System Dynamics National Model Interest Rate Formulation: Theory and Estimation

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Abstract

Within the MIT System Dynamics National Model, the risk-free interest rate is determined jointly by the normal interest rate and by liquidity. The normal rate is the rate which agents believe would obtain under normal circumstances, in the absence of transitory pressures. The normal rate continually adjusts to new interest rate conditions. During times of deficient liquidity, agents will increase the risk-free rate above the normal rate. The converse also holds. The risk-free rate will continue to adjust until pressures in the system are relaxed. Estimation results support the national model theory of interest rate formation.

1. Introduction

The MIT system dynamics national model is a general disequilibrium simulation model of the U.S. economy. The model has been designed to investigate causes of and policies addressed to cyclical and secular variation of prices and economic activity (Forrester 1979).

This paper is concerned with the risk-free interest rate which is determined endogenously within the financial sector of the national model. In the national model the risk-free interest rate or yield determines the interest payments made on government debt.[1] The risk-free rate is also the foundation upon which other, risky rates are formed. Hence, in the model the risk-free rate influences, either directly or indirectly through the risky rates, investment and consumption decisions as well as the actual interest payments on the debt and bonds of producers and consumers.

This paper presents the theory underlying the equations which determine the risk-free interest rate in the national model, as well as empirical work in support of the theory.

2. Theory

The Normal Rate. The risk free rate is determined by two factors: The normal or underlying interest rate and the effect of excess or deficient liquidity in the economy.

$$RFR_t = NRFR_t * EFLR_t \qquad (1)$$

RFR - Risk-free rate
NRFR - Normal risk-free rate
EFLR - Effect of liquidity on risk-free rate

We shall consider each of these factors in turn, taking the normal rate first:

$$\dot{NRFR}_t = (RFR_t - NRFR_t)/TANR \qquad (2)$$

NRFR - Normal risk-free rate
RFR - Risk-free rate
TANR - Time to adjust normal rate
the dot (˙) denotes a time derivative

The normal risk-free rate is the rate which actors in the financial system regard as the underlying rate of interest, the rate which would hold if all transitory pressures, represented by EFLR in equation 1, were eliminated.

The normal rate is not a quoted rate. Nonetheless, it is more than a convenient fiction. Actors in the financial system may be aware of imbalances in their own or others' portfolios; they may be aware of pressures which shape interest rates. But, these pressures do not themselves reveal the value of the appropriate interest rate. Instead, people translate these apparent pressures into a modification of a rate (ie. the normal rate) which they sense would be reasonable in the absence of such pressures.

The normal rate is not constant. It adjusts to new interest rate environments: The normal rate in 1960, a time of low interest rates, presumably was quite different from the normal rate during the late seventies and early eighties when interest rates were genrally high. In the formulation above, the normal rate is continuously changing at a speed proportional to its distance from the current risk-free rate. The parameter TANR determines how quickly the normal rate adjusts to new interest rate conditions.

The notion of a normal rate dates at least from Keynes (1936) who suggested it was determined by past experience. Modigliani and Sutch (1966) suggest that Keynes' normal rate can be approximated by a moving average of past rates of interest.[2] Quite similarly, the normal rate of interest defined by equation (2) is an exponential average of past values of the risk-free rate.

Liquidity and Free Reserves. The normal rate is modified by pressures in the financial system to produce the risk-free rate. These "pressures" may be produced by imbalances between the supply of and demand for financial resources. However, the interplay of the forces of supply and demand cannot be experienced by any economic actor or observer directly. What is experienced directly is excess or deficient liquidity.

The desire to lend or to buy securities is prompted by an excess of liquid funds; a need to borrow or to sell securities is prompted by illiquidity (or its anticipation). A situation of net positive liquidity (over all agents) will produce a decrease in the interest rates. Bankers, for example, faced with large excess free reserves will be willing to shave the normal rate on loans in response to requests by customers. Similarly, a situation of net deficient liquidity will tend to boost the interest rate above normal.

In its most general interpretation one may view the aggregate amount of excess or deficient liquidity in the financial system as made up of (1) free reserves proper, that is the free reserves of institutions required by the Fed to maintain reserves, (2) the free reserves of other financial institutions and (3) the excess (or deficient) currency and demand deposits (or near-demand deposits) held by companies and the public at large. In the presence of excess liquidity, interest rates will be driven downward; that is, the actual risk-free rate will fall below the normal risk-free rate. In the presence of illiquidity the interest rate will rise above the normal risk free rate.

This argument may be considered a dynamic or disequilibrium interpretation of Keynes' 1936 argument that the equilibrium interest rate is that rate at which the quantity of money available in the economy equals the desired quantity of money. An implication of Keynes' argument is that a condition of excess (deficient) money must result in a fall (rise) in interest rate.

The measurement of net aggregate excess or deficient liquidity in the national model is made easier by the assumption that all economic agents must transact with the "financial intermediary" in order to adjust their portfolios. This assumption eliminates the need for separate financial links between each pair of sectors. Instead, all financial resources are accumulated by the financial intermediary and parcelled out to the sectors demanding funds. This simplification eliminates unnecessary structure while retaining the essence of credit supply and demand in a modern economy with centralized financial markets. Of more importance to the present discussion, however, this simplification allows the aggregate state of excess liquidity to be measured as free reserves of the financial intermediary.

To see this more clearly consider a case of excess liquidity in the household sector of the model. The household, desiring to hold less of its wealth as demand deposits, will purchase "bank bonds"[3]. Since "bank bonds" have a lower reserve requirement than deposits, this action increases the free reserves of the financial intermediary. Hence, excess liquidiy of the household is transformed into increased free reserves of the financial intermediary. The bank, seeking to lend out these free reserves will reduce the interest rate.

In the model, therefore, pressure from net liquidity over all agents is measured by free reserves (FR) of the financial intermediary:

$$EFLR = f(RAR) \quad (3)$$

$$RAR = FR/R \quad (4)$$

$$FR = R - DR \quad (5)$$

Where:

EFLR - Effect of liquidity on risk-free rate
RAR - Relative available reserves
FR - Free reserves
R - Reserves
DR - desired reserves
f - non-linear function

RAR, ratr than FR, is used as a measure of liquidity in the banking system in ordeto make the formulation insensitive to changes in scale. "f" is a non-incasing function which assumes a neutral value of 1 when RAR is zero; that iswhen desired reserves equal actual reserves.

Equilibum and Dynamic Implications. Equations 1 through 5 represent a theory how interest rates are formed. Specifically, liquidity in the system AR) determines the percentage deviation of the risk-free rate from the nol or underlying interest rate. The underlying interest rate, in turn, iconstantly moving toward the risk-free rate.

The press by which interest rates are formed is represented diagramatically in figu 1. The risk free rate is jointly determined by relative available reserve and by the normal rate. The normal rate, in turn is determined by the ri free rate.

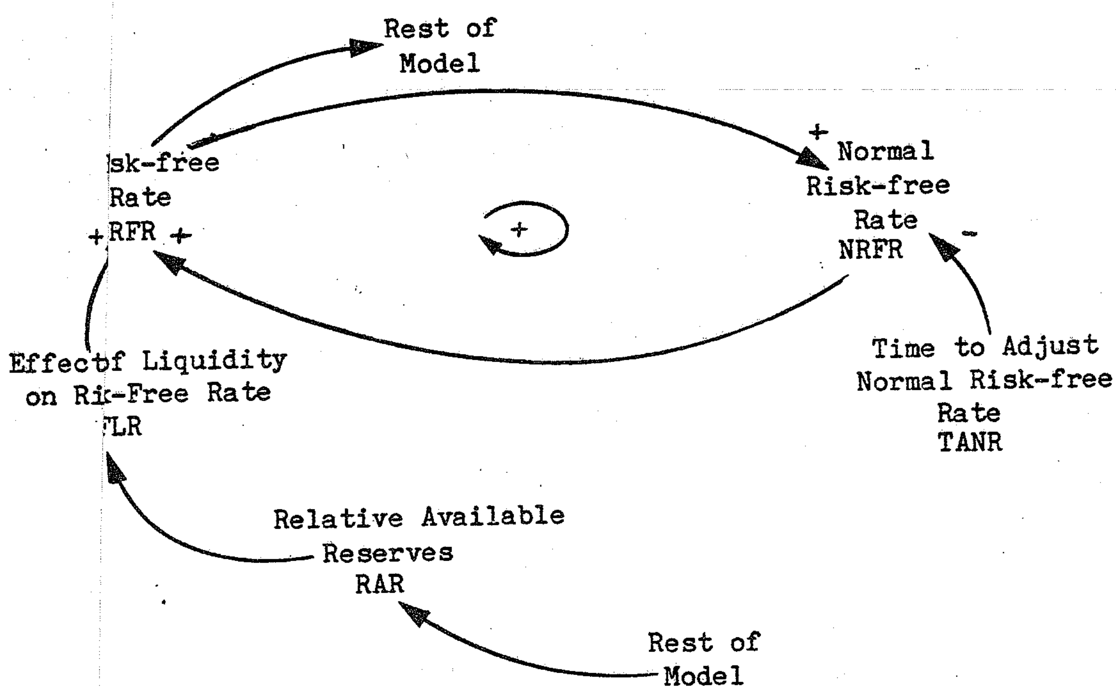

Figure 1: Causal Loop Diagram of Interest Rate Formulation

In theabsence of pressures caused by liquidity or illiquidity, the risk free rate ll equal the normal rate and there will be no forces causing change in eitherthe risk-free rate or the normal rate. In this sense the normal rate represnts a (potential) stress free equilibrium interest rate for the financal sector considered in isolation. A risk free rate equal to any given normalrate, however, may produce movement in other parts of the economic systemwhich eventually will find expression as excess or deficient liquidity in thefinancial sector. Hence, while the normal rate may be considered as the ecilibrium rate within the financial sector, the normal rate may not be (and ually will not be) a stress free equilibrium rate for the system consicred as a whole.

The beavior of the interest rate structure outside of equilibrium in the financal sector is controlled (1) by the effect of liquidity on the risk-free rate (EFLR) and (2) by the positive loop connecting the normal rate th the risk-free rate. While the percentage deviation of the risk free ratfrom the normal rate will be constant for a given continuing level of illiquity, the actual level of the risk-free rate will continue to rise as the normarate continuously adjusts towards the risk free rate. A continuing state illiquidity will result in a continuously increasing risk free rate. The converse also holds. The effect of continuing excess or deficient liidity on rates will be greater the greater is the direct response to excessr deficient liquidity (determined by EFLR) or the greater is the speed th which the normal rate adjusts to the risk free rate (determined by TA).

The dynamic response to excess or deficient liquidity is adaptively rional. No one knows how far the rate must be increased in order to eliminate n imbalance between supply and demand. Rather, rates will continue movg in the proper direction, until the imbalance is relieved.

3. Estimation

Derivation of Estimating Equation. While care must be taken in interpting the results (see section 4 below), estimation of the national model inrest rate equations is possible. In order to estimate the equations it is desirable to cast them into a different form. Substituting equation 1nto equation 2 gives:

$$\dot{NRFR}_t = (NRFR_t * EFLR_t - NRFR_t)/TANR \quad \text{or} \qquad (6)$$

$$\dot{NRFR}_t / NRFR_t = (EFLR_t - 1)/TANR \quad \text{or} \qquad (7)$$

$$\frac{d}{dt}\,(\ln(NRFR_t) = (EFLR_t - 1)/TANR \quad \text{or} \qquad (8)$$

$$\ln(NRFR_t) = \ln(NRFR_{to}) + \int_{to}^{t} (EFLR_s - 1)/TANR \; ds \qquad (9)$$

Noting from equation 1 that NRFR = RFR/EFLR, we have

$$\ln(RFR_t) = \ln(NRFR_{to}) + \ln(EFLR) + \int_{to}^{t} (EFLR_s - 1)/TANR \; ds. \qquad (10)$$

In the national model EFLR (equation 3) is linear in the usual operatir region and, hence, may be approximated by

$$EFLR_t = 1 + b * RAR_t \qquad (3')$$

Where b is a parameter (to be estimated below).

This function has a neutral effect when RAR is zero as required (see p.6). Substituting equation 3' into 10:

$$\ln(RFR_t) = \ln(NRFR_{to}) + \ln(1+b*RAR_t) + (b/TANR)\int_{to}^{t} RAR_s\, ds. \qquad (11)$$

Equation 11 is a nonlinear equation in three unknowns: b which determines the degree to which a liquidity imbalance affects the risk-free rate, TANR which determines the speed with which the normal rate adjusts to the risk-free rate, and $NRFR_{to}$ which is the initial value of the underlying interest rate.

The equation to be estimated, then, is:

$$\ln(RFR_t) = a + \ln(1+b*RAR_t) + (b/TANR)*CRAR_t + d_t \qquad (12)$$

$$\text{where } d_t = (e_t - E(e)) \qquad (13)$$

e is a disturbance,
E(e) is the expected value of the disturbance,
a is $\ln(NRFR_{to})$ plus the expected value of the disturbance and
$CRAR_t$ is a month by month accumulation of RAR from t_o to t.

<u>Data and Estimation Results</u>. It remains to specify data to be used for the risk-free rate (RFR) and relative available reserves (RAR). In the estimations which follow RFR has been approximated by a continuous-time yield calculated from monthly averages of daily figures of secondary market rates (bank-discount basis) on 90-day T-Bills.[4] Estimation results are substantially the same using the discrete time yield, the one-year yield on a constant maturity portfolio of government bonds, or the auction yield on 90-day bills.

RAR is defined as

$$RAR = \frac{(\text{Nonborrowed Reserves} - \text{Required Reserves})}{\text{Nonborrowed Reserves}}$$

Nonborrowed and required reserves are monthly averages of daily figures from the federal reserve system. The figures have been seasonally adjusted and adjusted for changes in reserve requirements to remove discontinuities. Results are substantially the same when using reserves not adjusted for discontinuities.[5]

Least squares estimation of equation 12, using monthly observations from January 1959 to June 1983, yields the following results.

$\hat{a} = 1.40082$ (.01454) $\qquad \hat{b} = -10.66100$ (.54507) $\qquad \widehat{TANR} = 57.83310$ months (4.70659)

$R^2 = .81365 \qquad DW = .15$

The fit, as judged by the R^2 statistic, is good, particularly in view of the fact that neither past values of the dependent variable nor other interest rates appear on the right hand side. The variables determining the risk-free rate in econometric models (Evans, Klein, Schink 1967, McCarthy 1972, Liu and

Hwa 1974, and Sinai 1981) frequently include other interest rates, particularly the discount rate, as exogenous inputs even though the direction of causality is open to serious question (Riefler 1930). The generation of interest rates in the national model does not rely on any exogenous interest rate inputs.

The slope ($\hat{b}$) of the function determining EFLR is negative, as it should be, indicating that as liquidity goes down, interest rates go up. The time constant of 58 months or 4.8 years has the correct sign, but seems a bit long. As discussed above, a too-long time constant may be compensated for by a too-steep estimate of the slope of EFLR. Consequently, both the estimated time constant and the magnitude of the estimated b may be too large, resulting in a good fit achieved because the errors are offsetting.

The low Durbin Watson statistic suggests the errors may exhibit first order autocorrelation. The autocorrelation and partial autocorrelation functions of the least squares residuals appear below:

AUTOCORRELATIONS

LAGS	ROW SE	Correlation Coefficients											
1 to 12	.06	.92	.84	.75	.67	.61	.57	.55	.55	.55	.55	.54	.53
13 to 24	.19	.49	.45	.41	.37	.33	.30	.28	.25	.24	.23	.23	.22
25 to 36	.22	.20	.18	.15	.13	.12	.10	.09	.07	.07	.07	.06	.03
37 to 48	.22	.02	-.01	-.02	-.05	-.07	-.09	-.09	-.10	-.11	-.10	-.09	-.10
49 to 60	.22	-.11	-.14	-.16	-.18	-.21	-.23	-.22	-.20	-.19	-.16	-.14	-.12
61 to 70	.23	-.13	-.14	-.15	-.17	-.20	-.20	-.20	-.18	-.17	-.16		

PARTIAL AUTOCORRELATIONS STD. ERR. = 0.058321

LAGS	Partial Autocorrelation Coefficients											
1 to 12	.92	-.02	-.10	.06	.01	.08	.16	.07	.02	.00	.04	.04
13 to 24	-.16	.01	-.04	.01	.02	-.04	-.02	-.08	.12	-.00	.02	-.04
25 to 36	-.04	-.05	.06	.01	.01	-.05	-.01	.02	.02	-.01	-.03	-.12
37 to 48	.08	-.08	.01	-.08	-.03	.03	.03	-.04	-.03	.08	.03	-.06
49 to 60	-.07	-.01	-.03	-.01	-.05	.04	.05	.08	-.04	.04	.03	.01
61 to 70	-.04	-.01	-.02	-.05	-.02	.06	-.00	.04	-.07	-.09		

The autocorrelation function declines gradually from a value of .92 while the partial autocorrelation function exhibits a spike of .92 at the first lag. This indicates that the structure of the errors can be well described by a first order auto-regressive process (Box and Jenkins 1976). More efficient estimates can be obtained by taking account of the structure of the errors. The following results were obtained using generalized nonlinear least squares to estimate

$$\ln(RFR_t) = a + \ln(1+b*RAR_t) + (b/TANR)*CRAR_t + d_t \qquad (12')$$

$$\text{Where } d_t = RHO*d_{t-1} + k_t \qquad (13')$$

k_t is an non-autocorrelated disturbance with mean zero.

$\hat{RHO} = .9711$ $\hat{a} = 1.39182$ (.14169) $\hat{b} = -3.89065$ (.51834) $\hat{TANR} = 19.808$ (7.30899)

$R^2 = .98$ DW = 1.64

The time constant has been reduced to a more reasonable 20 months (1.7 years). And, as expected, the magnitude of b has also been reduced. The Durbin Watson statistic is, strictly speaking, no longer meaningful, but is reported for completeness.

4. Using the Estimates to Support the Theory

The estimated coefficients are significantly different from zero, of the proper sign, and of reasonable magnitudes. In order to claim that they support the theory, however, it is necessary to establish that they are coefficients based on time series which are appropriately related to the national model concepts of risk-free rate and relative available reserves.

The Risk-free Rate. The rate on any government security is riskless only in nominal terms. Inflation or deflation may cause the real return to deviate from its expected value. In the United States, however, inflation risk over a period of ninety days is not significant. Consequently, use of the yield on 90-day T-Bills as an approximation to the risk-free rate is probably justified.

Relative Available Reserves. The time series used for relative available reserves (RAR) is the free reserves of members of the Fed divided by total member reserves. The use of data from a wider definition of banks and depository institutions would result in an increase of free reserves in about the same proportion as the increase in reserves; hence, there would be little change in RAR: RAR calculated with data which includes all depository instituitions is very close to RAR calgulated from data which includes only members of the Federal Reserve System.

Thus:

$$\frac{FR_{member\ banks}}{R_{member\ banks}} \simeq \frac{FR_{dep}}{R_{dep}} \qquad (14)$$

where $FR_{member\ banks}$ and FR_{dep} denote the Free Reserves of members of the Fed and of all depository institutions respectively. And, $R_{member\ banks}$ and R_{dep} represent reserves of member banks and of all depository institutions respectively. The denominator of the ratio on the right hand side of equation 14 represents the same concept as the denominator of the corresponding ratio in the national model. However, the numerator of RAR in the national model is actually excess liquidity of the entire economy, not only the free reserves of depository institutions (see p. 5) and,

$$\frac{FR_{dep}}{R_{dep}} \neq \frac{FR_{dep} + ELE}{R_{dep}} \qquad (15)$$

where ELE is Excess Liquidity Elsewhere in the economy.

In the real world excess or deficient liquidity can exist outside of depository institutions and can affect interest rates without affecting the reserves or free reserves of the Federal Reserve System. As a consequence, a portion of excess (or deficient) liquidity in the real world will not show up as free reserves of depository institutions.

To take one example of this possibility, consider a case of two men one of whom feels that he has too much of his wealth tied up in deposits and not enough in investments, while the other believes his portfolio to be well balanced between deposits and investments. This describes a situation of net excess liquidity which can exist even if free reserves in the banking system are zero. In order to obtain more investments the first man will offer the second a price above the current investment-price for a portion of his investments. A higher price is, of course, equivalent to a lower rate of interest. A transaction may occur in which investments pass from the first man to the first and deposits pass from the first to the second. The interest rate has been lowered through the existence of excess liquidity, however, the banking system as a whole experiences no net change in deposits and hence no net movement in free reserves (or RAR_{dep}).

While the national model formulation takes account of ELE, the time series used in the estimation proceedure does not do so explicitly. However, the measure of excess liquidity is ultimately used in the expression for the effect of excess liquidity on the risk-free rate (EFLR). And, the effect of liquidity on the risk-free rate may be approximated using data on free reserves of depository institutions in place of a broader measure of liquidity. More specifically, as will be derived below,

$$EFLR = 1 + b^* \frac{FR_{dep} + ELE}{R_{dep}} \simeq 1 + \hat{b} \frac{FR_{dep}}{Rdep} \qquad (3'')$$

Where $\hat{b}$ is the slope estimated in the GLS proceedure and b is the slope called for in the national model formulation. (Time subscripts have been eliminated for notational clarity).

While previous writers (Riefler 1930, ch. 2; Currie 1935, 83-103; Burgess 1936 & 1946, 181-207; Tinbergen 1939, 92-98), stressing the importance of bank reserves alone in the setting of interest rates, have attached little importance to the excess or deficient liquidity outside of the banking system, the position taken within this paper is not as extreme. Rather, the following paragraphs advance the notion that the coefficient b does, in fact, reflect movements in ELE as well[7] as FR_{dep} because these two components of liquidity are highly correlated.

Banks' and other depository institutions' balance sheets are influenced by factors that affect the balance sheets of all financial institutions, and, hence, banks' and other depository institutions reserve positions will be positively correlated with those of non-bank, non-depostitory financial institutions. Further, excess liquidity in the household or corporate sectors generated via deficit government spending, transfer payments, or sales or purchases of government bonds will be mirrored by free reserves in the banking system to the extent that such activities occur through the banking system.

Similarly, to the extent a change in desired household liquidity involves a conversion of cash to deposits, the excess liquidity will show up in the banking system (Cf. Cagan, 1965). Finally, there is a great amount of trading and arbitrage in the financial markets. It is difficult to believe that a significant pocket of excess liquidity (say, among banks) would exist simultaneously with a significant pocket of deficient liquidity (say, among households). Hence, correlation between excess liquidity of depository institutions and excess liquidity elsewhere should be high.

Assuming, then, that ELE is positively correlated with FR_{dep}, we can write

$$ELE \simeq h * FR_{dep} \qquad (14)$$

where h is positive. Using 3'' we have

$$EFLR \simeq 1 + b * \frac{(1+h) * FR_{dep}}{R_{dep}}$$

From which we see that

$$b * (1+h) = \hat{b} \qquad (15)$$

Since theory and the "correlation argument" above require b to be negative and h to be positive, the significantly negative GLS estimate of b supports the theory.

Given the relationships of equation 3'', the interpretation of TANR as an estimate of the theoretical TANR continues to hold. Hence, the significantly positive estimate of TANR also supports the theory.

In addition to showing that the GLS estimates support the theory developed here, the preceeding discussion also makes clear the relationship between the estimated coefficients and the corresponding parameters in the national model.

The estimate of TANR is an estimate of the model's TANR. The slope coefficient of the empirical work (b) is larger in magnitude (ie. more negative) than the corresponding concept (b) in the national model as equation (15) indicates. Hence the slope estimate is an upwardly biased estimate of the magnitude of the slope for EFLR in the national model.[9]

5. Summary

In the system dynamics national model, the normal interest rate is modified by the effect of liquidity to produce the risk-free rate. The notion of a normal rate, dating at least from Keynes, is more than a convenient fiction. It is believed that something very like a normal rate exists in the minds of those actually engaged in buying and selling securities in the financial markets.

The normal rate, as its name implies, is the rate which agents believe would obtain under normal circumstances; that is, it is the rate which would hold in the absence of transitory pressures. The normal rate, formed on the basis of past experience and continually adjusting to new interest rate conditions, is represented in the model by an exponential average of past riskless rates.

Pressures in the financial system are experienced as excess or deficient liquidity which may be measured in the national model as free reserves of the model's finanical intermediary. During times of excess liquidity, economic agents will lower the risk-free interest rate below the normal rate in the attempt to buy investments or to lend. Conversely, in a situation of deficient liquidity the risk-free interest rate will be increased above the normal rate as agents attempt to sell securities or to borrow.

The dynamic response of interest rates to continuing illiquidity is determined jointly by the magnitude of the effect of illiquidity on the risk-free rate and the speed with which the normal rate adjusts to the risk-free rate. Continuing illiquidity will result in a continually increasing risk-free rate. The converse also holds. This response is adaptively rational in the sense that the interest rate will continue to adjust until the supply and demand for financial resources balance.

Estimation results support the national model theory of interest rate formation. An estimate for the time constant controlling the formation of the normal rate rate was obtained. An upper bound for the magnitude of the slope of the function determining the effect of liquidity on the risk-free rate was also estimated.

Notes

1. The national model does not generate a term structure of interest rates. It appears sufficient for the purposes of the model to generate the rise and fall of the level of rates; changes in the term structure do not substantially affect the paticular behavior modes embodied in the national model.

2. Modigliani and Sutch (1966) were ultimately concerned with the term structure of interest rates. They were attempting to uncover the expected future spot short rate which would form part of the current long rate. This expected rate differs from the normal rate developed here since an expected rate presumably includes expected disturbances which may combine with the normal rate to determine the risk free rate, but which are distinct from the normal rate. Modigliani and Sutch (1966) included a extrapolative component as well as an averaging component in their formulation of expected rates. Their notion of an extrapolative component may make more sense in the context of an expected rate than a normal rate since current disturbances may be expected to continue for varying periods thereby affecting the expected future spot rate, but not the current normal rate. In any event, there is no extrapolative element in the system dynamics formulation of the normal rate.

3. There are two financial assets which actors in the national model may choose to hold: deposits and "bank bonds". Deposits represent transaction balances, while bank bonds represent all other financial assets. While, bank bonds are technically liabilities to the national model's financial intermediary only, they (indirectly) represent claims on the other sectors which have been financed by the financial intermediary. Hence, bank bonds correspond to an aggregate of corporate bonds, CD's, commercial paper, etc.

4. FYGM3 is CITIBASE's secondary market rate on 90-day treasury bills, calculated on a bank-discount basis. To convert this to continuous-time yield (CTY) the following calculations were performed. First, the percentage discount (DISC) is

$$DISC = FYGM3/4.$$

The decimal discrete-time yield (DTY) may now be written as

$$DTY = DISC/(100-DISC).$$

This may be converted to a continuous-time yield (CTY) as

$$CTY = \ln(1+DTY).$$

Finally, prior to performing regressions, this was converted to a yearly per centage:

$$CTY' = CTY*(365/90)*100$$

(Source for bank-discount rates: CITIBANK series FYGM3. See also Federal Reserve Bulletin table 1.35.)

5. (Source: CITIBASE, series FMRNBA and FMRQA. See also Federal Reserve Bulletin, table 1.20).

Use of required reserves rather than desired reserves as specified in equation (5) suggests that 3' might not be quite so simple. If, for example, desired reserves were actually required reserves plus some cushion we would have

$$DR_t = RR_t + c_t.$$

Where DR - Desired reserves
RR - Required reserves
c - Cushion

In this case

$$\begin{aligned} EFLR_t &= 1 + b*((R_t-DR_t)/R_t) \\ &= 1 + b*((R_t-(RR_t+c))/R_t) \\ &= 1 + b*RAR_t - b*c_t/R_t. \end{aligned}$$

Where EFLR - Effect of liquidity on risk free rate
R - Reserves
b - Slope coefficient

Using this last expression in place of 3' and assuming c constant produces the following estimating equation.

$$\ln(RFR_t) = a + \ln(1+b*RAR-b*c/R_t) + (b/TANR)*CRAR_t - b*c*CIR_t + d_t$$

Where $d_t = (e_t - E(e))$

e is a disturbance
RFR is the risk free rate
TANR is the time to adjust normal rate
CRAR is a month by month accumulation of RAR
CIR is a month by month accumulation of $1/R_t$.

Estimation of this last equation, with a first order autocorrelation correction as described below, yields an estimate of c which is not significantly different from zero. The other parameters change relative to the results to be described below, but not significantly. Other specifications of the cushion term do not lead to rejection of the hypohesis that RR = DR. However, extremely large standard errors result from colliniarity between accumulated free reserves (or accumulated RAR) and time, which these other specifications introduce. Estimation with a longer time series would likely alleviate these problems. This data is not currently available. Until more data is available, the simpler specification of equation 12 seems justified.

6. Source: CITIBASE series FCMRNB, FCMRQM, FMRNBA, and FMRQA.

7. A third justification, offered and dismissed by Cagan (1969), is that free reserves determine the level of deposits, hence, liquidity in the economy.

8. More precisely, positive correlation implies

$$ELE = q + h^{*}FR_{dep} + e$$

where q and h are constants, h>0, and e is an error term.

If the correlation is high, the error term (e) will be small. Further, the interest rate equations and particularly the specification of EFLR in (3'') carries dynamic implications for the value of q. A positive q would imply that perfect liquidity (ie. a perfect balance between supply and demand for financial resources) calls forth a continually declining interest rate. Similarly, a neqative q would imply that a balance between supply and demand for financial resources calls forth a continually increasing interest rate. It is more reasonable, to suppose that q is equal to zero, implying that a balance between supply and demand will result in an unchanging interest rate.

9. Due to the closed-loop or feedback nature of the national model, its sensitivity to changes in parameters is much slighter than might be expected. For example, flattening the slope of EFLR by two standard deviations from -3.9 to -2.85397 while increasing the time constant on NRFR by two standard deviations from 20 months to 34 months would substantially reduce the movement of the risk free rate in an open loop model. However, such an experiment in a deterministic run of the national model (version N328) alters the range in which the riskfree rate moves over the long wave (Forrester 1979) by only one percentage point from a range of about 6.7% - 3.8% to a range of about 6.1% to 4.25%. More importantly, the shape and period of the time path of the fluctuation is almost unchanged. (See also Graham and Hines 1983).

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