FORMAL EXPRESSION OF THE EVOLUTION OF KNOWLEDGE

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Abstract

In this paper we present a formal system S_Δ, in order to characterise the evolution of knowledge. In addition to the connectors of classical logic, we introduce two dynamic connectors - the mediate future and the immediate future - expressing the transformations that may affect data in the course of time. The axiomatisation of these connectors and their semantic characterisation lead us to define a model of interpretation for the formal system which is comparable to that of Kripke for modal logic. With this model we prove the intrinsic consistence and the validity of S_Δ. Similarly we demonstrate completeness and other propositions connecting the immediate future and the mediate future.

The formal system S_Δ is one of the component modules of the ARCHES system, a symbolic system for the representation and treatment of knowledge whose objective is to produce new knowledge through two modes of reasoning - deduction and analogy - based upon specific processes of inference.

1.0. INTRODUCTION.

This paper concerns the formal description and the logical properties of the formal system S_Δ which allows the expression of the modalities of knowledge evolution. S_Δ is one of the component modules of the ARCHES system, a symbolic system for the representation and treatment of knowledge [5]. Note that designing ARCHES required not only an analysis of work carried out in the representation of knowledge and reasoning (semantic networks, frames, expert systems, first order logic, etc... see [1], [7], [10], [11], [13]) but also looking at several case studies related to expert domains of human sciences [2]. To be more precise the ARCHES system allows the representation of any set of facts (objects of material culture, factual assertions, events, etc...) whose description and organisation are adequate to its architecture. For instance, statements like "Peter sleeps peacefully", "Mary's dress is made of light red wool", "the car V1 is located in the garage G" and "elephants have a trunk-like nose" represent elementary facts susceptible to be conveyed by ARCHES. The system aims at obtaining new knowledge from the facts which have been recorded in the corresponding knowledge base. The production of knowledge is carried out by two modes of reasoning-deduction and analogy - based upon specific processes of inference (see [6]).

The representation of facts is determined by composing entities constructed from the general notion of concept. The concepts are sets whose elements, called individuals, denote unambiguous and distinct observable terms producing the facts (for instance in the previous statements, PETER and V1 are two individuals belonging respectively to the concepts PERSONNE and CAR). Each fact is represented by a formula giving a description of an individual (see § 2.2) ; so if A denotes a concept, x a variable or a constant individual and y a description then A(x,y) is a formula of ARCHES (for more detail see [4]).

One of the originalities of ARCHES is to integrate, in its conception, operations which allow the expression of the evolutive character of data : acquisition of new knowledge, representation of events in terms of changes of states, modification in the description of phenomena resulting from calculations concerning them and/or objects related to them. Nevertheless the transformations which could occur on data involved in calculations must leave ARCHES in a consistent state. For instance, in an application of ARCHES describing the behavior of individuals, we must be able to assert at a given moment that "JOHN IS EATING" and, the moment later, that "JOHN IS NOT EATING" ; these statements would be contradictory in a system based on classical logic where properties and relations are definitively asserted or negated, irrespectively of the notions of time and

and space. In the same way, some natural language formulations of statements expressing changes of states have to be represented in ARCHES as the transition from a present descriptive state to a future one. For instance the statement "MARY IS AWAKENING" can be interpreted in ARCHES as a change of state between SLEEP and AWAKENESS ; these are still problems of consistency in that the two activities cannot occur simultaneously.
In order to represent the evolution of studied facts we present a non classical logic in which temporal connectors are defined, [8] and [15]. This paper aims to present the formal system described by the modalities of evolution of the descriptions characterising the individuals. To be more precise, the descriptions conveyed by ARCHES define a formal system S_Δ which has four components, to express the two aspects "Representation" (see § 2) and "Deduction" (see § 3.) which respectively define and organise them :

$$S_\Delta = \{\mathcal{A},\Delta,\Delta_T,\texttt{===>}\}$$

$\mathcal{A}$ is an alphabet on which is generated the set Δ of descriptions, $\Delta_T \subset \Delta$ representing the set of theses of S_Δ. These three sets aim to define and convey the representation of descriptions (see § 2.2, and § 2.3). They are built from a primitive data structure able to convey the elementary descriptions called descriptive terms (see § 2.1). The relation ===> is a deductive relation defining inferential activity among the descriptions. It is defined by rewriting rules on descriptive terms and from specific conditions associated with two categories of connectors. These are <u>static connectors</u> which define addition, disjonction and negation, and <u>dynamic connectors</u> which define the immediate future and the mediate future (see § 3.). We give a semantic characterisation of this temporal formal system, defining a model of evolution similar to that of Kripke for modal logic ([9], [12], [14]) (see § 4.) and we show that the deductive relation ===> is valid in this interpretation (see § 5.). Chapter 6 presents propositions related to dynamic connectors together with a decision procedure for the relation ===>.

2.0. <u>REPRESENTATION SYSTEM OF DESCRIPTIONS</u>.

2.1. <u>The primitive elements</u>.

Descriptive terms permit the representation of properties and more generally of state relations characterising individuals. They are built from four basic entities : features, classes, operators and the functional symbol \$, [3]. Features permit the representation of distinctive characters of

individuals , as in the previous example (see § 1.) the words PEACEFUL, RED, LIGHT, NOSE, TRUNK, G. Thus features may be <u>information without extension</u> (PEACEFUL, RED, LIGHT), <u>concepts</u> (NOSE, TRUNK), or <u>individuals</u> (G). Features having a single semantic nature are grouped into sets called classes. The class symbols express the semantic scope of the state relations attested in the elementary descriptions. Thus the features RED, TRUNK and G refer respectively to the classes COLOR, FORM and LOCALISATION. The relationships which exist between the classes and the features are expressed by particular functional symbols, in general n-ary, called operators. The operators allow specification of the semantic nature of the state relations which characterise individuals. For example, the statement "Automobile whose seating capacity is inferior or equal to 5" shows the kind of relationship which can exist between classes and features : the feature "number of seats" (i.e. numerical value 5) is related to its class CAPACITY by the arithmetical operator .LE.($\leqslant$). Moreover properties and state relations can be described locally, as in the following two statements : "Amphoras stamped T_1 of type P_1" and "Automobile of a dark blue colour". The characterisation of features by descriptive terms permits the precise representation of this descriptive situation (properties of properties, state relations made precise by these properties ; state relations of state relations, etc.). The relation which expresses the attribution of a descriptive element to a feature is represented by the functional symbol \$, thus permitting the representation of local descriptions of state relations. When a state relation is not locally described, we consider that the corresponding feature is characterised by the empty description Λ (see § 2.2.).

More precisely a descriptive term is any expression of the form $op_n(T,t_\ell)$ in which op_n is an n-ary operator, T a class symbol, and t_ℓ a tuple of degree n whose elements are either of the form $\$(t_i,\Lambda)$, or of the form $\$(t_i,op_{ni}(T_i,t_{\ell i}))$ where t_i is a feature referring to the class T, Λ is the empty description and $op_{ni}(T_i,t_{\ell i})$ a descriptive term characterising t_i.
We call <u>undescribed descriptive term</u> any descriptive term for which the elements of the tuple t_ℓ are of the form $\$(t_i,\Lambda)$. Otherwise it is a <u>described descriptive term</u>. Thus in the statement "the dark blue car V1 is in the garage G", the individual V1, instance of the concept CAR, is characterised by the two descriptive terms ISA(COLOR,\$(BLUE,ISA(SHADE,DARK))) and IN(LOCATION,INS(GARAGE,G)), the first being described and the second undescribed.

2.2. The set of descriptions Δ.

The description of an individual x belonging to the extension of the concept C is the organised set of descriptive terms which characterises it.

More precisely the descriptions are generated from an alphabet composed of four categories of symbols : /1/ a countable set $\mathcal{L}$ of descriptive terms ; /2/ the empty string Λ ; /3/ two categories of connectors, the state connector $\neg$, $*$, $+$ respectively called Negation, AND of addition and non exclusive OR, and the dynamic connectors $\circ$ and $\circledast$ respectively called immediate future and mediate futrue ; /4/ the parentheses (and) .

$$\mathcal{A} = \mathcal{L} \cup \{\Lambda\} \cup \{\neg, *, +, \circ, \circledast\} \cup \{(,)\}$$

The countable set Δ of descriptions is a formal language built on $\mathcal{A}$. The descriptions which are its well formed formulas follow the rules (R) of construction :

- R1 Λ is a description (called the "empty description") ;
- R2 Every descriptive term is a description ;
- R3 If a is a description then $\neg a$ is a description ;
- R4 If a and b are descriptions then $(a * b)$ is a description ;
- R5 If a and b are descriptions then $(a + b)$ is a description ;
- R6 If a is a description, then $\circ a$ is a description ;
- R7 If a is a description, then $\circledast a$ is a description ;
- R8 Any description is derived from descriptive terms by application of the previous rules.

$\mathcal{L}$ is the base of Δ ($\mathcal{L} \subset \Delta \subset \Delta^*$).

2.3. The theses of the formal system S_Δ.

Let $\mathcal{I}$ be the finite set of individuals and $\mathcal{D}$ the mapping which associates with every $x \in \mathcal{I}$ its description $\mathcal{D}(x)$. If the individual x is an instance of the concept C, then the association of $\mathcal{D}(x)$ to x is represented through the structure $C(x,\mathcal{D}(x))$ (see § 1.) : we say that $\mathcal{D}(x)$ is a thesis of the formal system S_Δ.

Let $\Delta_T \subset \Delta$ be the set of descriptions such that $\mathcal{D}$ be a one to one mapping between $\mathcal{I}$ and Δ_T : the structures stored in ARCHES are the well formed formulas $C(x,\mathcal{D}(x))$ in which x, instance of the concept C, has $\mathcal{D}(x) \in \Delta_T$ for description.

Δ_T is the set of theses of the formal system S_Δ of characterisation of the descriptions.

3.0. DEDUCTIVE ORGANISATION OF DESCRIPTIONS.

3.1. The transformation rules of descriptive terms.

The representation and the logical properties of descriptive terms allow the definition of three kinds of rewriting rules : rules of decomposition, rules of inheritance and transitivity, and rules of extension. These rules express the semantic properties of classes. For example the semantic relations existing between the classes PART and MATERIAL are expressed by the following rule of inheritance :

ISA(PART,$(x,ISA(MATERIAL,y)))--->ISA(MATERIAL,y)

This rule means that if an individual has a part x made of material y, then the material employed in the manufacturing of the individual is also y.

Generally, for any application we have several possible rewriting rules like the previous one. These rules define the system of substitution reduction rules of ARCHES, from which are operated the transformations on descriptive terms.These transformations are determined by the rewriting relation --->* defined on the set of descriptive terms as the transitive and reflexive closure of the relation --->. Given two descriptive terms A and B, we say that B derives from A by application of substitution/reduction rules if and only if the relation A --->*B is verified. We have demonstrated that the results obtained from the processes of derivation do not depend on the order in which rewriting rules are applied. This allows us to define an original algorithm of decidability for the relation A---->*B. This algorithm can be split into two parts : first a procedure for the irreductible descriptive term of any descriptive term U and then a function of discordance between two descriptive terms V and W which searches for the first different undescribed descriptive term in V and W.

3.2. Definition of the relation ===>.

In order to complete the definition of S_Δ, we have to define the modalities of derivation of the descriptions by taking into account their formation rules from the descriptive terms and the connectors $*$, $+$, $\neg$, $\circ$, $\circledast$ (see § 2.2.) and the rewriting relations of the descriptive terms (see § 3.1.). So on the set ∆ we define a deduction relation between the descriptions, written ===> , which is the smallest transitive reflexive relation verifying the following condition (on this subject see [16]):

C1 a * b ===> a ; a * b ===> b
C2 if a ===> b then a * c ===> b * c
C3 if a *¬b ===> b then a ===> b
C4 if a --->* b then a ===> b
C5 a ===> ⊛a
C6 oa ===> ⊛a
C7 o⊛a ===> ⊛oa
C8 ⊛(a+b) ===> ⊛a+⊛b

The conditions C1 and C2, which establish the logical relation between the relation ===> and the connector *, give to the derivation generated by ===> the status of deduction (see § 3.4.). The condition C4 determines the deductive relations (from a specific element to a generic one) between descriptive terms and descriptions. The condition C5 and C6 indicate that the present and the immediate future belong to the future ; C7 defines the semantic relations between immediate future and mediate future.

3.3. The equality relation = .

We say that a = b (the description a "is equal to" the description b) if and only if a ===> b and b ===> a. We may show easily that the relation = is a relation of equivalence. The relation =, and hence the relation ===>, satisfies the following conditions :

C9 a*b=b*a ; a+b=b+a
C10 a*(b*c) =(a*b)*c ; a+(b+c)=(a+b)+c
C11 a*a=a ; a+a=a
C12 a*Λ=Λ*a=Λ ; a+Λ=Λ+a=a
C13 a*(a+b)=a ; a+(a*b)=a
C14 a*(b+c)=(a*b)+(a*c) ; a+(b*c)=(a+b)*(a+c)
C15 a*¬a=¬a*a=Λ
C16 o¬a=¬oa
C17 o(a*b)=oa*ob
C18 ⊛⊛a=⊛a
C19 ⊛Λ=Λ
C20 if a ===> b then oa ===> ob
C21 if a ===> b then ⊛a ===> ⊛b

Conditions C9 to C15 determine the semantic properties of the connectors *, + and ¬ giving to them the same formal status they have in classical logic. Consequently the quotient set ∆/= is a distributive and complemented lattice.

Condition C16 expresses the determinism of the evolution of descriptions.

Condition C18 points out that the connector ⊛ is transitive.

C19 Expresses the coherence of the evolution of descriptions in that the set ∆ has one and only one minimum (because ∆/= is a distributive and complemented lattice).

Conditions C20 and C21 mean that if the formula a ===> b is always true then it will always be true in the future. Note that conditions C1 and C5 to C19 are of the form a ===> b (or a=b which is equivalent to a ===> b and b ===> a). Therefore these conditions are the axioms of a formal system with formulas of the form a ===> b and they define the deduction relation ===> (i.e. the meta-linguistic symbol ===> becomes an element of the object language in this formal system). Otherwise the conditions C2, C3, C4, C20 and C21 of the form : IF a ===> b THEN c ===> d, are the inference rules of the same formal system.

3.4. The deductive nature of the relation ===>.

Theorem. The formula a ===> is verified if and only of the formula a*b=a is verified.

Proof. If a ===> b is verified, then a*a ===> b*a (from C2) ; from C11 we have a ===> a*a then a ===> b*a (transitivity). Besides a*b ===> a (from C1) ; it results that a*b=a. Reciprocally if a*b=a then a ===> a*b (definition of =) ; since a*b ===> b from C1, then by transitivity a ===> b.

We immediately infer the equality a+b=b from the relation a*b=a (from C13), and reciprocally.

This theorem says that if the formula a ===> b is verified then the description a conveys a more "specific" information than the description b does. The relation ===> is inferential and always turns an information into a more general one ; it is an inferential operation of deductive type. From this we can show that the formula Λ ===> a is always true ∀a∈∆ ; the empty description Λ can be interpreted as the always false description, i.e. Λ cannot characterise any individual. Finally this theorem allows us to show that every described descriptive term generated from n undescribed descriptive terms is more specific than the addition of these n undescribed descriptive terms taken separately.

Example :

Consider the following system of rewriting rules :

$$(S)\quad \begin{cases} U \dashrightarrow \Lambda \text{ (rule of decomposition)} \\ ISA(COLOR,\$(x,ISA(SHADE,y))) \dashrightarrow ISA(SHADE,y) \end{cases}$$

From C4 and the previous theorem we can prove

ISA(COLOR,$(BLUE,ISA(SHADE,LIGHT)))===>ISA(COLOR,BLUE)*ISA(SHADE,LIGHT)

But we cannot prove the symetrical relation :

ISA(COLOR,BLUE)*ISA(SHADE,LIGHT)===>ISA(COLOR,$(BLUE,ISA(SHADE,LIGHT)))

4.0. SEMANTIC CHARACTERISATION OF S_Δ.

Definition. We call interpretation of the formal system $S_\Delta = \{\mathcal{A}, \Delta, \Delta_T, ===>\}$ the structure $\mathcal{M} = \{D, \mathcal{C}\}$ whose elements D and $\mathcal{C}$ respectively represent a set and a function to be defined.

4.1. Definition of D.

D is a non empty set called the interpretation range of S_Δ. The set D is the union of two sets D_I and D_C : the elements of D_I correspond to symbols of individuals belonging to the set $\mathcal{I}$, and the elements of D_C correspond to symbols of concepts.

These two sets have the two following properties : $D_C \subset \mathcal{P}(D_I)$ ($\mathcal{P}(D_I)$ denotes the power set of D_I) because every concept may be interpreted as a set of individuals ; and $D_C \subset D_I$ because every concept may be interpreted as an individual.

4.2. Definition of $\mathcal{C}$.

$\mathcal{C}$ is a correspondance function which associates to each component of S_Δ its interpretation in the range D.

(1) Interpretation of individuals

Each individual x is associated with its interpretation, noted $\mathcal{C}(x)$, belonging to the set D_I.

(2) Interpretation of concepts

The interpretation of any concept A, regarded as a set of individuals, is defined as follows :

$$\mathcal{C}(A) \in D_C \subset \mathcal{P}(D_I)$$

(3) Interpretation of information without extension

Every feature t representing an information without extension (see § 2.1.)

is interpreted as the set of the interpretations of the individuals characterised by this feature. So every information without extension t is interpreted as a part of the set D_I :

$$\mathcal{C}(t) \in \mathcal{P}(D_I)$$

As every feature is either an information without extension, or a concept, or an individual (see § 2.1.), then the set of features is interpreted by the set T such that :

$$T = D_I \cup \mathcal{P}(D_I)$$

(4) Interpretation of the functional symbol $ (see § 2.1.)

When the local description is represented by the empty description Λ, the functional symbol $ is interpreted as the identity function :

$$\mathcal{C}(\$(t,\Lambda)) = \mathcal{C}(t)$$

When the local description is not represented by the empty description Λ, the features locally described can only be either an information without extension or a concept (see § 2.1.). Nevertheless the interpretation of the functional term $\$(t,op_n(C,t_\ell))$ remains the same whatever the type of the feature t because it is always interpreted as a set of individuals (see § 4.2., (2) and (3)).

We define the interpretation of the descriptive term $op_n(C,t_\ell)$ as the set of interpretations of the individuals characterised by the descriptive term.

In other words, the interpretation of any couple <class,operator> of the form $<C,op_n>$ introduces a mapping of T^n into $\mathcal{P}(D_I)$:

$$\mathcal{C}(<C,op_n>) : \quad T^n \dashrightarrow \mathcal{P}(D_I)$$

with $\quad \mathcal{C}(<C,op_n>)\ (\mathcal{C}(t_\ell)) \in \mathcal{P}(D_I)$

Hence the functional term $\$(t,op_n(C,t_\ell))$ is interpreted as the subset of the interpretations of the individuals belong to the set $\mathcal{C}(t)$, characterised by the descriptive term $op_n(C,t_\ell)$.

More precisely, the functional symbol $ is interpreted as the operation of intersection :

$$\mathcal{C}(\$(t,op_n(C,t_\ell))) = \mathcal{C}(t) \cap \mathcal{C}(<C,op_n>)\ (\mathcal{C}(t_\ell))$$

Note that if $\mathcal{C}(t)$ is always an element belonging to D_C when t is a concept, this is no longer true in the general case for $\mathcal{C}(\$(t,op_n(C,t_\ell)))$. In other words, $\mathcal{C}(\$(t,op_n(C,t_\ell)))$ is a "homogenous" set of individuals in that it is included in the concept $\mathcal{E}(t)$; but this set does not form a proper concept explicitly represented by a symbol in ARCHES. Note that the interpretation of the functional symbol \$ does not introduce the notion of evolution. In other words, the local descriptions allowing the characterisation of features do not evolve independently of the main descriptions in which they play a role. It is the elements of description characterising the individuals which globally evolve (see (5) and (6) above), and this is consistent with the syntactic definition of the connectors o and ⊛ (see § 3.).

(5) Interpretation of the descriptive terms which form the base $\mathcal{L}$ of Δ (see § 2.2.).

The enumerable set Δ of descriptions is mainly built from the enumerable set $\mathcal{L}$ of descriptive terms. Every element of $\mathcal{L}$ is of the form $op_n(C,\#(t_1,\ldots,t_i,\ldots,t_n))$ in which op_n is an n-ary operator, C a class and $\forall i\ 1\leqslant i\leqslant n\ t_i$ is a described or undescribed feature (see § 2.1.). We define the interpretation of the elements belonging to $\mathcal{L}$ in respect to the modalities of evolution of individuals.

The modalities of evolution of individuals are determined from the definition of a set Ω isomorphic to N. Every element of Ω is called a state. The states indicate the different transformations which may occur on the descriptions characterising the individuals. With each state which expresses the descriptive situation of the individuals at a given moment, we associate an integer. In general, a state may generate new states which themselves evolve and so on. In our model of interpretation, we suppose that evolution is a determinist process because Ω is isomorphic to N (see § 3.3. : condition C16) ; by this we mean that each possible state can evolve towards one and only one following state. So, from an initial state E_0 the interpretations of the descriptions characterising the individuals in the state E_0 (see also (6)) evolve in such a way that the transformations they undergo generate a linear sequence of states $E_1, E_2, \ldots E_n, \ldots$ organised by the structure $\mathcal{M}$, organisation similar to the one defined by Kripke for modal logic [14].

The interpretation of a descriptive term belonging to $\mathcal{L}$ is defined as the set of interpretations of individuals characterised by this descriptive term ; and these interpretations take account of the evolution of the individuals which will be expressed by means of the set N in accordance with the definition of Ω.

In other words the interpretation of any couple <class,operator> of the form $<C,op_n>$ maps T^n into $(N \dashrightarrow \mathcal{P}(D_I))$:

$$\mathcal{E}(<C,op_n>) : T^n \dashrightarrow (N \dashrightarrow \mathcal{P}(D_I))$$

with
$$\mathcal{E}(op_n(C,\#(t_1,..,t_n)))(i) = \mathcal{E}(<C,op_n>)(\mathcal{E}(t_1),\ldots\mathcal{E}(t_n))(i) \in \mathcal{P}(D_I)$$

The i^{th} state is formed by the union of sets of the form $\mathcal{E}(op_n(C,\#(t_1,\ldots,t_n)))(i)$ for all interpretations of couples <class,operator>. In particular if we have $\mathcal{E}(op_n(C,\#(t_1,..,t_n)))(i)=\emptyset$ in the interpretation of the couple $<C,op_n>$ then there is no individual characterised by the descriptive term $op_n(C,\#(t_1,..,t_n))$ in the state i.

(6) Interpretation of the descriptions (see § 2.2.).

Every interpretation a is interpreted as the set of the interpretations of the individuals characterised by a : more precisely, it is interpreted as a mapping of N into $\mathcal{P}(D_I)$ so as to take account of the evolution of individuals :

$$\Delta \dashrightarrow (N \dashrightarrow \mathcal{P}(D_I))$$

with
$$\mathcal{E}(a)(i) \in \mathcal{P}(D_I)$$

$\mathcal{E}(a)(i)$ determines the set of interpretations of individuals characterised by the description a in the state i.

We denote by $\mathcal{E}_0$ the mapping which defines the interpretation of descriptions in the initial state E_0 :

$$\mathcal{E}_0(a)=\mathcal{E}(a)(0)$$

The formation rules of the descriptions from the connector $*$, $+$, $\neg$, o and ⊛ and the definition of $\mathcal{E}_0$ contribute to the definition by iteration of the correspondence function $\mathcal{E}$ through the following rules (see § 3.2. and 3.3.)

1 $\mathcal{E}(a*b)(n) = \mathcal{E}(a)(n) \cap \mathcal{E}(b)(n)$

2 $\mathcal{E}(a+b)(n) = \mathcal{E}(a)(n) \cup \mathcal{E}(b)(n)$

3 $\mathcal{E}(\neg a)(n) = \complement\mathcal{E}(a)(n)$ ($\complement$: complement in D_I)

4 $\mathcal{E}(\circ a)(n) = \mathcal{E}(a)(n+1)$

5 $\mathcal{E}(⊛ a)(n) = \bigcup_{p=0}^{\infty} \mathcal{E}(a)(n+p)$

6 $\mathcal{E}(\Lambda)(n) = \emptyset$

The first three rules correspond to the classical and intuitive interpretation of the connectors $*$, $+$ and $\neg$.

Rules 4 and 5 determine the semantics of the evolution of descriptions : the interpretation of the connector $\circ$ (rule 4) shows that this latter expresses the evolution between two consecutive states E_i and E_{i+1} ; the interpretation of the connector $\oplus$ (rule 5) expresses the evolution between two successive but not necessarily consecutive states E_i and $E_j (j \geqslant i)$. From rule 5 we may also deduce that $\mathcal{E}(\oplus\oplus a)(n) = \mathcal{E}(\oplus a)(n)$, which proves the transitivity of the connector $\oplus$. Finally, we may remark that unlike the connector $\circ$, the connector $\oplus$ assures that the present is a part of the future (because $j \geqslant i$).

In an other way rule 6 shows that, whatever the state E_i, there is no individual x characterised by the empty description (see § 3.4.).

The connector $\oplus$ must be introduced in order to derive descriptions in which the sequence of states is not explicit (the aim is to look for the existence of at least one state in which an individual x has a given group of properties ; this connector is similar to the existential quantifier in classical logic).

5.0. VALIDITY OF THE RELATION OF DEDUCTION ===> UNDER ANY INTERPRETATION $\mathcal{M}$.

Definition 1 : Given two descriptions a and b of Δ, the formula a ===> b is defined to be valid if for every interpretation $\mathcal{M}$ and for all i, $\mathcal{E}(a)(i)$ is included in $\mathcal{E}(b)(i)$:

$$\mathcal{E}(a)(i) \subset \mathcal{E}(b)(i)$$

Definition 2 : The deduction relation ===> is valid if all the component formulas are valid.

Theorem . The deduction relation ===> is valid.

Proof. To prove the validity of ===>, it is necessary to prove the validity of every condition satisfying this relation.

The validity of conditions C1 to C4 and C9 to C15 which determine the relations between the connectors $*$, $+$ and $\neg$, and the relation ===>, is established immediately because the quotient set $\Delta/=$ is a distributive and complemented lattice (see § 3.3.).

Let us prove the validity of conditions C5 to C8 and C15 to C21 (see § 3.2. and 3.3.) :

Proof of C5 : $\mathcal{E}(\oplus a)(i) = \bigcup_{p=0}^{\infty} \mathcal{E}(a)(i+p)$ from rule 5 ;
hence $\mathcal{E}(a)(i) \subset \mathcal{E}(\oplus a)(i)$, which proves the validity of the formula a ===> $\oplus$a.

Proof of C6 : $\mathcal{E}(\oplus a)(i) = \bigcup_{p=0}^{\infty} \mathcal{E}(a)(i+p)$; or
$\mathcal{E}(\circ a)(i) = \mathcal{E}(\circ a)(i+1)$, which proves the formula a ===> $\oplus$a.

Proof of C7 : simultaneously we have

$$\mathcal{E}(\oplus a)(i) = \bigcup_{p=0}^{\infty} \mathcal{E}(\circ a)(i+p) = \bigcup_{p=0}^{\infty} \mathcal{E}(a)(i+1+p)$$

and

$$\mathcal{E}(\circ\oplus a)(i) = \mathcal{E}(\oplus a)(i+1) = \bigcup_{p=0}^{\infty} \mathcal{E}(a)(i+1+p).$$

consequently the formula $\circ\oplus$a ===> $\oplus\circ$a is verified.

Proof of C8 : we have simultaneously :

$$\mathcal{E}(\oplus(a+b))(i) = \bigcup_{p=0}^{\infty} \mathcal{E}(a+b)(i+p),$$

and

$$\mathcal{E}(\oplus a + \oplus b)(i) = \mathcal{E}(\oplus a)(i) \cup \mathcal{E}(\oplus b)(i) = \bigcup_{p=0}^{\infty} [\mathcal{E}(a)(i+p) \cup \mathcal{E}(b)(i+p)] = \bigcup_{p=0}^{\infty} \mathcal{E}(a+b)(i+p).$$

consequently the formula $\oplus$(a+b) ===> $\oplus$a+$\oplus$b is verified.

Proof of C16 : to prove the validity of the formula $\circ\neg a = \neg\circ a$ it is necessary to prove that $\forall i \ \mathcal{E}(\neg\circ a)(i) = \mathcal{E}(\circ\neg a)(i)$.
From rule 4 we have $\mathcal{E}(\circ\neg a)(i) = \mathcal{E}(\neg a)(i+1)$; thus

$$\mathcal{E}(\neg a)(i+1) = \complement\mathcal{E}(a)(i+1) \text{ (rule 3).}$$

Consequently $\mathcal{E}(\circ\neg a)(i) = \complement\mathcal{E}(\circ a)(i)$ rule 4,
whence : $\mathcal{E}(\circ\neg a)(i) = \complement\complement\mathcal{E}(\neg\circ a)(i)$ or

$$\mathcal{E}(\circ\neg a)(i) = \mathcal{E}(\neg\circ a)(i).$$

Proof of C17. $\mathcal{E}(\circ(a*b))(i) = \mathcal{E}(a*b)(i+1)$ rule 4 ;
whence $\mathcal{E}(\circ(a*b))(i) = \mathcal{E}(a)(i+1) \cap \mathcal{E}(b)(i+1)$ rule 1.
Consequently $\mathcal{E}(\circ(a*b))(i) = \mathcal{E}(\circ a)(i) \cap \mathcal{E}(\circ b)(i)$, so $\mathcal{E}(\circ(a*b))(i) = \mathcal{E}(\circ a * \circ b)(i)$

Proof of C18. Obvious

Proof of C19. Obvious because $\forall i, \mathcal{E}(\Lambda)(i) = \emptyset$.

Proof of C20 and of C21. Obvious from definition 1 and rules 4 and 5. Consequently all the conditions satisfied by the relation ===> are valid. From definition 2 we can deduce that the relation ===> is valid.

6.0. METHOD OF RESOLUTION FOR THE FORMULA H===>C.

6.1. Some propositions.

From conditions C5 and C6 and from the transitivity of the relation ===> we easily establish the three following propositions :

$$\begin{cases} \neg\circledast\neg a \Longrightarrow a \\ \neg\circledast\neg a \Longrightarrow o a \\ \neg\circledast\neg a \Longrightarrow \circledast a \end{cases}$$

These propositions show that the complex operator "$\neg\circledast\neg$" may be interpreted as an universal quantification in the framework of time expression (analogous to interpretation with $\exists$ and $\forall$ in classical logic : $\neg\exists\neg \equiv \forall$) ; thus we may express the permanence of descriptions whatever the state of knowledge : if $\exists i \in N$ such that $x \in \mathcal{C}(\neg\circledast\neg a)(i)$ then $\forall j \quad x \in \mathcal{C}(a)(j)$ and conversely.
From the axiomatisation of the relation ===> we can also establish the three propositions :

$$\begin{cases} o(a+b) = oa+ob \\ \circledast(a+b) = \circledast a+\circledast b \\ \circledast(a*b) \Longrightarrow \circledast a*\circledast b \end{cases}$$

The converse proposition of the last line is not verified : this is easy to prove from the semantic interpretation of the formula $\circledast a*\circledast b \Longrightarrow \circledast(a*b)$ (see § 4.2., (6)). So, in order to represent the description in a normal form expressed as an addition of disjonctions,we make the hypothesis that descriptions of the type $\circledast(a*b)$ and $\circledast\neg(a+b)$ are not formulas of the formal system S_Δ.
Finally we proved, provided that the induction hypothesis is verified, six propositions expressing the semantic relations between the connectors $\circledast$ and o in any state n :

$$\begin{cases} \forall n \in N \quad o^n \circledast a \Longrightarrow \circledast a \\ \forall n \in N \quad o^n a \Longrightarrow \circledast a \\ \forall n \in N \quad o^n o a \Longrightarrow o\,o a \\ \forall n \in N \quad \circledast o^n a \Longrightarrow \circledast a \\ \forall n \in N \quad \neg\circledast\neg a \Longrightarrow o^n a \\ \forall n \in N \quad \neg\circledast\neg a \Longrightarrow \circledast o^n a \end{cases}$$

6.2. Decision procedure for the relation ===>.

We have elaborated a decision procedure in order to solve the following problem : given a couple of descriptions (H,C), determine if the relation H===>C is verified. Obviously, this problem is of prime importance in order to demonstrate theorems of the ARCHES system (see § 1.). The definition of this procedure based on the formal properties of the relation ===>, uses the methodology of problem solving by decomposition and construction of the corresponding AND/OR graphs. More precisely, this procedure builds up two AND/OR trees $\mathcal{A}_H$ and $\mathcal{A}_C$ associated with the hypothesis H and the conclusion C, the modalities of construction being determined from derivation schemes of the description and of their properties. Then, the procedure builds up the AND/OR graph $\mathcal{A}_R$ by "appending" to each terminal of $\mathcal{A}_H$, the tree $\mathcal{A}_C$ without its root and tries to validate the relation H===>C by searching for at least one valid AND sub-tree of $\mathcal{A}_R$ through the utilisation of the algorithm of decidability for the relation --->* (see § 3.1.).

7.0. CONCLUSION.

The conception of the formal system S_Δ, whose logical properties have been systematically studied, appears as a methodological and theoretical contribution to the study of evolution in knowledge. We essentially focused on the representation and treatment of "future" by means of connectors whose semantic characterisation shows that data evolution is deterministic, i.e. the transformations they undergo generate a linear sequence of successive states organised in accordance with the model of interpretation of S_Δ.

This formal system is one of the component modules of the ARCHES system, symbolic system of representation and treatment of knowledge whose utilisation in domains of empirical knowledge such as the human sciences, is potentially of great importance.

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