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THE CAUSAL STRUCTURE OF THE WHARTON-UAM MODEL OF THE SPANISH ECONOMY

0. ABSTRACT

The analysis of the causal structure of economic models is a tool for understanding the functioning of existing models, particularly in their interdependent component, and also for assisting the modelors in the process of constructing or modifying large econometric models. In this paper the authors briefly describe the methods of causal analysis and apply them to the Wharton-UAM model of the Spanish Economy. The paper, in English, discusses suggested modifications of the model as a result of the structural analysis and is completed by two Appendices, in Spanish, discussing the theoretical construct of the model and the detailed results of the causal analysis; in an Annex are also reproduced the variables and the equations of the model thus introducing the reader to a fully documented version of the model.

1. CAUSALITY AND CAUSAL STRUCTURES

In his seminal paper on "Causal Ordering and Identifiability" (1), Herbert Simon limits the notion of causality to the study of causal relations in a given model structure. He doesn't attempt to tackle the more philosophical or statistical (depending on the schools) problem of the choice-of-regression or of the initial choice of existence or non existence of relations between variables. This elementary notion has been further developed by Simon (2) and extended and completed by Mesarovic (3) and Mc Elroy (4) in order to include both linear and non-linear models.

The analysis of causal structures of economic models has followed this general line of thought and has in recent years developed a number of operational algorithms to simplify the search for the causal mechanisms both in recursive and interdependent models (5).

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(1) In Studies in Econometric Method, W.C. Hood and T.C. Koopmans, ed., Wiley, New York, 1953

(2) H.A. Simon, Causation, in International Encyclopedia of Statistics, W.H. Kruskal and J.M. Tanur, Free Press, New York, 1978

(3) M.D. Mesarovic, Mathematical Theory of General Systems and some Economic Problems, in Mathematical Systems Theory and Economics, eds. H.W. Kuhn and G.P. Szegö, Springer Verlag, Berlin, 1969

(4) F.W. Mc Elroy, A simple method of causal ordering, International Economic Review, Vol. 19, No 1, 1978

(5) E. Fontela and M. Gilli, The Causal structure of economic models, Futures, London, dec. 1977 and, in Spanish, Estadistica Española, Madrid, dec. 1980. M. Gilli, Analysis of static and dynamic structures in economic models : methodological and practical aspects, Applied Mathematical Modelling, Vol. 5, 1981; M. Garbely and M. Gilli, Two approaches in Reading Model Interdependencies, in Analysing the Structure of Economic Models, J.P. Ancot, ed., Nijhoff, The Hague 1984

An economic model may be formally represented by a system of n structural relations:

$$h\,(y,\ z) = 0 \qquad [1]$$

which translate the relations existing between the endogenous variables given by $y \in R^n$ and the exogenous given by $z \in R^m$.

The causal structure of model [1] is defined by the presence or absence of the variables in each relation, i.e. by the content in zero or non-zero elements of the matrix

$$\left\| \frac{\partial h}{\partial y'} \;\vdots\; \frac{\partial h}{\partial z'} \right\| . \qquad [2]$$

The essential analysis of the causal structure consists first in searching partitions of the set of relations and the set of variables such as the matrix $\frac{\partial h}{\partial y'} = A$, rearranged on its rows and columns in accordance with these partitions, takes the form

$$A = \left\| \begin{matrix} A_{11} & 0 & & 0 \\ A_{21} & A_{22} & 0 & 0 \\ \vdots & & \ddots & 0 \\ A_{p1} & \dots & \dots & A_{pp} \end{matrix} \right\|$$

where all matrices Aii, i = 1,2 ....p, are square and non decomposable. In this presentation, the matrix A_{11} corresponds to the smallest subset of relations which enables us to determine the corresponding subset of variables. These variables may then be considered as exogenous with respect to the rest of the system. In the next step, the matrix A_{22} defines the smallest subset of relations defining the corresponding subset of variables. This reasoning may be pursued until A_{pp}. The matrices A_{ii}

which correspond to sets of variables and relations are called blocks. Each non zero matrix A_{ij}, i j, shows the existence of a causal link, called a direct cause, from the variables of block j to the variables of block i.

Thus, the matrix A furnishes the causal configuration which shows the resolution scheme of the model.

If: $p = 1$ the structure is totally interdependent

$1 < p < n$ the structure is block recursive

$p = n$ the structure is completely recursive

The ordering of the blocks A_{ii} in matrix A is not always unique. In the case of absence of a direct cause between two blocks their order is indifferent. Thus one may define the notion of level as the largest set of blocks none of which verify any direct causal relationship.

In the case where we have a matrix $A_{ji} = 0$ there is no direct cause from block i to block j; nevertheless the variables of block i could have an indirect relation with those of block j even in this case if there exists a set of non zero matrices

$$A_{l_1 i} , A_{l_2 l_1} , \ldots\ldots\ldots A_{j l_n}$$

satisfying $i < l_1 < l_2 \ldots\ldots\ldots\ldots < l_n < j$.

We may then say that there exists an indirect cause from the variables of block A_{ii} to those of block A_{jj}.

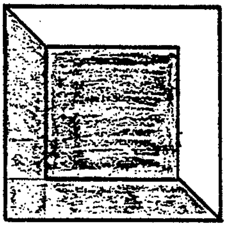
2. RECURSIVE AND INTERDEPENDENT STRUCTURES

Initiated using mainly Boolean algebra, the operationality of the analysis of causal structures of economic models has made substantial progress with the introduction of graph theory. Each model can indeed be represented by a directed graph $G = (X, W)$ in which X is a set of vertices representing all the variables y and z, and W a set of edges.

Starting from the information contained in this directed graph it is possible to compute the blocks of the system defined as strong components of the graph and obtained by the intersection of the set of descendants with the set of ascendants of a vertex; it is also possible to establish a hierarchy of the vertices of the graph, defining the relevent levels and relations between them (1).

The interesting aspect of the analysis, irrespective of the mathematical tool used for its determination, lies obviously in the interpretation of concrete structures of economic models.

The majority of economic models are block recursive but in general they include a very large single interdependent block. Thus most models have a matrix A whose pattern can be portrayed in the following way:



There are logical reasons for the appearance of this type of structure : many economic variables are causally related to others with feed-backs operating directly or indirectly in time-periods shorter than those usually considered by economic models (the year or the quarter); similarly many economic variables are defined by specific aggregation processes (i.e. the national accounts framework) and the interdependence of aggregated models often conceals detailed causal processes at the level of the components of the aggregated variables.

Whatever the reason, it is obvious that if the analysis of causal structures was only aimed at establishing this particular type of structures then we could say that its interest for applied model builders would be rather limited.

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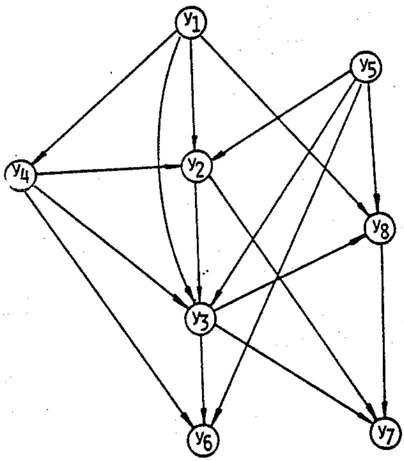
(1) M. Gilli, TROLL PROGRAM CAUSOR, A program for the Analysis of Recursive and Interdependent Causal Structures, MIT-CCREMS, Troll Technical Report 37, has developed a set of computer programs for all the computations referred to in this paper.

This is why a particular attention is to be given in the analysis of causal structures to the study of the causal processes inside the interdependent blocks themselves.

For this purpose one fundamental tool is the minimal graph or skeleton of the interdependent structure.

The minimal graph in the framework of a recursive structure has been studied by Warfield (1); the set is obtained algorithmically starting from the vertices at the bottom of the hierarchy, y_p, and suppressing all the edges $y_i \longrightarrow y_p$ when an indirect path from y_i to y_p exists; the new graph so obtained is again reconsidered in a similar way up to completing the operation for all vertices of the system. It is obvious that many different structures can produce the same minimal graph in which case the structures only differ in their use of direct versus transitive causality for each relation.

If we consider the following graph:

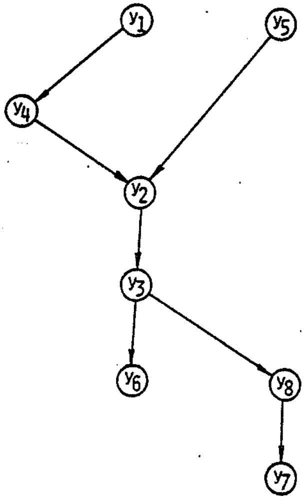


(1) J.N. Warfield, Structuring Complex Systems, Battelle Memorial Institute monograph, No 4, Columbus 1974, later published in J. Warfield, Societal Systems: Planning, Policy and Complexity, Wiley, NY 1977

the minimal graph is given below together with a matrix indicating:

- B in case of an essential direct relation
- D in case of a Direct relation non essential
- I in case of an Indirect relation

The B relations are those of the minimal graph.



	y_1	y_5	y_4	y_2	y_3	y_8	y_6	y_7
y_1								
y_5	.							
y_4	B	.						
y_2	D	B	B					
y_3	D	D	D	B				
y_8	D	D	I	I	B			
y_6	I	D	D	I	B	.		
y_7	I	I	I	D	D	B	.	

The same analysis applied to an interdependent structure has only recently been treated (1) defining minimal graphs, which are not necessarily unique; it has been possible, however, to identify also:

(1) M. Garbely, La Causalité en Econométrie : analyse des structures interdépendantes, Ph. D. thesis, University of Geneva, 1985

a) arcs belonging to all possible minimal graphs of the interdependent structure;

b) arcs not belonging to any minimal graph and therefore of limited interest for the inner functioning of the model;

c) arcs that in certain cases could belong to the minimal graph.

The analyst can then decide upon the combination of c) arcs that gives a plausible economic interpretation to the minimal graph and therefore better explains the causal structure of the interdependent block.

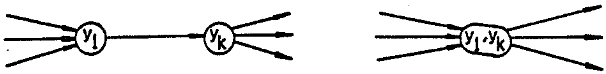
It should be noted that the computation of the minimal graph of an interdependent structure is very heavy and therefore it is recommended to proceed, prior to it, to a simplification of the structure; some of the easier simplification processes are the following:

Condensation

Elimination of intermediate vertices

is replaced by

Contraction



A process of condensation and contraction of the set of vertices can substantially reduce the size of the interdependent block. More comprehensive systems for simplification, including the replacement of entire subsets by a single variable of the interdependent block, require a careful analysis of their economic meaning in order not to loose in the simplification process, the original theoretical ideas embodied in the model.

For the simplified interdependent block, it may be interesting also to compute the essential sets.

The problem of finding the elements responsible for interdependence (they can be either arcs or vertices) corresponds to that of finding a minimal cover in a given graph. A method and the algorithm for the construction of such a cover is described in Gilli and Rossier (1). The set of elements of interest has to be such that any circuit of the block contains at least one of these elements. It is clear that only minimal sets having the property of cutting off all circuits are to be searched. These sets of elements are called essential sets. In the case of arcs, they are called essential feedback arc sets; in the case of vertices, they are called essential feedback vertex sets.

(1) Understanding Complex Systems, Automatica, Vol. 17, 1981

3. THE ANALYSIS OF CAUSAL STRUCTURES AND THE MODELING PROCESS

Up to now the analysis of causal structures, following the original suggestion of H. Simon, has been applied to the better understanding of the causal functioning of existing models. (1). However, the fact that efficient algorithms and computer programmes allow for an easy computation of all the relevent elements of the analysis (blocks, levels, hierarchies, condensable variables, contractable arcs, minimum essential sets, essential relations) makes it possible to use structural analysis during the process of model building itself.

Considering the fact that most models are built by research teams, with a particular internal division of work (some dealing with price relations, other dealing with foreign trade, etc.), the tools of structural analysis could in real time provide insights into the nature of indirect links progressively introduced and their role in the operation of the model and in particular of its interdependent block. It is then prossible to use the overall potential structure in the process of choice of certain relations (i.e. when statistical tests may not provide a satisfactory difference between alternative explanations of a phenomenon) or to modify some modeling "styles" (i.e. the choice of the indexation formula for many aggregated price indices) with a final objective of increasing the recursiveness of the overall model (if, for instance, excessive interdependence provides to the model a black-box characteristic).

4. THE INTERDEPENDENT BLOCK OF THE WHARTON-UAM MODEL OF THE SPANISH ECONOMY

Confirming the findings of other causal structures of large econometric models, the Wharton-UAM model of the Spanish Economy is characterized by a very large interdependent block. As indicated in Appendix A and B the model has 369 variables (317 endogenous), 88 behaviourial relations (229 identities), and two interdependent blocks : one of 173 variables and the second of 4 variables. The hierarchical structure shows 14 levels, with the main interdependent block at level 6. The five previous levels are made up by single-variable blocks and, including the exogenous variables, there are 106 variables above the main interdependent block. Table 3 in Appendix B portrays the main relations between the 14 levels of the model. If the dynamic structure is analysed, it is found (Appendix B, p. 13) that only the variables on t-1 add new relations to the system (15 new relations) and that these new relations considerably increase the size of the interdependent block bringing it to 195 variables, reducing the number of previously determined variables to 99.

(1) G.M. Gallo and M. Gilli, How to strip a model to its essential elements?, Colloque ISMEA-Université d'Ottawa, Oct. 1984.

The main interdependent block in the static structure can be said to determine simultaneously :

- consumption
- household income
- investment
- corporate financing
- foreign trade
- taxes
- price deflators
- social security
- employment
- value-added by sectors.

Including lagged variables, the interdependent block also computes employment in agriculture, activity and unemployment rates, and all public administration accounts.

An in-depth analysis has been made in Appendix B of the nature of the main interdependent block of the static structure (173 variables).

Thus, by a process of condensation and contraction the size of the block has first been reduced to 71 variables (the names of the new set of variables are on page 27 of Appendix B, table 9, while the content of the condensed-contracted variables is to be found in the Annex, on the last table).

In order to give an economic interpretation of this interdependent block, it has been rearranged in 8 sub-systems:

S_1 : Consumption

S_2 : Investment

S_3 : Exports

S_4 : Imports

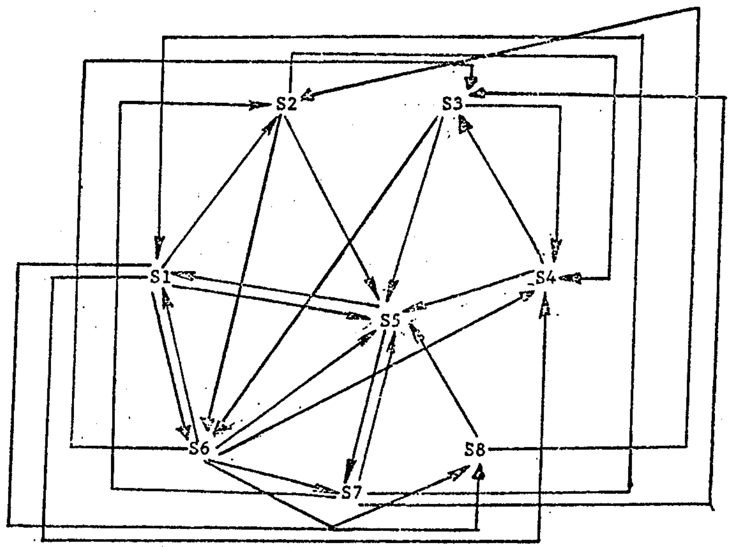
S_5 : GDP (Constant prices)

S_6 : Value-added, employment, productivity and prices (by sectors)

S_7 : Price deflations of consumption and investment

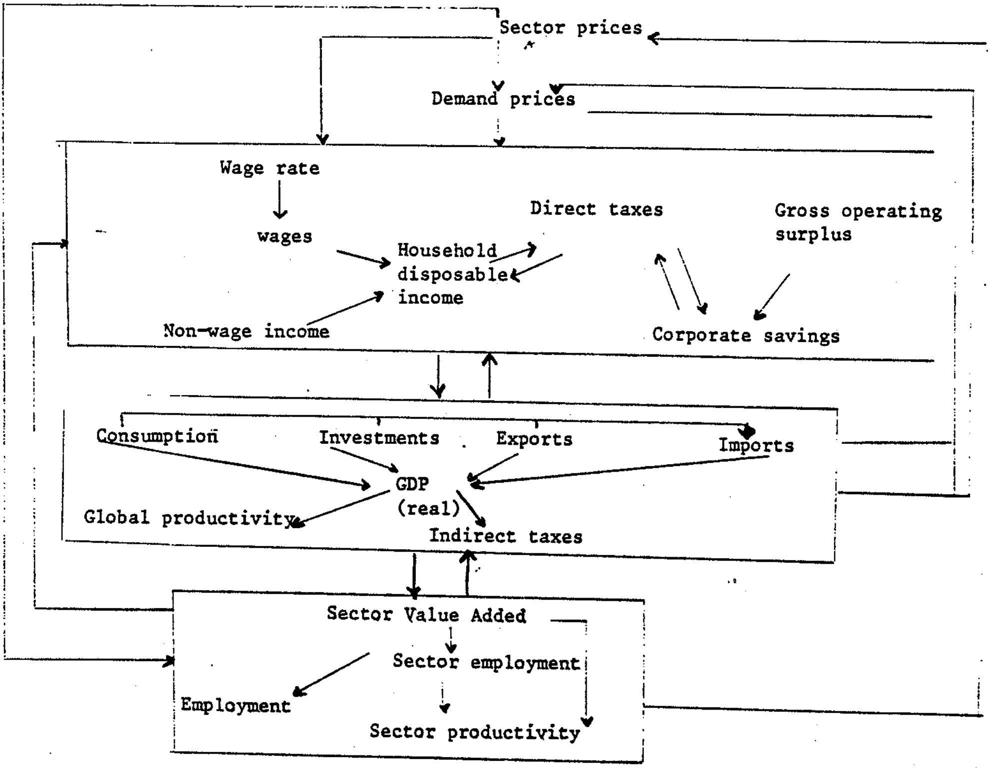
S_8 : Gross savings of corporations.

Tables 10 to 17 in Appendix B describe the structural content of these seven sub-systems and their relations inside the interdependent block are the following



GRAPH 1

MAIN RELATIONS IN THE INTERDEPENDENT BLOCK



Graph 1 summarizes the main structural organization of the interdependent block.

5. FINAL CONSIDERATIONS ON FUTURE DEVELOPMENTS OF THE WHARTON-UAM MODEL OF THE SPANISH ECONOMY

The tools of structural analysis have been used in order to plan changes in the Spanish model reducing the size of the main interdependent block. The model builders wanted to modify data and equations and in this process of model change have used structural analysis as a permanent instrument allowing

a) to identify before making some changes the potential causal consequences, thus orienting the process of change from the point of view of overall complexity;

b) to detect the variables and relations that structurally increase the complexity of the model and have to justify in economic terms this key computational position; this allows to suppress irrelevant complexities.

As examples of this use of structural analysis in the process of introducing changes in the Wharton UAM model of the Spanish economy, we indicate the following:

- the analysis of the essential set of variables of the interdependent block (variables that if made exogeneous will introduce full recursivity) has shown the great importance of price variables. One of the main reasons has been found to be the fact that, in the model, many aggregated price indices are computed from current over constant values of the aggregated variables themselves and thus require a global solution of the model. Computing these average prices indices directly as weighted average indices (Laspeyres) could thus considerably reduce complexity;
  Similarly using for public investment the same price deflator then for total investment introduces an additional interdependence in the investment subsystem that can easily be avoided introducing an equation computing a specific price index for public investment.

- Other structurally important changes but of lesser relevance from the economic point of view include:

  - the change of the wage pressure variable acting upon the price deflators of sectorial output;
  - the definition of imports and exports prices;
  - the use of exogenously defined average tax rates in order to compute some market price indices;
  - the redefinition of global average productivity.

All these changes have been suggested by the structural analysis as a way of reducing the complexity of the model and in particular the size of its interdependent block; none of them implies a modification of the theoretical base of the model; but if they are introduced the size of the main interdependent block is lowered to 77 variables (original model : 173) that can be reduced after contraction and condensation at only 8 groups of variables or computational subsystems. The functioning of the interdependent block is now more transparent in terms of economic interpretation as can be seen in Graph 2, which is to be compared to Graph 1.

GRAPH 2

MAIN RELATIONS OF THE INTERDEPENDENT BLOCK AFTER SUGGESTED STRUCTURAL CHANGES