SYNTHETIC DESIGN OF POLICY DECISIONS IN SYSTEM DYNAMICS MODELS - A Modal Control Theoretic Approach[1]

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ABSTRACT

Researchers and practitioners in System Dynamics usually follow a trial-and-error process to design new policy decisions. They mainly use causal loop diagrams for this purpose. However, these diagrams portray 'direction' of influence and not its 'strength'. Therefore, the process of policy design becomes time consuming especially for a beginner and those working with insufficient computer facility. This paper presents an alternative approach for policy design using Modal Control Theory. In this approach, policy variables are treated as control variables by delinking them from other variables. This generally leads to greatly simplified models which are free from many nonlinearities. Providing that this reduced system is linear and controllable, it is possible to synthetically generate control policies by modal control theory to ensure any prescribed degree of stability. These theoretical control policies then can be used to design realistic policy decisions. The Chapter-8 problem of Coyle [1] is used here as a test example. It is shown that policies designed in the light of modal control theoretic results are superior to those suggested by Coyle.

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1 Part of this work was carried out when the first author was a visiting fellow at the System Dynamics Research Group at the University of Bradford, England, under a Commonwealth Scholarship programme. He is particularly grateful to Dr. R. G. Coyle for his many useful suggestions during the early phase of the work.

INTRODUCTION

The current practice in designing policies in system dynamics models is rather unstructured. One usually follows a 'trial-and-error' procedure for policy design. Realistic policies are tested. Influence diagrams are used as aid to analyse and understand system behaviour. Previous policies are modified and tested again. This process continues till acceptable system behaviour is obtained. Generally this final policy is presented. Though not evident to an outsider, surely, anybody carrying out this exercise is aware of the agony which he undergoes to arrive at the final policy.

Influence diagrams or causal loop diagrams have great merit in analysing system behaviour. But the fact that they indicate direction of relationships and not their strength limits their full usefulness.

Modal control theory provides an attractive method with which it is possible to synthetically design control variables by linear feedback of state or output vector in a linear system so that eigenvalues may be assigned to any desired locations. This makes it possible for the closed loop system to generate the desired behaviour.

The assumption of linearity in modal control theory is quite restrictive. However, it is shown in this paper that synthetically generated control variables provide

important guidelines to design realistic policies which provide more desirable system behaviour than that which is obtained from policies set intuitively. The Chapter-8 problem of Coyle [1] has been used as a test example in this paper.

VIEWING SYSTEM DYNAMICS MODELS FROM A DIFFERENT PERSPECTIVE

Basic variables in system dynamics models are levels and rates. Level equations are very easy to write. But developing rate equations require both skill and understanding of behaviour of system components. This makes the designing task quite difficult.

An alternative method for policy design is to first delink the policy variables from other variables. This helps in simplifying the model structure and in studying 'controllability' of the system. One can then apply the procedure of modal control theory to design policy decisions to achieve desired system behaviour. Delinking rate variables from other variables converts them to 'control variables'. One can, therefore, dispense with almost all the auxiliary variables, and the resultant system is simple and in most cases linear.

The basic system considered in Chapter-8 by Coyle is represented in the influence diagram shown in Fig. 1.

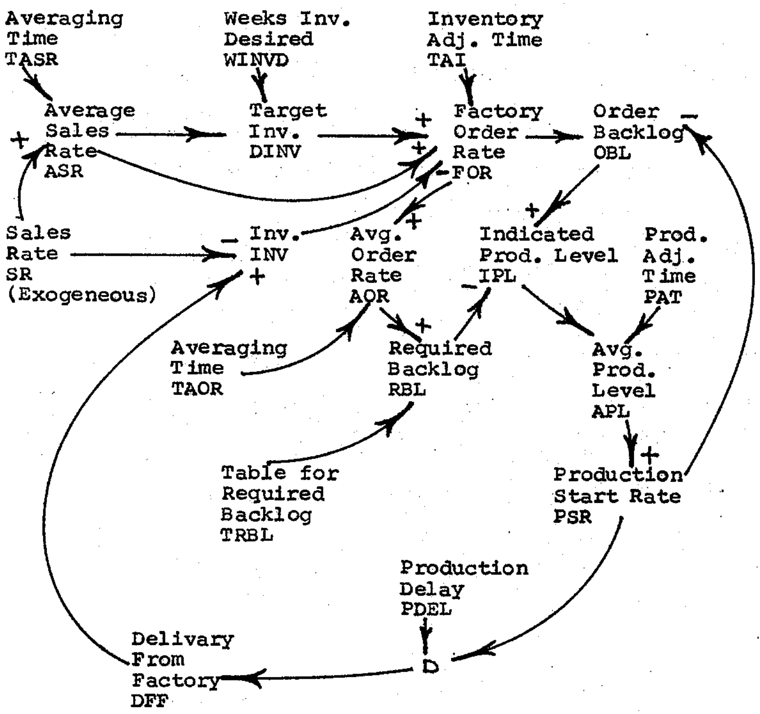

Fig. 1 Influence Diagram of the Basic System

The basic model is an eighth-order system with two pure integrations (Inventory, Order Backlog), three smoothed levels (Average Order Rate, Average Production Level, Average Sales Rate) and three levels inside the third-order delay. The model also consists of only one table function

nonlinearity (RBL(t) = f (AOR (t))). The policy variables of the system are Factory Order Rate (FOR) and Production Start Rate (PSR).

A close look at the diagram indicates that the three smoothed levels and the nonlinearity have been introduced only to define the rate variables FOR and PSR. Since a policy design problem requires the policy variables to be defined in new ways, one can neglect the old links which are used to define FOR and PSR. The reduced basic system then takes the form shown in Fig. 2

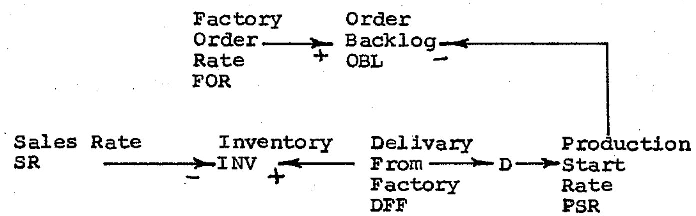

Fig. 2 The Reduced Basic System

It may be noted that the auxiliary variables (Desired Inventory, Indicated Production Level and Required Backlog), and the three smoothed level have now been omitted resulting in a very simplified system which is linear. If we assume the delay to be of first-order, then the system is of third-order. The exogeneous variable is Sales Rate (SR) and the control variables (policy variables) are Factory

Order Rate (FOR) and Production Start Rate (PSR). It may be noted that although DFF is a rate, it is not a policy variable since it is determined completely be the delay process of the system; the variable is, therefore, retained. Fig. 3 presents the analogue representation of this reduced system.

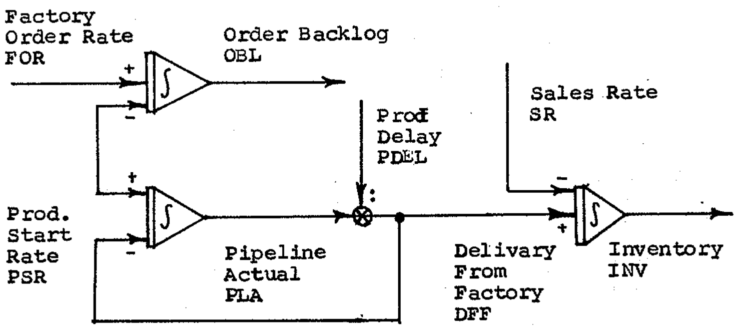

Fig. 3 Analogue Representation of the Reduced Basic System

Following standard procedure, one obtains the following state differential equation :

$$\underline{x}(t) = A\underline{x}(t) + B\underline{u}(t) + \underline{C}z(t) \qquad \ldots(1)$$

where,

$$\underline{x} = \begin{bmatrix} PLA \\ OBL \\ INV \end{bmatrix}, \quad A = \begin{bmatrix} -\frac{1}{PDEL} & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{PDEL} & 0 & 0 \end{bmatrix}$$

$$\underline{u} = \begin{bmatrix} FOR \\ PSR \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 1 & -1 \\ 0 & 0 \end{bmatrix}, \quad \underline{C} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, \quad Z = SR$$

$\underline{x}$ is the vector of state variables, $\underline{u}$ the vector of control variables, and Z the uncontrollable exogeneous variable.

CONTROLLABILITY OF THE REDUCED SYSTEM

It is obvious from Fig. 2 and Fig. 3 that, in this reduced format, FOR has no influence over PLA and INV, whereas PSR firectly affects OBL and PLA, and indirectly affects INV also. Thus all the three models are controllable by the two input variables.

One may also follow the standard mathematical procedure to derive the above-mentioned result.

Eigenvalues of the A-matrix are

$$\lambda_1 = -\frac{1}{PDEL}, \qquad \lambda_2 = \lambda_3 = 0.$$

In the presence of confluent eigenvalues, one forms the Jordan Matrix, J, with two blocks :

$$J = J_1\left(-\frac{1}{PDEL}\right) \oplus J_2(0) = \left[\begin{array}{c|cc} -\frac{1}{PDEL} & 0 & 0 \\ \hline 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

The modal matrix, V, of A^T (A transposed) must be such that it should satisfy

$$A^T V = V J^T \qquad \ldots(2)$$

Assuming $V = \left\{ v_{ij};\ i,\ j = 1,2,3 \right\}$, Eqn. (2) gives

$$v_{13} = v_{21} = v_{23} = v_{31} = v_{33} = 0$$

$$v_{12} = v_{32}$$

Other elements of V may be arbitrarily chosen :

$$v_{11} = v_{12} = v_{32} = 1$$

$$v_{22} = 2$$

Thus,

$$V = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad \ldots(3)$$

The mode controllability matrix, P, is then given by

$$P = V^T B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \hdashline 2 & -1 \\ 0 & 0 \end{bmatrix} \qquad \ldots(4)$$

The first row corresponds to the first Jordan block while the second and third rows correspond to the second Jordan block. Following standard controllability criteria developed for the case of confluent eigenvalues, one can say, on inspection of P matrix, that

a) the first Jordan block is uncontrollable by the first input variable, FOR, but controllable by the second input variable, PSR

b) the second Jordan block is only partially controllable by both the input variables, FOR and PSR.

These results conform to our earlier observations.

SYNTHETIC DESIGN OF POLICIES FOR STABILITY

The reduced system depicted in Eqn. (1) may be written in a different fashion :

$$\frac{d}{dt}\begin{bmatrix} PLA(t) \\ OBL(t) \\ INV(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{PDEL} & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{PDEL} & 0 & 0 \end{bmatrix} \begin{bmatrix} PLA(t) \\ OBL(t) \\ INV(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} FOR(t) \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} PSR(t) \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} SR(t) \qquad \ldots(5)$$

The control variables, FOR and PSR, are designed in modal control theory as linear feed back of state variables. A sequential design is presented here wherein the input variables are designed one after another in sequential manner treating the system as a single-input system at each stage. The procedure given in Porter and Crossley [2] is followed here.

DESIGN OF FACTORY ORDER RATE (FOR)

The mode controllability vector corresponding P-matrix given in Eqn. (4) :

$$\underline{p} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

Since the first element of the second Jordan block is non-zero ($p_1^{(2)} = 2$), FOR can control the first mode of the second Jordan block.

Using modal control theory for the case of confluent eigenvalues [ref. 2] it may be shown that

$$FOR(t) = K_1^{(2)}\, \underline{v}_1^{(2)^T}\, \underline{x}(t) \qquad \ldots(6)$$

where,

$K_1^{(2)}$ = gain of the proportional controller

$v_1^{(2)}$ = eigenvector corresponding to the first mode of the second Jordan block. It is also the second column in the mode controllability matrix, V, defined in Eqn.(3).

Further,

$$K_1^{(2)} = \frac{\mu_1^{(2)} - \lambda_1^{(2)}}{p_1^{(2)}} \qquad \ldots(7)$$

where,

$\mu_1^{(2)}$ = desired eigenvalue for the first mode in the second Jordan block

and $\lambda_1^{(2)}$ = Eigenvalue of the A matrix which corresponds first mode in the second Jordan block.

Obviously,

$$\lambda_1^{(2)} = \lambda_2 = 0$$

$$\underline{v}_1^{(2)^T} = [1 \quad 2 \quad 1]$$

$$p_1^{(2)} = 2.$$

Assuming that the designer wishes to have the new eigenvalue location at -2,

$$\mu_1^{(2)} = -2$$

Hence, using Eqn. (7),

$$K_1^{(2)} = -1$$

Finally, Eqn. (6) gives

$$FOR(t) = -PLA(t) - 2OBL(t) - INV(t) \qquad \ldots(8)$$

Using Eqn. (8), the state differential equation (5) may be as below :

$$\frac{d}{dt}\begin{bmatrix} PLA(t) \\ OBL(t) \\ INV(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{PDEL} & 0 & 0 \\ -1 & -2 & -1 \\ \frac{1}{PDEL} & 0 & 0 \end{bmatrix} \begin{bmatrix} PLA(t) \\ OBL(t) \\ INV(t) \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} PSR(t) + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} SR(t) \qquad \ldots(9)$$

It may be checked that the eigenvalues of the new plant matrix are

$$\mu_1 = -\frac{1}{PDEL}, \quad \mu_2 = -2, \quad \mu_3 = 0.$$

Thus it is ensured that only the second eigenvalue has changed, the other two remaining unchanged.

DESIGN OF PRODUCTION START RATE (PSR)

The plant matrix, A_1, of the system defined in Eqn. (9) contains distinct eigenvalues, as noted earlier. The modal matrix, W , of such a system must satisfy the following

$$A_1^T W = W\Lambda \qquad \text{...(10)}$$

where Λ is the eigenvalue matrix containing the eigenvalues of A_1 in its principal diagonal and zeroes elsewhere. Eqn. (10) is satisfied by

$$W = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix} \qquad \text{...(11)}$$

The mode controllability vector is given by

$$\underline{q} = W^T \underline{b} \qquad \text{...(12)}$$

$\underline{b}$ is the coefficient vector of the control variable, PSR given by $\underline{b}^T = [1 \quad -1 \quad 0]$

Using Eqn. (12)

$$\underline{q} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Since no element of this vector is zero, PSR can control all the three modes. However, since the second vector has already been controlled by the other control variable, FOR, PSR may be designed to alter the first and the third modes.

Since A_1 has distinct eigenvalues, one has to follow the modal control theory for the case of distinct eigenvalues where it is desired to shift more than one eigenvalues. Following the same, one obtains

$$PSR(t) = \sum_{j=1}^{2} \left[\frac{\prod_{K=1}^{3} (\rho_K - \mu_j)}{q_j \prod_{\substack{k=1 \\ k \neq j}}^{3} (\mu_K - \mu_j)} \underline{w}_j^T \right] \underline{x}(t) \qquad \text{...(13)}$$

where, ρ's are the eigenvalues of the closed loop plant matrix and $\underline{w}_j$ is the jth eigenvector, or the jth column of the W-matrix.

Assuming $\rho_1 = \rho_3 = -3$ (ρ_2, of course, equals $\mu_2 = -2$), Eqn. (13) gives

$$PSR(t) = -\frac{35}{6} PLA(t) - 54\, INV(t) \qquad \text{...(14)}$$

It may be checked that the closed loop plant matrix has indeed the desired eigenvalues.

If it was wanted to shift also the second eigenvalue to $\rho_2 = -3$, then the following is obtained :

$$PSR(t) = -\frac{415}{66} PLA(t) + \frac{6}{11} OBL(t) - \frac{888}{11} INV(t) \quad ...(15)$$

DISCUSSION ON THE SYNTHETICALLY DESIGNED POLICIES

Eqn. (8) gives the expression for Factory Order Rate, FOR. It indicates that as level in the production delay, order backlog and inventory rise, factory order rate must be cut back and vice-versa. This seems to be quite realistic. A higher weightage to order backlog is only expected since it is directly affected by factory order rate. However, no definite conclusion can be drawn from the comparison of their coefficient values, since certain elements of the eigenvectors have been selected arbitrarily.

Eqn. (14) gives the expression for Production Start Rate, PSR. One immediately notices in Eqn. (14) the absence of an order backlog term, and heavy negative dependence of production start rate on inventory and pipeline orders in the production delay.

Eqn. (15) gives an alternative expression for PSR when an additional second mode was also controlled. An order backlog term appears in this expression, but its coefficient is negligibly small compared to the other

coefficients. The coefficients of PLA and INV are now more negative than the corresponding coefficients in Eqn. (14). This is possibly to offset the disturbing features arising out of the new design link between OBL and PSR.

Thus the important features of production start rate are its almost total independence of the order backlog and its heavy dependence on inventory and pipeline content actual. Such a design makes the order backlog redundant and questions the utility of the system of creating a factory order rate. Such structural changes are indeed very welcome features of the design. Also it may be mentioned here that no new policy attempted by Coyle [1] had the feature of heavy dependence on inventory and pipeline content actual.

DESIGNING REALISTIC POLICY DECISIONS

Factory order rate(FOR) and Production Start rate are designed by modal control theory and given in equation Eqn (8) and Eqn. (14) (and/or Eqn. (15)) respectively. Unfortunately, unlike a physical system, policies cannot be designed and used in such a straight forward and deterministic way. Therefore, realistic policies can be designed only intuitively after taking cognizance of Eqn. (8) and Eqn. (14) (or (15)).

Considering Eqn. (8) and Eqn. (14) the following equations for FOR and PSR are suggested.

POLICY - I

FOR = ASR + (DINV - INV)/TAI + (RBL - OBL)/TACI + (DPLA - PLA)/TAL ...(16)

and

PSR = ASR + (DINV - INV)/TAIP + (DPLA - PLA)/TALP ...(17)

the parameter values are suggested to be the following

TAI = 12
TACI = 6
TAL = 12
TAIP = 4
TALP = 4

DPLA and PLA are desired pipeline content actual and pipeline content actual respectively. These are defined in the usual way.

Alternatively, one may retain the original policy equations as formulated initially as a good representation of reality but use multipliers defined in a way such that it takes into consideration the modal control theory results. The following equations for FOP and PSR are suggested.

POLICY II

FOR = (ASR + (DINV - INV)/TAI)*(FOML)*(FOMB) ...(18)

and

PSR = (APL)(PMI)(PML) ...(19)

where,

FOML : Factory Order Rate Mult. from pipeline content in production delay

FOMB : Factory Order Rate Mult. from Order Backlog

PMI : Production start rate Mult. from Inventory consideration

PML : Production start rate Mult. from pipeline content in production delay.

These multipliers are defined by table functions shown in Fig. 4 through Fig. 7 respectively. The actual numerical values have been taken almost arbitrarily in these table functions. But the shapes of these functions are designed in consonance with Eqn. (8) and Eqn. (14). These multipliers may be assumed to be discouragement or encouragement factors.

OBL has not been used in designing PSR since its coefficient in Eqn. (15) is negligibly small.

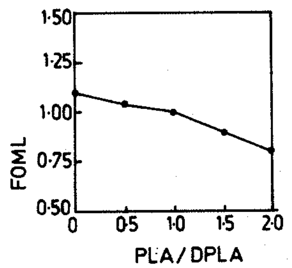

Figure-4

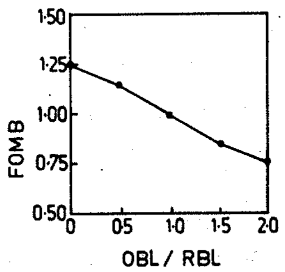

Figure-5

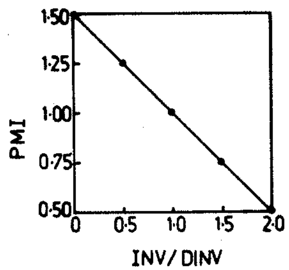

Figure-6

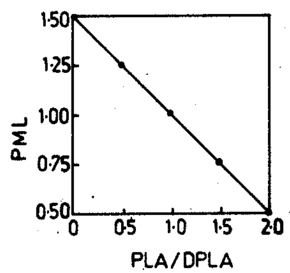

Figure-7

RESULTS AND DISCUSSION

The model has been simulated with the revised policies and the results are compared with the behaviour of the model obtained by Coyle [1] with his final revised policy.

Fig. 8 reproduces the model behaviour obtained by Coyle [1] . Fig. 9 and 10 show the model behaviour when the policy set - I and II are followed respectively when Sales rate (SR) is given step increase of 40 per cent at the 10th week. Same scale has been used for all the three figures for the sake of easy comparison. The model with policy set I and II was run using DYMOSIM - a Fortran based software package for simulating system Dynamics models which is developed at the Indian Institute of Technology [3] .

From the figures, it is quite evident that the revised policies (Figs. 9 and 10) give better results than those finally obtained by Coyle. It is observed that the variations of INV and PSR are much smoother for the newly designed policy sets than those for Coyle. Only FOR for Policy I shows an increase in its maximum value than that by Coyle. This has not affected the variation of PSR or INV since the PSR equation does not contain either FOR or OBL. Although the variables fluctuate a

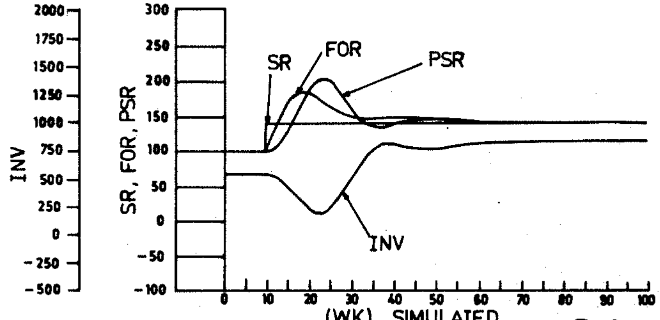

Fig-8

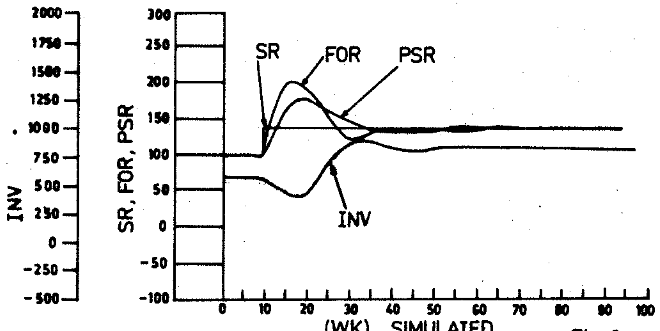

Fig-9

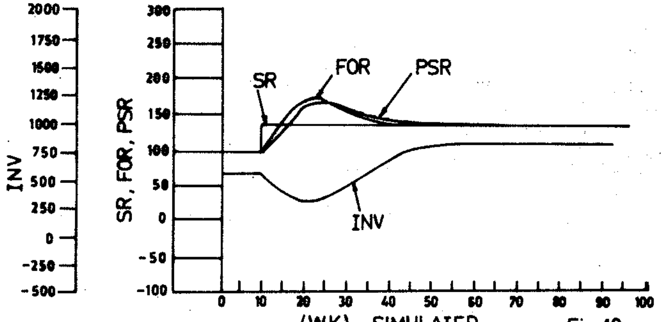

Fig-10

little, they settle down in almost at same time as that by Coyle.

Policy II does not show any fluctuating tendency and has the minimum setting times for all the variables. FOR and PSR also do not have high peak values. Noting that policy II is a modified version of the original policy (which has been shown to be undesirable by Coyle), we can say that the modification brought about by the results of the modal control theory can help in designing better policies.

Thus modal control theory can help in providing guidelines to design realistic policies which can give more desirable model behaviour then those obtained for policies set intuitively. However, this demands a knowledge the mathematical background of the theory and we hope that this is worth developing.

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