System Dynamica Generalized Modeling for Forecasting Multiproduct Substitution

Zhao Chong Jie
Zhejiang University PRC

ABSTRACT

The System Dynamics Generalized Substitution Modeling is presented. This modeling considered the influence factors of circumstance by introducting action function. The methodelogy is based on the System Dynamics with econometrics,combining three postulates in product substitution and decomposing multi-product into several two-product substitution. Parameter estimation, which existed in all System Dynamics Modelings,is one important but still unsolved problem. Now this problem has been solved in our paper by orthogonal simulation, it is based on the orthogonal theory and generalized least squares(GLS)

INTRODUCTION

The history of technical development tell us:in nature there is no technique that is forever advanced,because advance and backwardness are relative. It is the dialectical relation between advance and backwardness that a series of technological substitution can be produced. So man can speak that the history of technical development actually is the history of technological substitution. Moreover,the substitution in technology must be showed by products. The substitution of products is as a result of technological substitution.

That people demand products is only for their function not for the products themselves. Different forms of products may have same function. Based on this principle, product substitution can be produced and continued.

Appearence of large amount of substitution products makes the forecasting for single product lose sense, because forecasting is related not to products but to their function. So the study for multi-products substitution appear.

BACKWARD

1) Logistic Curve

Although introduction of Pearl curve was only for approximating the growth curve in biological phenomana . Because of the similarity in their outside properties,it's equation is used as startpoint of products substitution model.

It's form is:

$$y=a/(1+b\cdot e^{-c\ t}) \quad (1,1)$$

2) Mansfield model

Mansfield took substitution of multi-technology in multi-department as analysis object and tried to explain the nature of technological substitution from causality.

His conclusion was that :

$$m(i,j) = n(i,j)/(1+e^{-l(i,j)\cdot -b(i,j)\ t}) \quad (2,1)$$

$$b(i,j) = a(i,1)+a(i,2)\cdot p(i,j)+a(i,3)\cdot s(i,j) \quad (2,2)$$

here :

p: win capability s : infest scale

m/n : user share a,l : constant

i :department j : technology

3) Fisher model :

The form is :

$$dy/(y\ dt) = k\ (1-y) \quad (3,1)$$

or : $$y = 1/(1+e^{-k(t-t(1/2)}) \quad (3,2)$$

4) Sharif model for multi-product substitution

The model for forecasting two-product substitution in Sharif model was still used. The difference was that two-product in odler models were transformed into two-group product. The principles of substitution can be stated as following :

a) The oldest product in the market p_1 being technologically least advanced will lose it's market share to all other products (p_2 to p_n).

b) The newest product in the market p_n being advanced will obtain it's market share to all other products (p_1 to p_{n-1}).

c) The intermediate product (p_2 top_{n-1}) obtain its market share to older products and lose its market share to newer products.

In order to determine the share of intermediate products (px), following analytical procedure was used.
a) Group all older products (p_1 to p_{x-1}) as p_{g1} , considering the substitution between p_{g1} and others :

$$df(p_{g1})/dt = c_1 f(p_{g1}) (1-f(p_{g1})) \quad (4,1)$$

b) Group all newer products (p_{x+1} to p_n) as p_{gn} considered the substitution between p_{gn} and others;:

$$df(p_{gn})/dt = c_2 f(p_{gn}) (1-f(p_{gn})) \quad (4,2)$$

From $f(p_{g1})$ and $f(p_{gn})$ we can obtain:

$$f(p_x) = 1-f(p_{g1})-f(p_{gn}) \quad (4,3)$$

for $f(p_1)$ and $f(p_n)$ following result can be directly obtained;

$$df/dt = c f (1-f) \quad (4,4)$$

Only f is transformed into $f(p_1)$ and $f(p_n)$.

Discuss

These models contain several supposes;

1) All products have same properties:

In Sharif's model products are divided into two groups and then one group products are considered as one product. It means that products in one group have same properties. But that isn't true. It is because these products have different character that we study the problem of multi-products substitution.

2) Unchangeableness of factors

All models showed above consider influences of circumstance only statically. It is showed that in equation constant are used as properties factors. Suoh as b(i,j) (in equation (2,1)), K (in equation(3,1)) and c_1 c_2 (in equation (4,1) (4,2)) etc. In fact circumstance factors are not always constant. There are not only statical but also dynamical factors. All static factors can be combined together and represented by constant, but dynamic factors must be alon considered.

3) One direction substitution

In these models influ ence factors are constant, so man cann't change the symble of coefficient (positive or negative). It means that it is new or old of product that determine the substitution of products. But in fact , only the characters of products can determine the substitution, although several new products at first are very good. After a period of timm because of the change of circumstance factors, their advantage may be not advantage but shortcoming. So at this time newer products may be substituted by olders.

BUILDING OF GENERALIZED MODEL

1) Substitotion of multi-product

Comparing with two-product substitution, because of introduction of intermediate products, multi-product substitution is more complicated. Usually intermediate products in one side substitute older products, but in the other side they can be substituted by newer products. in fact the substitution between different products can be determined by product character and their adopted degree to circumstance. Owing to the change of circumstance, in a group of products intermediate products can substitute several products and at same time can be substituted by others. Now we have known that it is unilateral that man only pay attention to new or old of products.

In the other side each product in one group of products react to the other products. The amount of substitution not only based on circumstance factors, but also the absolute amount of their market shares. It means that for example substitution between product i and product j influence the substitution between product i and product k.l... (except i.j).

2) Suppose of model

a) Decomposability :

In multi-product substitution in one unit time the amount of substitution for one product can be decompose as the amounts of substitution of the product and other products.

b) Timelessness ;

In any unit time substitution between two products only based on the stale of two-products and circumstance factors at that time, not based on past process.

c) Independence :

At any time the amount of substitution between product i and product j only depend on influence factors and market shares of product i and j (f_i and f_j).

3) Restrain condition

We use market share to represent the level of product substitution. Specialization of market share produce following restrain condition:

$$\sum_{i=1}^{n} f(i) = 1 \qquad (5,1)$$

here f(i): market share of product i
n : product number

4) Diagram

For the restrain condition " the sum of all market share equal to 1 " i.e $\sum_{i=1}^{n} f(i) = 1$. This condition certainly can be replaced into following condition:

$$\sum_{i=1}^{n} df(i)/dt = 0 \qquad \sum_{i=1}^{n} f(i)\Big|_{t=t_0} = 1 \qquad (5,2)$$

or: $$\sum_{i=1}^{n} df(i)/dt = \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} df(i,j)/dt = 0 \qquad (5,3)$$

here df(i,j) : the substitution amount that product $j_{(pj)}$ obtain from product $i_{(pi)}$
df(i) : the substitution amount that product i obtain from the other (in time interval dt)

Because the substitution amount that pi obtain from pj is also the amount that pj (through pi) lose.

Hence: $$df(i,j)/dt + df(j,i)/dt = 0 \qquad (6,1)$$

We use this condition to diagram, the restrain $\sum_{i=1}^{n} df(i)/dt = 0$ can be satisfied. After introducting suppose " Independence" and "Decomposability" , we obtain:
(taking three products as example)

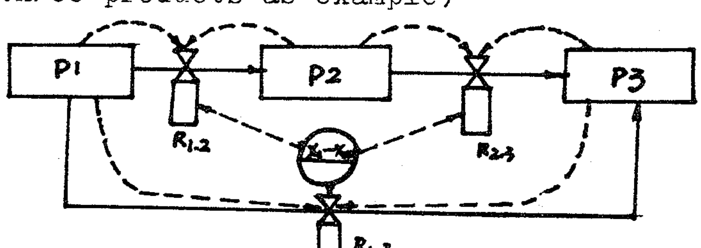

Fig. 1 diagram

here x_i: influence factor i=1.2.···m
$R_{i.j}$: R_{ij} =df(i,j)
P_1:P_2.P_3 : product 1.2.3.

5) Building of model

After determining diagram of the system, multi-products substitution has been divided into several two-products substitution. Now our task is to determine their action form. Considering the analysis of Mansfield and Fisher (mentioned above), We notice that replace only m(i,j)/n(i,j) given by Mansfield's model into f(i,j) (the meaning is same) b(i,j) (in equation(1,3)) introduced by Mansfield actually is coeffiences k (in equation (1,4)) in the Fisher's model. It means that for two-products substitution the proportion factor k comprehend function acted by all influnence factors to products substitution, but in Mansfield's model only contain ststistic "win capability " and "infest scale". Now we represent these influence factors by one function , their variables are $x_1 . x_2 . \cdots x_m$ given by diagram mentioned above. these not only camtain statistic influence factors, but also dynamic factors. Using the suppose "timelessness", the variables in the function don't contain time varible.

Otherwise, because of the difference between substitution of multi-products and two-products (for latter, two-product's sum of market share equal to one constantly) the factor (1-f) represent latent capability of whole market in one side, also represent the other product's market share in the other side. For multi-product substitution, the first explanation wasn't suitable but we can use "independence" suppose and combine with second explanation. So we can obtain:

$$df(i,j)/dt \propto f(i)\ f(j) \qquad (7,1)$$

If action function of influence factors will be considered, the equation become:

$$df(i,j)/dt = G_{(i,j)}(x_1 x_2 \cdots x_m)\ f(i)\ f(j) \qquad (8,1)$$
$$i=1.2.\cdots n \qquad i \neq j$$

here $G_{(i,j)}(x_1 x_2 \cdots x_m)$: general action function form of influence factors $x_1 . x_2 ,\ \ x_m$.

(for df(i,j) the meaning are the same as above)

For n kinds of product substitution, use "decomposability" suppose its general equation should be writen as:

$$df(i)/dt = \sum_{\substack{j=1 \\ j \neq i}}^{n} G_{(i,j)}(x_1 \cdots x_m)\ f(i)\ f(j)$$
$$G_{(i,j)}(x_1 \cdots x_m) = -G_{(j,i)}(x_1 \cdots x_m) \qquad i=1,2\cdots n \qquad (9,1)$$

Taking three products as example:

$$df(1)/dt = G_{(1,2)} \cdot f(1) \cdot f(2) + G_{(1,3)} \cdot f(1) \cdot f(3)$$
$$df(2)/dt = - G_{(1,2)} \cdot f(2) \cdot f(1) + G_{(2,3)} \cdot f(2) \cdot f(3) \quad (9,2)$$
$$df(3)/dt = - G_{(1,3)} \cdot f(3) \cdot f(1) - G_{(2,3)} \cdot f(3) \cdot f(2)$$

The Relation Between General Model and Others Model

1) When the amount of product equal to two (n=2)

a) If let $G_{(1,2)}(x_1 \cdots x_m)$ =k, it will become the Fisher's model.

b) If let $G_{(1,2)}(x_1 \cdots x_m) = a(i,1)+a(i,2) \cdot r(i,j) + a(i,3) \cdot s(i,j)$ it will become Mansfield's model.

2) When the amount of product larger than two (n>2)

Let $G_{(i,j)}(x_1 \cdots x_m)$ =constant, now the general equation become the Sharif's multi-product substitution model.

Take threeproducts as example, the model become:

$$df(1)/dt = c \cdot f(1) \cdot (f(2) + f(3))$$
$$df(2)/dt = -c \cdot f(1) \cdot f(2) + c \cdot f(2) \cdot f(3) \quad (10,1)$$
$$df(3)/dt = cf(3) \cdot (f(1) + f(2))$$

It satisfies obviously the equation:

$$df(2)/dt = -df(1)/dt - df(3)/dt$$

By use the initial condition:

$$\sum_{i=1}^{3} f(i) \Big|_{t=t0} = 1$$

We can obtain $f(2) = 1 - f(1) - f(3)$ immediatly.

The equations are the represents of Sharif's suppose of "All Products Have Same Properties",but Sharif represented these only by c_1 and c_2. Coefficient c_1 obtained by equation (4,1) usually don't equal to $-c_2$ obtained by equation (4,2). In fact, they satisfy refrain condition $\sum f(i)=1$ only by $f(2)=1-f(1)-f(3)$,but they don't ensure to satisfy refrain condition in subsystem:

$$df(i,j)/dt = - df(j,i)/dt$$

In the former it can satisfy the conditions only through taking errors into f(2).

Now we can see that because of introducing action function of influence factors,suppose "decomposability" "independence" and "timelessness" our model not only contains advantages of former models, but also overcome shortcomings of three sides (All products have same properties,unchangeablenss of factors and one direction substitution) and make the model more reasonable.

INFLUNENCE FACTOR ANALYSIS

The action function general form of influence factors is mentioned above. Now let's discuss these factors and their effections in detail further.

For product substitution,related factors usually following:
a) profitability for the manufacturer of the newer product
b)size of investment necessary for economic production,
c) stage of perfection in production technology due to time and experience,
d)overall growth of industrial production (expansion of economy),
e)utility adjusted price ratio between competing producs,
f)sales and promotional efforts for the newer product,
g)useful life of older product and capital rquipments for manufacturing (durability and obsolescence),
h)quality characteristics of the newer product
i)stage of diffusion characterized by adopted population and potential adopters.

THese factors are generally divided into three kinds. The first acts on producer the second acts on user, the third acts on the both. Obviously,all factors act on either producer or user.

Based on this character and combine with further analysis, we can get these following conclusion:
Influence factors are subjective and product substitution isn't introduced by them. It is the producer and user that determine the product substitu tion. All factors make action only through producer and user.

Because producer provide and user realize the posibility of product substitution,so the factors influencing both are acted at the same time. Also, as the amount of products substitution has only one result,it shows that there are interaction between producer and user. So the principle on which we consider is that: for product substitution only these factors that have influence to producer and user should and must be considered.

When thinking their action,we should consider from statistic and dynamic aspects.Some factors(such as function of product)have been fixed,when products just enter into market,these factors can be combined together and represented mit one constant.But dynamic factors (such as price and cost),which aren't fixed over whole living period of product,must be represented alone.At same time we must consider their influence form to product substitution.

For example,for textile product substitution we consider average income as dynamic factor.The result is:

$$G(x)=a_1(y/x-k(n-y))\Delta x$$

here x:average income/man.
y:textile product/man.
k,n,a_1:constant.
(I have discussed in detail at another paper)

PARAMETER ESTIMATION IN THE GENERAL MODEL

After giving the general model of n products substitution,we considered how to estimate parameter in action function in actual model.

For this kinds of model, there are several difficulties in parameters estimation. They are:

a) The model consists of differential equations.
b) There are same parameters in different equations.
c) Crossequation are interrelated.

As a matter of fact, as estimating parameters in system dynamic models,we often meet difficulties above. (df(i,j)/dt =-df(j,i)/dt) In the past,parameters of system dynamic model are determined by subjective to do qualitative analysis. After setting his polynomial model, Sharif considered that because of difficulties in determining parameters, parameters in polynomial model usually can be determined by subjective.So advatages of the model are that they can help us to do qualitative analysis.

For these problems our ideal showed following:

1) differential equations are represented by difference equation.

2) Estimating all equation's parameters at same time and using general least square to eliminate the interrelation in equations.

a) When parameters are linear, we can solve large equation. (I have related it in detail at another paper.)

b) When parameters have general forms (linear or nonlinear) we can solve these by orthogonal simulation combining with general least square. Computer orthogonal simulation actually is that: choosing optimum parameters combination using orthogonal test, simulating object value in each group of parameters and then choosing optimum parameter group in given region, based on the theory of orthogonal design to search for the best way.

For our general model, simulation model are:

$$\text{Min.} \sum_{k=1}^{l} (\tilde{F}_k-\hat{F}_k)^T (H)^{-1} (\tilde{F}_k-\hat{F}_k)$$

$$\text{s.t} \quad \hat{f}(i,k) = \sum_{\substack{j=1 \\ j\neq i}}^{n} G_{(i,j)}(x_1 \cdots x_m)\, \hat{f}(i,k-1)\, \hat{f}(j,k-1) + \hat{f}(i,k-1) \qquad (11,1)$$

let $f(i,o) = f(i,o)$ $\qquad i = 1.2.\cdots n$

here $\tilde{F}_k$:

$$\tilde{F}_k = \begin{bmatrix} \tilde{f}(1,k) \\ \tilde{f}(2,k) \\ \vdots \\ \tilde{f}(n,k) \end{bmatrix}$$ actual value matrix of f(i) at time k,

$\hat{F}_k$:

$$\hat{F}_k = \begin{bmatrix} \hat{f}(1,k) \\ \hat{f}(2,k) \\ \vdots \\ \hat{f}(n,k) \end{bmatrix}$$ calculated value matrix of f(i) at time k.

H : covariance matrix.

The computer simulation process is : (please to see next page)

This method has widely suitibility and flexibility :

1) The problem of determining nonliear parameters can be solved.

2) Choosing parameters region can be controled qualitatly, it ensure the reasonable of model parameters.

3) When all observation data are not obtained, it has not much influence.

4) Crossequations are considered , so it increases the efficience of parameters estimation.

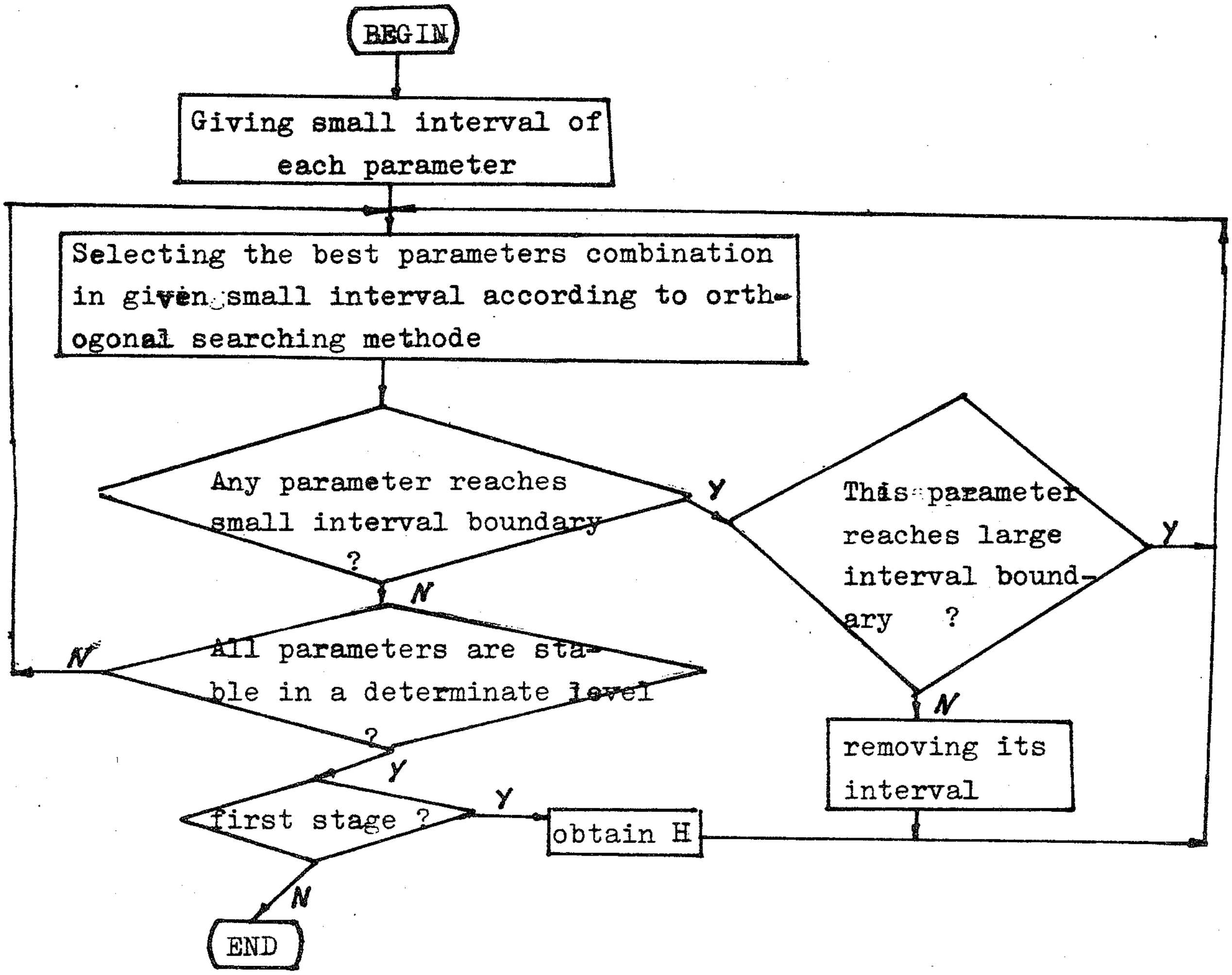

Fig.2 computer diagram

REFERENCES

1)Halold A. Linstone & Devendra Sahal "Technological Substitution" 1976 by American Elsevier

2)M. Nawaz Sharif & Chowdhury Kabir "System Dynamics Modeling for Forecasting Multi-level Technological Substitution " Tech. Fore Socia Chang Vo.8 no.4 1976.

3)M. nawaz Sharif & A.K.M.Abdul Haq. "Determinants for Forecasting Technological Substitution " Tech. Fore Socia Chang No.13 1979.

4)M.N.Sharif & K.Ramanathan "Polynomcial Innovation Diffusion Models " Tech. Fore Socia Chang No.21 1982.