STUDY BY SYSTEM DYNAMICS OF THE PROBLEM OF THE EQUALIZATION OF THE GAIN RATE

Rafael Pla-López
Faculty of Mathematics
Universitat de València
Spain

Abstract. We study the problem of the adequation of the market prices to some "production prices" with which an intersectorial equalization of the gain rate was obtained in an ideal capitalism of free competition. We criticize the treatment of this problem by Marx, Sweezy and Salama-Valier, by its static or aprioristic character. And we propose a system of dynamic regulation utilizing the intersectorial coefficients of an input-output table and supposing full mobility of the capital. The study of this system permits to conclude that the equilibrium values of the gain middle rate and of the production prices depend exclusively of the intersectorial coefficients of the directly or indirectly productive sectors. From this, we study the evolution of the market prices, of the production prices, of the capital organic composition, of the gain sectorial rates and of the gain middle rate from a modification of these coefficients in an equilibrium situation.

INTRODUCTION

The concept of "production price" was introduced by Marx (1894) to explain the functioning of the capitalist systems of free competition and with full mobility of the capital: the equilibrium of such systems demands that gain rate is the same in every the production sectors; so, the amount of the prices must permits an equalization of the gain rate: these are the "production prices".

But Marx do not explain the formation of the production prices when the different sectors have different capital organic compositions (quotient between the capital invested in production and means and the capital invested in work power): he suppose only the existence of these production prices without to explain its genesis.

Sweezy (1972) want to explain this genesis through the movement of capitals; but his explanation carries to conclude that this movement goes to the sectors with lower capital organic composition. Salama-Valier criticize correctly this explanation by to be in contradiction with the real history, in which the capital organic composition increases: more and more, the capital is invested in greater proportion in machines, and not in wages.

But Salama-Valier, to avoid this contradiction, suppose that the equalization of the gain rate is independent of the movement of capitals, and in the sectors with greater capital organic composition the greater productivity carries directly to increase the price until the production price. But this reasoning is aprioristic, and supposes the sane thing which must be explained.

In fact, in the treatment of this authors there is a confusion between the logical process and the real dynamic process: in the marxist theory, the "production price" is a deviation from the "value" of the product (measured by the work time socially necessary for its productions); but in the reality the production prices evolve from the prior production prices.

REGULATION BETWEEN PRICES AND AMOUNTS

To explain the formation of the production prices, we use a system of multiple regulations between amounts and prices from the classical supply and demand curves (see for example Lipsey (1967)). So, we introduce the following sub-systems:

System 1: $Of^{I}=k_{1}pr^{E_{O}}$ (1)

where Of^{I}=ideal supply, pr=price, E_{O}=supply elasticity.

System 2: $Of(t)=Of(t-1)+(Of^{I}(t-1)-Of(t-1))/t_{1}$ (2)

where Of=real supply, t=time, t_{1}=adjustment time.

System 3: $Dem^{I}=Of$ (3)

where Dem^{I}=ideal demand.

System 4: $pr^{I}=\sqrt[E_{D}]{Dem^{I}/k_{2}}$ (4)

where pr^{I}=ideal price, E_{D}=demand elasticity.

System 5: $pr(t)=pr(t-1)+(pr^{I}(t-1)-pr(t-1))/t_{2}$ (5)

where t_{2}=adjustment time.

System 6: $k_{1}^{I}=k_{2}pp^{E_{D}-E_{O}}$ (with $|E_{O}|<|E_{D}|$) (6)

where k_{1}^{I}=ideal supply constant, pp=production price.

System 7: $k_{1}(t)=k_{1}(t-1)+(k_{1}^{I}(t-1)-k_{1}(t-1))/t_{3}$ (7)

where t_{3}=adjustment time.

System 8: $Dem=k_{2}pr^{E_{D}}$ (8)

where Dem=real demand.

System 9: m=minimum of (Of, Dem).

System 10: $k^{i}=\sum_{j}\alpha^{ij}pr^{j}$ (9)

where k^{i}=unitary cost of the product "i", α^{ij}=intersectorial coefficients.

System 11: $g_{m}=\left(\sum_{i} pr^{i}\, m^{i} \,/\sum_{j} k^{j}\, Of^{j}\right)-1$ (10)

where g_{m}=gain middle rate.

System 12: $pp^{i}=(1+g_{m})k^{i}$ (11)

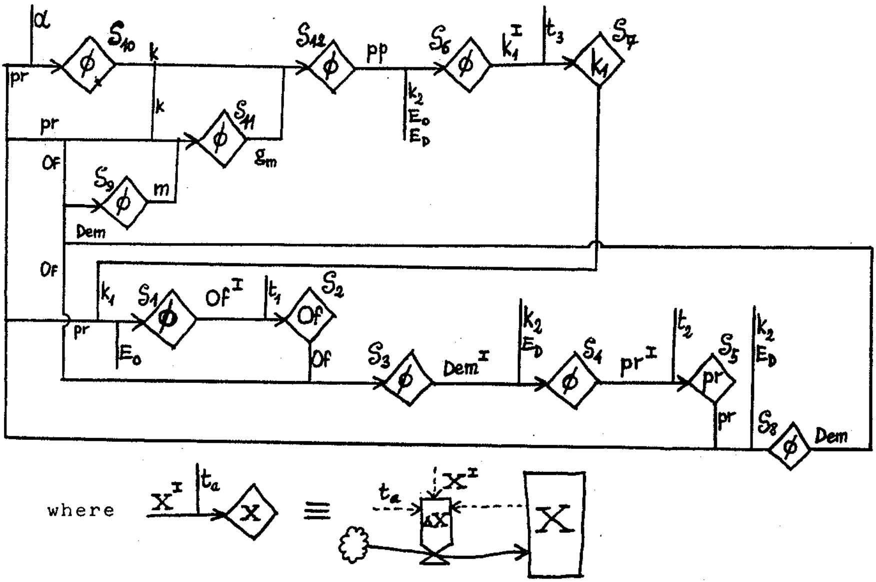

Fig. 1 Multiple regulations between prices, supply and demand.

The inter-relations between the Systems 1 to 12 are showed in the Figure 1. Note that the intersectorial movement of the capitals is described by the Systems 6 and 7 through the change of the supply curve (characterized by the constant k_1).

For a k_1 gived, if the stability condition $|E_0|<|E_D|$ is fulfilled, the Systems 1, 2, 3, 4, 5 and 8 tend to an equilibrium situation in which $pr=pr^I$ and $Dem=Dem^I=Of=Of^I$. If we name "p" and "q", respectively, the equilibrium values of the price and the amount, we have

$$p=\sqrt[E_0-E_D]{k_2/k_1} \qquad (12)$$

$$q=\sqrt[E_0-E_D]{k_2^{E_0}/k_1^{E_D}} \qquad (13)$$

from the equations 1, 3, 4 and 8.

If we suppose the intrasectorial regulations (described by the Systems 2 and 5) are very faster than the intersectorial regulation (described by the System 7), that is to say, t_1 and t_2 are very smaller than t_3, we can write

<u>System 6':</u> $k_1^I=k_1(p/pp)^{E_0-E_D}$ (14)

<u>System 10':</u> $k^i=\sum_j \alpha^{ij}p^j$ (15)

<u>System 11':</u> $g_m=\left(\sum_i p^i q^i/\sum_j k^j q^j\right)-1$ (16)

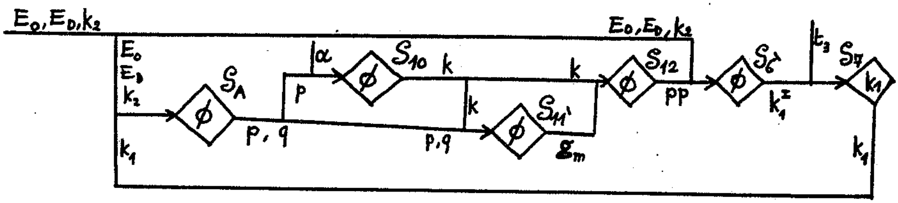

Fig. 2 Formation of the production prices through an intersectorial regulation.

Now, if we suppose that the price "p" and the amount "q" have the values corresponding to an intrasectorial equilibrium, we can show the inter-relations for the intersectorial regulation in the Figure 2. There, the Sistem A is defined by the equations (12) and (13).

From the equations (7) and (14), there is intersectorial equilibrium when $k_1=k_1^{\,i}$, and therefore the actual price "p" is equal to the production price "pp".

So, for the equations (11) and (15), the intersectorial equilibrium condition will be

$$\sum_j \alpha^{ij}\, pp^j = pp^i/1+g_m \qquad (17)$$

That is to say, $1/1+g_m$ is the **self-value** of the intersectorial coefficients matrix, and (pp^i) is its **self-vector.** So, the gain middle rate and the production prices in equilibrium depend only of the intersectorial coefficients. Its values can be obtained from the characteristic equation

$$\mathrm{Det}(\alpha^{ij}(1+g_m)-\delta_{ij}) = 0 \qquad (18)$$

The indetermination of the production prices "pp" for each solution of (18) is not significant, since the only important from an economic point of view is the relative proportions between the different production prices.

THE EVOLUTION OF THE EQUILIBRIUM CONDITIONS

To study the evolution of the situations of intersectorial equilibrium, we are going to suppose 3 sectors of the production:

1: **directly productive** sector (which produces the production means).

2: **indirectly productive** sector (which produces consumption objects for the workers).

3: **improductive** sector (which produces consumption objects for people which do not work).

So, the coefficients α^{i3} are equal to zero, that is to say

$$\begin{Bmatrix} k^1 \\ k^2 \\ k^3 \end{Bmatrix} = \begin{Bmatrix} \alpha^{11} & \alpha^{12} & 0 \\ \alpha^{21} & \alpha^{22} & 0 \\ \alpha^{31} & \alpha^{32} & 0 \end{Bmatrix} \begin{Bmatrix} p^1 \\ p^2 \\ p^3 \end{Bmatrix} \qquad (19)$$

And, if we define $\lambda=1/1+g_m$, from (18) we have

$$\mathrm{Det}(\alpha^{ij}-\lambda\delta_{ij}) = 0 \qquad (20)$$

and so

$$\mathrm{Det}\begin{Bmatrix} \alpha^{11}-\lambda & \alpha^{12} & 0 \\ \alpha^{21} & \alpha^{22}-\lambda & 0 \\ \alpha^{31} & \alpha^{32} & -\lambda \end{Bmatrix} = 0 \ , \qquad (21)$$

that is to say

$$(\alpha^{11}-\lambda)(\alpha^{22}-\lambda) - \alpha^{12}\alpha^{21} = 0 \qquad (\lambda \text{ is not zero}) \qquad (22)$$

Therefore

$$\lambda = \frac{\alpha^{11}+\alpha^{22} \pm \sqrt{(\alpha^{11}+\alpha^{22})^2 - 4(\alpha^{11}\alpha^{22}-\alpha^{12}\alpha^{21})}}{2} \ , \qquad (23)$$

that is to say

$$\lambda = \frac{\alpha^{11}+\alpha^{22}}{2} \pm \sqrt{\left(\frac{\alpha^{11}-\alpha^{22}}{2}\right)^2 + \alpha^{12}\alpha^{21}} \qquad (24)$$

But, from (17),

$$\lambda\sum_j \alpha^{ij}\, pp^j = pp^i \qquad (25)$$

and so, from (19),

$$pp^2=(\lambda-\alpha^{11})pp^1/\alpha^{12} \ , \quad pp^1=(\lambda-\alpha^{22})pp^2/\alpha^{21} \qquad (26)$$

But the production prices and the intersectorial coefficients must be positive numbers, and therefore

$$\lambda > \alpha^{11} \text{ and } \lambda > \alpha^{22} \qquad (27)$$

and so

$$\lambda > \frac{\alpha^{11}+\alpha^{22}}{2} \qquad (28)$$

Therefore, in (24) the sign plus is the correct one, and

$$g_m = \frac{\alpha^{11}+\alpha^{22}-2(\alpha^{11}\alpha^{22}-\alpha^{21}\alpha^{12}) - \sqrt{(\alpha^{11}+\alpha^{22})^2-4(\alpha^{11}\alpha^{22}-\alpha^{21}\alpha^{12})}}{2(\alpha^{11}\alpha^{22}-\alpha^{21}\alpha^{12})} \qquad (29)$$

Observe that the gain middle rate does not depend on the intersectorial coefficients of the improductive sector (number 3).

Now, if the intersectorial coefficients of the directly or indirectly productive sectors change, the equilibrium will be broken.

Evolution of the gain sectorial rates

We can define the **gain sectorial rates** as

$$g^i = p^i/k^i - 1 = p^i / \sum_j \alpha^{ij} p^j - 1 \tag{30}$$

If there is equilibrium in the instant "t",

$$1+g^i(t+1) = \frac{pp^i(t)}{\alpha^{i1}(t+1)pp^1(t)+\alpha^{i2}(t+1)pp^2(t)} =$$

$$= (1+g_m(t))\frac{\alpha^{i1}(t)pp^1(t)+\alpha^{i2}(t)pp^2(t)}{\alpha^{i1}(t+1)pp^1(t)+\alpha^{i2}(t+1)pp^2(t)} \tag{31}$$

So, if some intersectorial coefficient $\alpha^{i_o j}$ decreases for a sector "i_o", the gain sectorial rate g^{i_o} will be greater than the prior gain middle rate; if the others intersectorial coefficients do not change, then the others gain sectorial rates will be equal to the prior gain middle rate. So, g^{i_o} will be the greatest gain sectorial rate, and therefore the movement of capitals will happen toward the sector "i_o". The movement of capitals will finish when every the gain sectorial rates will be equals to the new gain middle rate according to (29).

Evolution of the market prices and amounts

In the instant "t+1" (going out from the equilibrium) $p^i(t+1)=pp^i(t)$ and $q^i(t+1)=q^i(t)$, and so, from (15), (16) and (17),

$$1+g_m(t+1) = (1+g_m(t))\sum_{i,s}\alpha^{is}(t)pp^s(t)q^i(t) / \sum_{j,l}\alpha^{jl}(t+1)pp^l(t)q^j(t) \tag{32}$$

Therefore, if some intersectorial coefficient decreases and the other ones stay with the same values, then $g_m(t+1) > g_m(t)$.

So, for $i \neq i_o$, from (15), $k^i(t+1)=k^i(t)$, and so, from (11), $pp^i(t+1) > pp^i(t)$.

Hence, from (14), $k_1^{iI}(t+1) < k_1^i(t+1)$, and, from (7), $k_1^i(t+2) < k_1^i(t+1)$.

Therefore, from (12), $p^i(t+2) > p^i(t+1) = p^i(t) = pp^i(t)$, that is to say, the **market prices** increase for the sectors $i \neq i_o$.

Also, from (13), $q^i(t+2) < q^i(t+1) = q^i(t)$ (since $E_p < 0$). That is to say, the amounts decrease in these sectors (for the movement of capitals toward i_o).

On the other hand, from (11), (15) and (32),

$$\frac{pp^{i_o}(t+1)}{pp^{i_o}(t)} = \frac{\sum_{i\neq i_o,s}\alpha^{is}(t)pp^s(t)q^i(t)+\sum_s\alpha^{i_o s}(t)pp^s(t)q^{i_o}(t)}{\sum_{j\neq i_o,l}\alpha^{jl}(t)pp^l(t)q^j(t)+\sum_l\alpha^{i_o l}(t+1)pp^l(t)q^{i_o}(t)} \cdot \frac{\sum_j\alpha^{i_o j}(t+1)pp^j(t)}{\sum_r\alpha^{i_o r}(t)pp^r(t)} < 1$$

Therefore, from (14), (7), (12) and (13),

$p^{i_o}(t+2) < p^{i_o}(t+1) = p^{i_o}(t) = pp^{i_o}(t)$ and $q^{i_o}(t+2) > q^{i_o}(t+1) = q^{i_o}(t)$. That is to say, the **market price** decreases and the amount increases for the sector i_o (for the movement of capitals toward this sector).

Evolution of the equilibrium production prices

For the new equilibrium production prices, from (26) and (24),

$$pp^2/pp^1 = \left(\frac{\alpha^{22}-\alpha^{11}}{2}+\sqrt{\left(\frac{\alpha^{22}-\alpha^{11}}{2}\right)^2+\alpha^{12}\alpha^{21}}\right) / \alpha^{12} \qquad (33)$$

And now, deriving with regard to α^{22} (for example),

$$\partial(pp^2/pp^1)/\partial\alpha^{22} = \left(\sqrt{\left(\frac{\alpha^{22}-\alpha^{11}}{2}\right)^2+\alpha^{12}\alpha^{21}} + \frac{\alpha^{22}-\alpha^{11}}{2}\right)/2\alpha^{12}\sqrt{\left(\frac{\alpha^{22}-\alpha^{11}}{2}\right)+\alpha^{12}\alpha^{21}} > 0$$

So, if α^{22} decreases (saving in work power in the sector 2), the equilibrium production price of the sector 2 decreases in relation to the sector 1, according to the happening for "t+2".

Evolution of the equilibrium gain middle rate

According to (32), the gain middle rate increases when the intersectorial coefficients decrease breaking the equilibrium. To study the evolution of the **equilibrium gain middle rate**, we are going to derive with regard to some intersectorial coefficient, for example α^{22}. So, from (24),

$$\partial\lambda/\partial\alpha^{22} = \left(\sqrt{(\alpha^{11}-\alpha^{22})^2+4\alpha^{12}\alpha^{21}} - (\alpha^{11}-\alpha^{22})\right)/2\sqrt{(\alpha^{11}-\alpha^{22})^2+4\alpha^{12}\alpha^{21}} > 0 \qquad (34)$$

Therefore, if α^{22} decreases, then λ decreases and the equilibrium gain middle rate increases.

Capital organic composition

According to Marx (1867), the **capital organic composition** of the sector "i" is

$$o^i = \frac{\text{cost of the production means}}{\text{cost of the work power}} = \frac{\alpha^{i1}p^1}{\alpha^{i2}p^2} \qquad (35)$$

Therefore, if the intersectorial coefficient α^{i2} decreases (saving in work power in the sector "i"), then the capital organic composition increase. But also a capital displacement toward this sector is produced, and the gain middle rate increases.

CONCLUSIONS

The equilibrium values of the gain middle rate and of the production prices depend exclusively of the intersectorial coefficients of the directly or indirectly productive sectors.

The diminution of intersectorial coefficients with output in a given sector determines a capital displacement toward this sector. It permits to explain the capital displacement toward sectors with greater capital organic composition.

Any diminution of intersectorial coefficients with output in productive sectors determines an increasing of the gain middle rate. It contradicts the supposition that the increasing of the capital organic composition involves a diminution of the gain middle rate: Marx was wrong in this point.

REFERENCES

Forrester, J.W. (1968). Principles of systems. Wright-Allen Press, Cambridge.

Lipsey, R.G. (1967). Introducción a la economía positiva. Vicens-Vives, Barcelona.

Marx K. (1867). El Capital, vol.I. Fondo de Cultura Económica, México, 1959.

Marx K. (1894). El Capital, vol.III. Fondo de Cultura Económica, México, 1959.

Salama P., and Valier, J. (1973). Une introduction a l'économie politique. Maspero, Paris.

Sweezy, P.M. (1972). Teoría del desarrollo capitalista. Fondo de Cultura Económica, México.