportion of the variation in judgments could be reliably predicted by the linear, additive models; nonlinear and nonmetric models requiring additional

FIGURE 2. Two of the 23 Alternative Profile Segments Depicting Criteria for the Evaluation of Job Availability and of Population Distribution.

PROFILES OF OVERALL SYSTEM PERFORMANCE

Job Availability	6
Housing Quality	15
Population Distribution	7
Industrial Conditions	5
Density Composition	9
Tax Structure	11
Job Availability	18
Housing Quality	6
Population Distribution	15
Industrial Conditions	18
Density Composition	11
Tax Structure	16

FIGURE 3. Two of the 23 Alternative Profile Segments Depicting Individual Ratings as a Basis for the Evaluation of Overall System Performance.

predictive terms were not tested due to the limited number of profiles available for the data base (details of this analysis have been presented in Rohrbaugh and Andersen, 1979, and Andersen and Rohrbaugh, 1979).

Dynamic Summary of the Sensory Field.

The construction of an empirically derived cross-sectional objective function as discussed above represents a fairly straightforward application of existing techniques of judgment analysis to the field of dynamic systems. However, extending the evaluation of system performance over an extended time frame poses several conceptual and technical problems. The system dynamics literature is filled with suggestions that longitudinal evaluation of system performance might prove difficult. For example, many systems exhibit unexpected or even counter-intuitive behaviors over time (Forrester, 1971). A system that initially shows relative improvement may soon reverse itself and show deterioration.

To address this difficulty of deriving a full longitudinal objective function, the following approach was developed. The 7 regression equations constituting the full judgment model for each of the participants (6 equations for each of the initial sets of judgment tasks, e.g., housing quality and job availability, and one equation for the overall evaluation) were attached to the Urban Dynamics model in order to create a new objective function sector. When the model subsequently was run, plotted output from the objective function sector showed how the individuals' preferences varied dynamically as the performance of the system fluctuated over time, as illustrated in Figure 4.

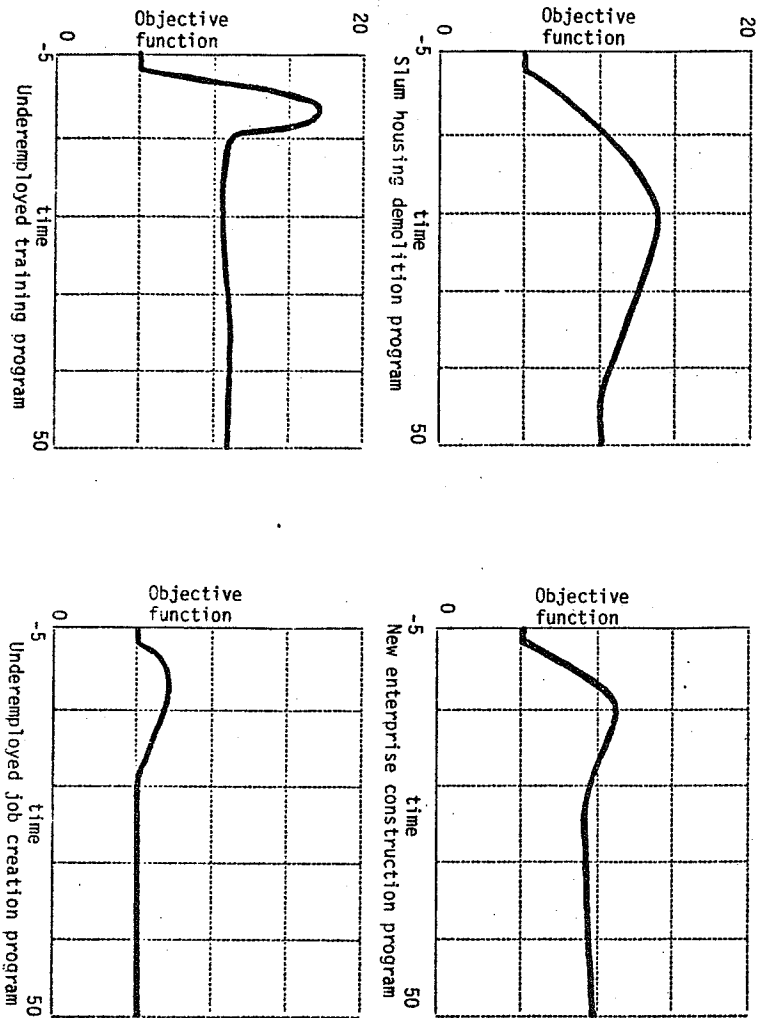


Figure 4: Summary Objective Functions for Four Urban Revival Policies

Sensory Organization--Objective Function Trajectories

How does one respond to the patterned stimuli that comprise an objective function varying through time? What sensory organization is brought to bear on the shifting characteristics of the curves such as those shown in Figure 4? For example, consider two systems, A and B, that begin at the same point (as measured by the objective function) and improve to the same equilibrium point that represents a 20% improvement in overall system performance. System A, however, rapidly improves by 50% in the first fifteen years, deteriorates to its initial conditions by the thirtieth year, and finally settles into equilibrium at the end of a fifty year period. In contrast to this oscillating improvement pattern, system B rises slowly but steadily to its final equilibrium, in the beginning showing much less drastic improvement than system A but never deteriorating as does system A. Whether a decision maker prefers the pattern of oscillating improvement or the pattern of gradual improvement is an additional problem requiring individual judgment. It would appear to be quite difficult to quantify and measure exactly what it is about these two dynamic objective functions that is preferable. The problem is further exacerbated when one considers the broad range of behavior that can emanate from a dynamic system.

A fundamental difficulty to be overcome in order to evaluate alternative trajectories is determining a set of dimensions that can be used to classify the various curves generated by the objective function sector of the Urban Dynamics model. Furthermore, do decision makers evaluate system performance through time by consistently using a fixed set of curve characteristics? To begin to investigate these problems, a class of analytically "well-behaved" curves that closely paralleled the trajectories generated by the objective function sector of the Urban Dynamics model were explored further. The curves

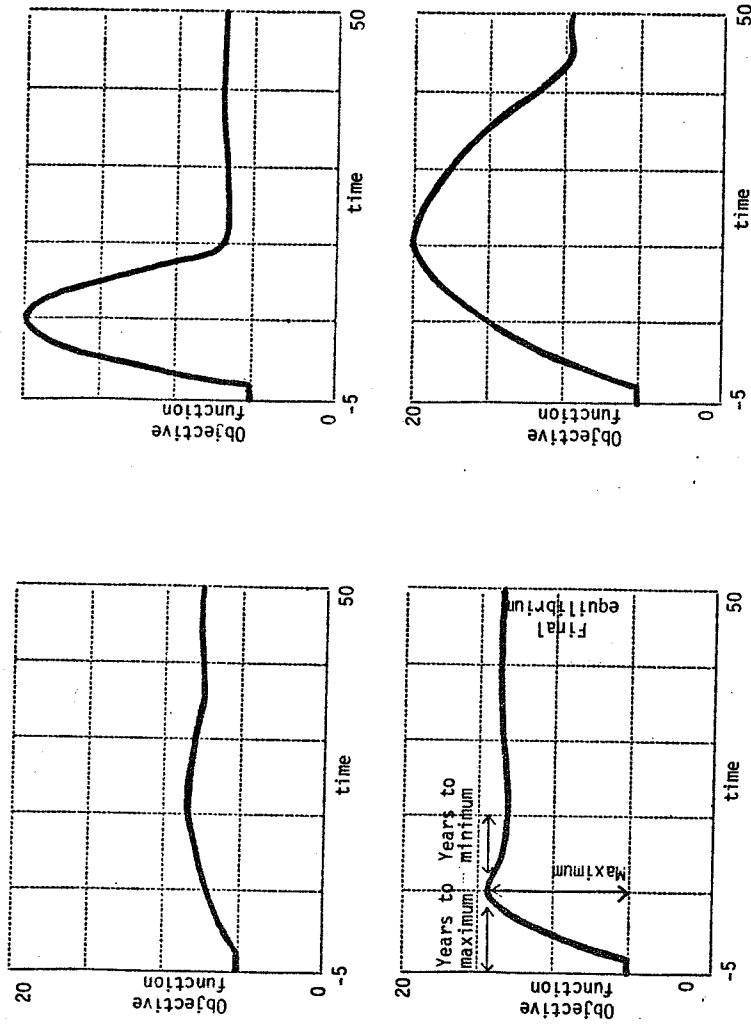


Figure 5: Sample Objective Functions Presented to Participants for Evaluation

analyzed in this portion of the study were generated by orthogonally varying four characteristics that almost fully defined their trajectories: number of years from initial equilibrium to maximum, number of years from maximum to second minimum, maximum point, and final equilibrium point. The systematic variation of the four characteristics at five levels (e.g., 5, 10, 15, 20, and 25 years from initial equilibrium to maximum) produced a set of 25 alternative curves, 4 of which are illustrated in Figure 5.

The set of 25 curves was presented to the 15 research participants (advanced graduate students in the field of management) who were instructed that these curves represented hypothetical objective functions tracing the longitudinal acceptability of urban system states. The participants were asked to evaluate the curves on a scale of 1 (a completely undesirable trajectory) to 20 (a very desirable trajectory). Again, through the use of stepwise multiple regression analyses, judgment models were derived which could be used to predict consistently the desirability of a wide range of curves. Characteristics of the curves were entered into the regression equations only if they were found to be statistically significant predictors of the participants' judgments ($p < .05$). The resulting multiple R s ranging from .89 to .97 indicated that a major proportion of the variation in judgments could be reliably predicted by the linear, additive models based on the four characteristics identified.

Table 1 presents the standardized regression coefficients in relative form (i.e., constrained to sum to 1.00) for each participant. The judgment models shown in Table 1 represent diverse cognitive approaches to the evaluation of the 25 alternative objective function trajectories. Six of the participants appear to be particularly concerned about the final equilibrium point, placing over 80% of their relative weight on that one characteristic of

Table 1

Relative Weights Comprising Judgment Models for Objective Function Trajectories

Cluster type	IA				IB	IC	IID						IIE	IIF	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Participant															
Final equilibrium	1.0	1.0	1.0	1.0	.82	.81	.77	.75	.68	.65	.62	.57	.56	.35	.30
Maximum point	---	---	---	---	---	---	.23	.25	.32	.35	.38	.43	.30	.40	.49
Years to minimum	---	---	---	---	.18	---	---	---	---	---	---	---	.14	.13	.11
Years to maximum	---	---	---	---	---	.19	---	---	---	---	---	---	---	.12	.09
Multiple R	.96	.97	.97	.96	.90	.89	.92	.95	.94	.95	.92	.91	.95	.91	.97

the trajectories. The remaining nine participants appear to be concerned both about the final equilibrium point and the maximum point of the trajectories, some placing almost 50% of their relative weight on the latter characteristic. Concern about the number of years from initial equilibrium to maximum and the number of years from maximum to second minimum also differentiate the sensory organization of the participants but to a lesser degree.

By grouping participants into clusters of individuals with similar methods of responding to the patterned stimuli provided by the objective function trajectories, it is possible to describe at least six discrete judgment policies reflected by the decision makers in the present study:

IA: concern for maximizing final equilibrium

IB: concern for maximizing final equilibrium and years to second minimum

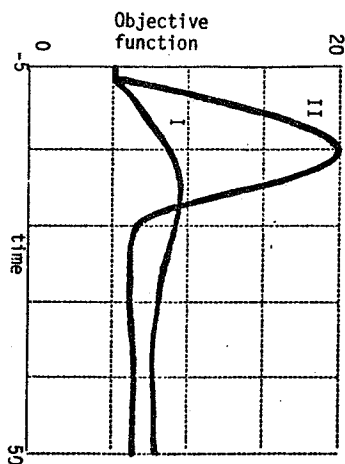
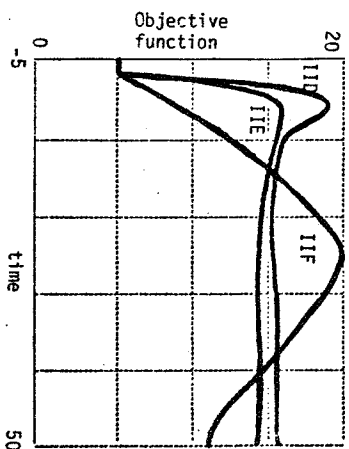
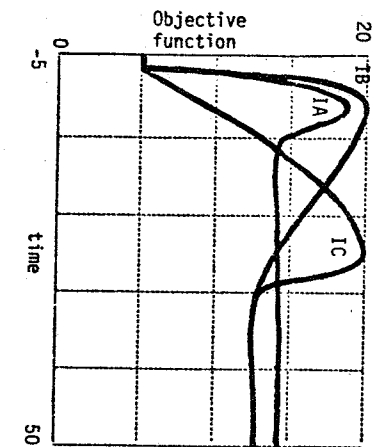
IC: concern for maximizing final equilibrium and years to maximum point

IID: concern for maximizing final equilibrium and maximum point

IIE: concern for maximizing final equilibrium, maximum point, and years to second minimum

IIF: concern for maximizing all four characteristics but particularly maximum point

A series of two choices between objective function trajectories will determine for any decision maker which of the six methods of sensory organization is being used. The first choice, shown at the top of Figure 6, indicates whether the decision maker is in cluster type I or cluster type II. If the decision maker prefers the trajectory associated with cluster type I, the second choice, shown at the lower left of Figure 6, indicates whether the cluster type is IA, IB, or IC. If the decision maker prefers the trajectory associated with cluster type II, the second choice, shown at the lower right of Figure 6, indicates whether the cluster type is IID, IIE or IIF.



Because a regression equation exists to mathematically model the sensory organization that each cluster of decision makers brings to the patterned stimuli created by the objective function trajectory, any run of the Urban Dynamics model can produce a single index of overall performance of the urban system with respect to a particular urban revival policy. For example, because the decision maker whose objective function trajectories shown in Figure 4 produced a judgment policy indicating a concern only for maximizing final equilibrium (cluster type IA), it is clear that he would prefer the effects of an underemployed training program (shown at the bottom left of Figure 4) to the effects of an underemployed job creation program, slum housing demolition, or new enterprise construction program.

Put in terms more familiar to system dynamicists, the results presented in Table 1 suggest that, when presented with output such as that found in Urban Dynamics, decision makers tend to fall into two broadly defined clusters. The first cluster consists of those who tend to place considerable emphasis on the steady state behavior of a system in response to a policy change. The second cluster consists of decision makers who place relatively more emphasis on the transient response of the system. The most important transient characteristic is the maximum point of the transient peak followed by the speed of decline (measured as time to minimum) and the speed of increase in overall system performance (measured by time to maximum).

These results suggest that, when faced with "worse before better" or "better before worse" types of system performance, these two clusters of decision makers would tend to split in their evaluation of preferred policies. Of course, these results strictly hold only for behaviors similar to those emanating from the Urban Dynamics study. Separate analyses would have to be conducted to identify similar clusters of preference structures for other classes of system performance.

An interesting speculation centers on what causes these differences in preference structures for various clusters of decision makers. One unexplored question in this study is the degree to which the participants tended to view the sample trajectories as definitive statements about what the future would actually be like versus a belief that the trajectories are fallible predictions about the future originating from a model or other predictive device. One hypothesis that might explain the differences between the two major cluster of decision makers is that those who favor steady state may believe that the trajectories are actually a statement of what will happen. They may be pessimistic about their ability to intervene and change the outcome in the long run, favoring a policy that leads to a presently established sure gain. On the other hand, those who favor the transient response may be skeptical of the accuracy of long term predictions and optimistic about their ability to intervene and reverse a bad or deteriorating situation in the long run. Hence these decision makers may favor short-term improvements at the expense of the longer run consequences of the policy being examined.

Conclusions

The work presented above has developed an empirically derived method, based upon social judgment analysis, for quantitatively deriving and modeling the implicit preference structure for individual decision makers. When coupled with a system dynamics simulation model, these preference structures can be turned into dynamically varying objective functions that summarize at any point in time how an individual decision maker ranks the overall system performance. This result is especially useful because it allows researchers to determine to what degree individual decision makers differ in the criteria

that they employ for evaluating overall system performance over time. For a sample of fifteen, important individual differences in the criteria used to evaluate system performance were found. Further, these individual differences were found to cluster into several distinct types. The major difference between the various cluster centered on whether individuals more heavily weighted the system's transient response versus the system's steady state response. Important characteristics of the system's transient response were found to be (in order of importance) the maximum point for the transient curve, the speed of decline, and the speed of improvement in the overall system performance. The paper concludes with some speculation on what might be the psychological determinants of the empirically observed clusters of individual differences.

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