CHAOTIC BEHAVIOR IN PREDATOR—PREY—FOOD SYSTEM DYNAMICS MODELS.

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1. Introduction.

A great interest has been raised recently on chaotic behavior in system dynamics models. This interest is largely justified. The discovery that deterministic systems can show chaotic behavior has deep consequences for the system dynamicist. Among other things, it is well known that strange attractors show a pathological sensitivity to initial conditions. This property impedes the use of a single trayectory (obtained by simulation) as representative of the system behavior. So, the traditional working way of the system dynamicist should be deeply reconsidered if these strange attractors are exhibited by their models. This last is higly possible due to the nonlinear character of these models. Therefore, the system dynamicist should be able to study whether or not those attractors appear in his models. If they appear, then the classical study through the analysis of the trajectory should be rejected, and studies of an stochastic nature -not yet well understood- should be undertaken.

In this paper we study some ecological system dynamics models where such strange attractors seem to appear.

2. Modeling predator-prey-food systems.

The problem of modelling the chain of relations between predator, prey and food (for the prey) can be solved using system dynamics. Of this three-levels structure the sub-structure formed by predator and prey, and the one formed by the prey and their food have been previously discussed in the literature.

The former was studied forty years ago by Lotka and Volterra in their classical model. Today it is well known that this model is unsatisfactory as far as it is structurally unstable (the only equilibrium is a center). Fortunately we have a system dynamics elaborated version which does not contain this restriction (Henize 71).

The qualitative analysis of this latter version of the predator-prey system shows that the model exhibits some interesting behaviors (point attractors and limit cycles) related by Hopf bifurcations, that can be fully analyzed (Toro et al. 84, Toro 86).

The second sub-structure (the prey and food one) can be exemplified by the Kaibab system dynamics model (Goodman 74). This model is well known by the system dynamicists, and it describes the collapse of the carrying capacity of the Kaibab plateau, due to the uncontrolled growth of the deer population (the prey) as a consequence of a substantial extirpation of the natural predators of the deer (cougars, wolves and coyotes). The qualitative analysis of this model, using a two time scales technique, permits understanding of the dynamic mechanism that explains how the collapse is produced (Toro 86).

Now, in this paper, modeling that merges both sub-structures into a single one is proposed. As a matter of fact not one but two merging models are introduced. Both rest upon different hypothesis, but both show many interesting behaviors, particularly chaotic motion. These two models will be discussed in what follows, with special emphasis on the chaotic motion they can show.

3. Three levels model with external forcing.

Consider a predator-prey model given by the equations:

$$\dot{x} = x(n_1\tau_1(x)-\tau_2(x)/b_1-n_2z\tau_3(x)) \qquad (1)$$

$$\dot{z} = b_3z(\tau_5(\theta_3(x))-\alpha_2\tau_6(\theta_3(x)))$$

The Forrester diagram of that system is shown in Fig. 1. The meaning of the variables and parameters is given in Appendix and the shape of the tables is given in Fig. 2. This model is a slight modification of one taken from (Henize 71) and for appropiate values of the parameters shows an oscillatory periodic behavior, which is characteristic of systems whose attractor is a limit cycle. The qualitative analysis of this model has been developed elsewhere (Toro et al. 84), where it has been shown

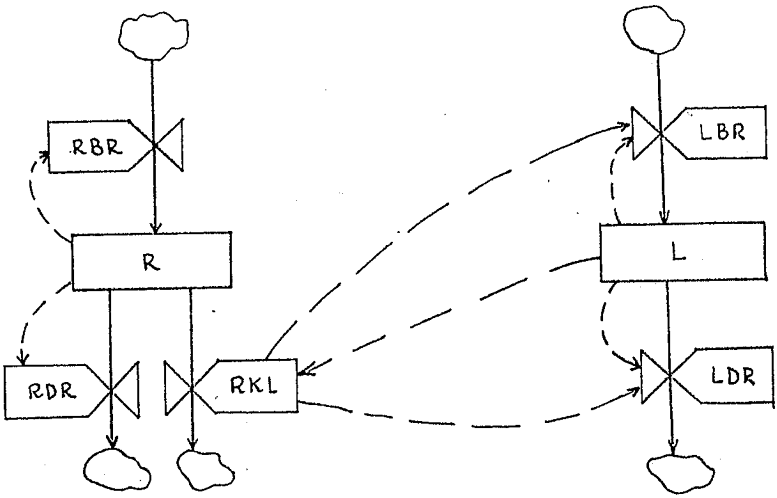

Figure 1. Forrester diagram of the model.

that this model shows a Hopf bifurcation for parameter α_2. Behaviour modes of (1) are resumed in Fig. 3a, 3b in the parameter plane (n_1, α_2).

In equations (1) neither the area of the habitat, nor the amount of food resources of that habitat (the prey food) appear explicitly. We shall introduce the following variables to take into account in the model those attributes of the habitat:

AREA = area of the region (habitat).

y = resources supplied by the region.

RCP = resources consumed by every prey.

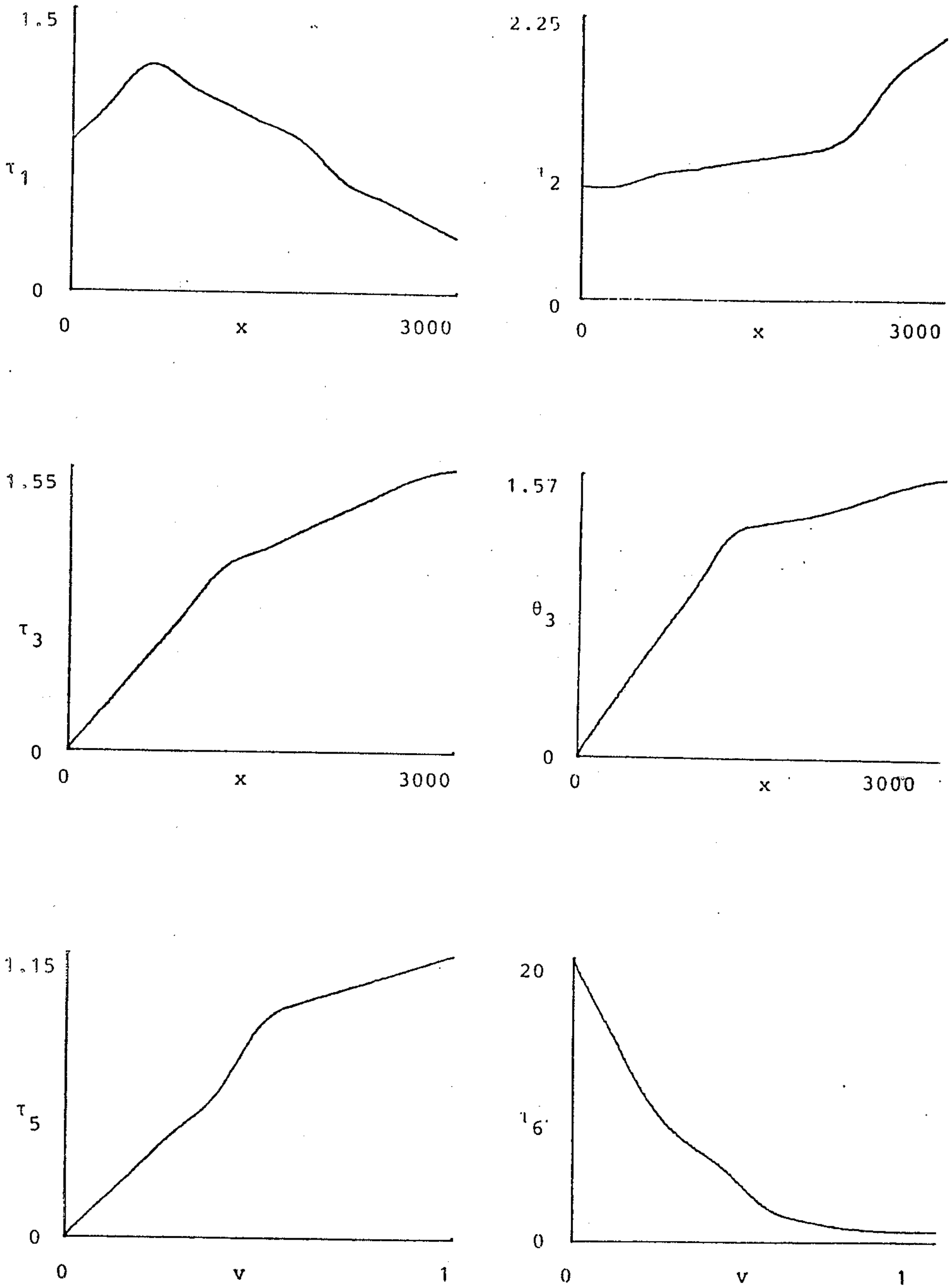

Figure 2. Tables of the model.

Furthermore, the tables TT1, TT2 and TT3 are introduced. Those tables are related with τ_1, τ_2 τ_3 (Fig. 2) throught a change of scale in the axis, in such a way that:

$\tau(0)$ = TT1(1), $\tau(3000)$ = TT1 and so for TT2, TT3.

Given appropiate values to the new parameters Eqs. (1) can be rewritten:

$$\dot{x} = x(n_1 TT1(RCPx/y) - TT2(RCPx/y)/b_1) - n_2 z TT3(x/AREA) \qquad (2)$$

$$\dot{z} = b_3 z(\tau_5(\theta_3(x)) - \alpha_2 \tau_6(\theta_3(x)))$$

Up to now the hypothesis is that the amount of resources is constant. If we change that hypothesis and assume that the amount of resources changes periodically (this can be related to cycles in nature due to annual cycles), then the variable y takes the

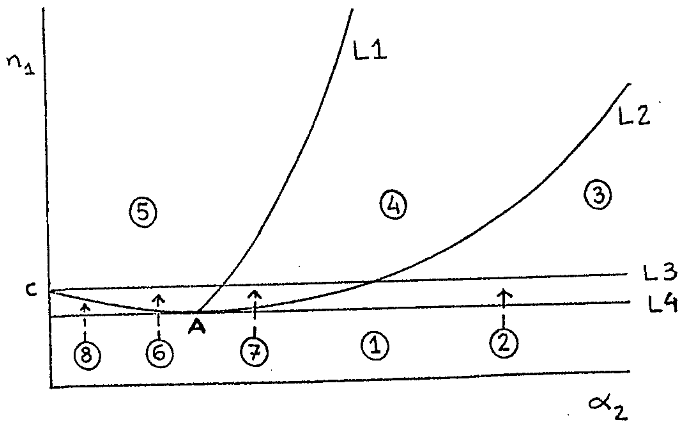

Figure 3a. Regions in the parameter plane n_1, α_2.

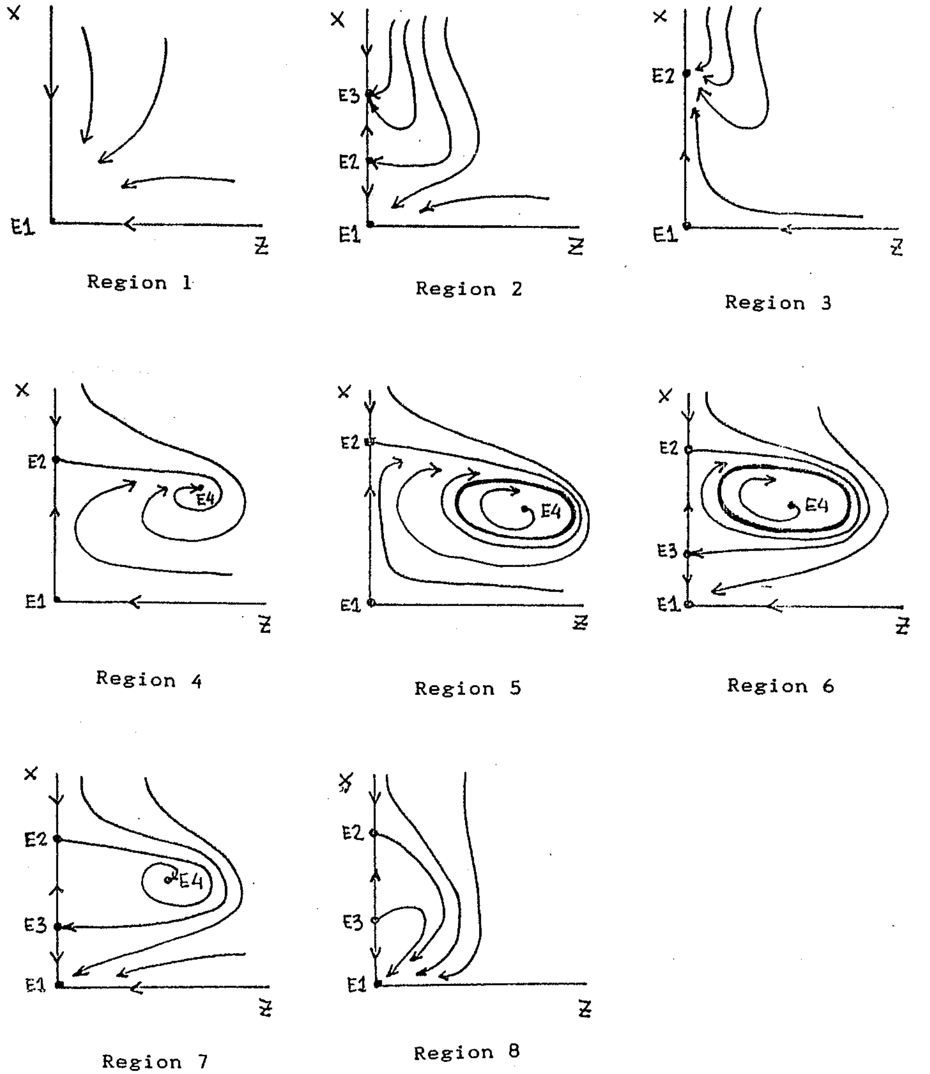

Figure 3b. Behaviour modes of the model.

form.

$$y = REN+AMP*SIN(2*PI*t/PERIOD) \tag{3}$$

where REN stands for the average value of the resources, and AMP and PERIOD for the amplitude and period of the oscillation respectively.

Equations (2), together with (3), form a nonautonomous differential equation with forced oscillations. These equations are written:

$$\begin{aligned} \dot{x} &= x(n_1 TT1(RCPx/y)-TT2(RCPx/y)/b_1)-n_2 zTT3(x/AREA) \\ \dot{z} &= b_3 z(\tau_5(\theta_3(x))-\alpha_2\tau_6(\theta_3(x))) \\ y &= REN+AMP*SEN*(2*PI*t/PERIOD) \end{aligned} \tag{4}$$

That system can show a great variety of behaviors. These behaviors for AMP = 0 are resumed in Fig. 3a, 3b.

For AMP > 0, PERIOD > 0, and values of α_2 belonging to the region to the right of curve L1 and above L4 (Fig. 3a) the system shows a periodic behavior of the same period as the forcing function y. In this case, for AMP = 0 the system (4) shows a point atractor.

However, for n_1 and α_2 in the regions 5 or 6 (Fig. 3a, 3b) the system (4) has a periodic attractor for AMP = 0. In that case, for AMP > 0 the system (4) shows complex behaviors, including strange attractors. Fig. 4a, 4b shows a plot the output

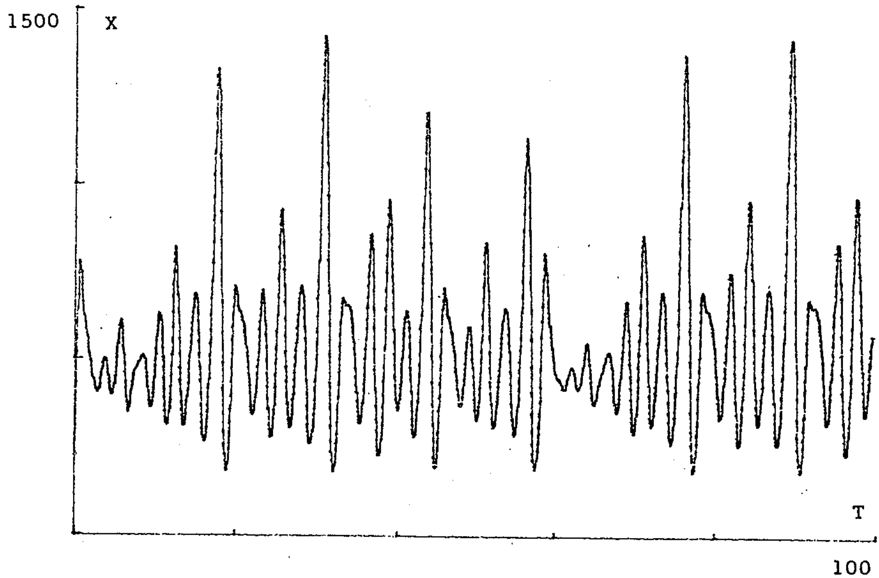

Figure 4a.

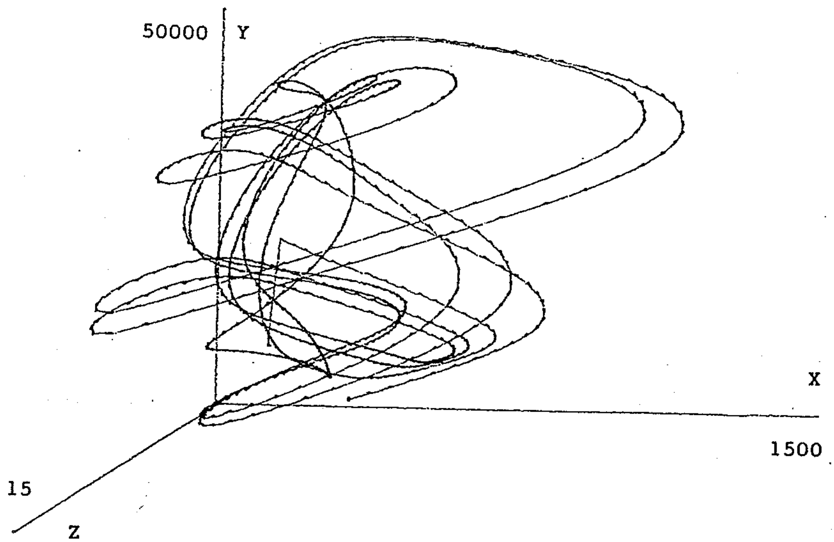

Figure 4b. Output of the system with PERIOD = 6.5, AMP = 20000.

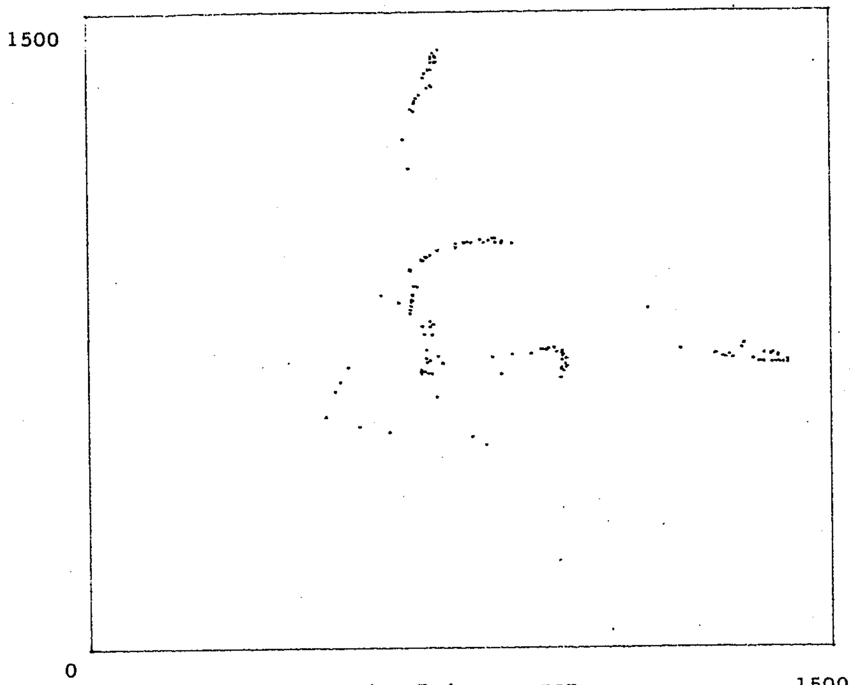

Figure 4c. Poincare map.

of the system for that case.

Fig. 4c shows the Poincare map for the system (4) with AMP > 0 and parameters n_1, α_2 in region 5 (Fig. 3a). The shape of Poincare map confirms system (4) has a chaotic attractor.

Fig. 5a shows a new output of (4) for other values of the parameter in region 5 also. Fig. 5b shows the Poincare map. This figure shows an attractor called a torus.

The results of the analysis of this model are very similar to the ones reached by Rasmussen and coworkers for a simple model of the economic long wave, where a Hopf bifurcation for the unforced system and an strange attractor for the forced system have been found (Rasmussen et al. 84).

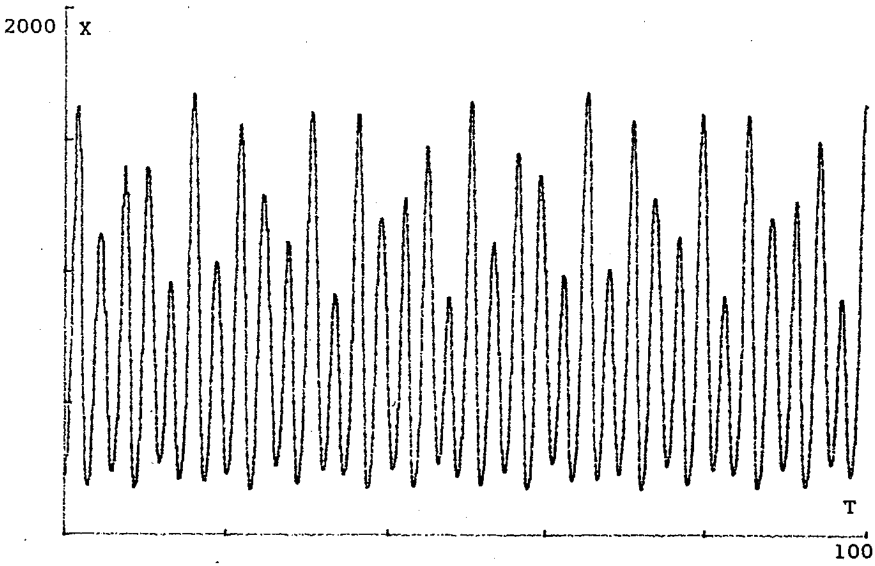

Figure 5a. Output of the system with PERIOD =7, AMP =10000.

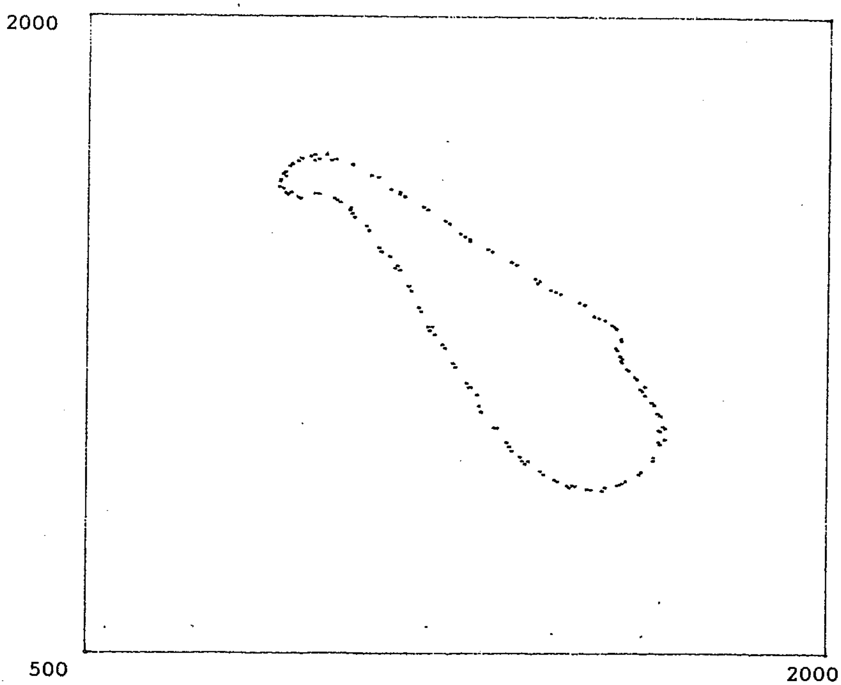

Figure 5b. Poincare map.

4. Autonomous three level system.

In the previous section the three-stage system considered was a predator-prey one with the prey food acting as a forcing function. Now we are going to consider a three-stage autonomous system. This system has been inspired by a mixing of a model of predator-prey type, as the one considered in the previous section, with one of the Kaibab type, this last regarding to relations between prey and their food. As a matter of fact, the model here introduced can be considered as a modification of the Kaibab type model, when the predator are not extinguished but it is allowed an interaction between predators and prey of the kind sugested by the Lotka-Volterra models.

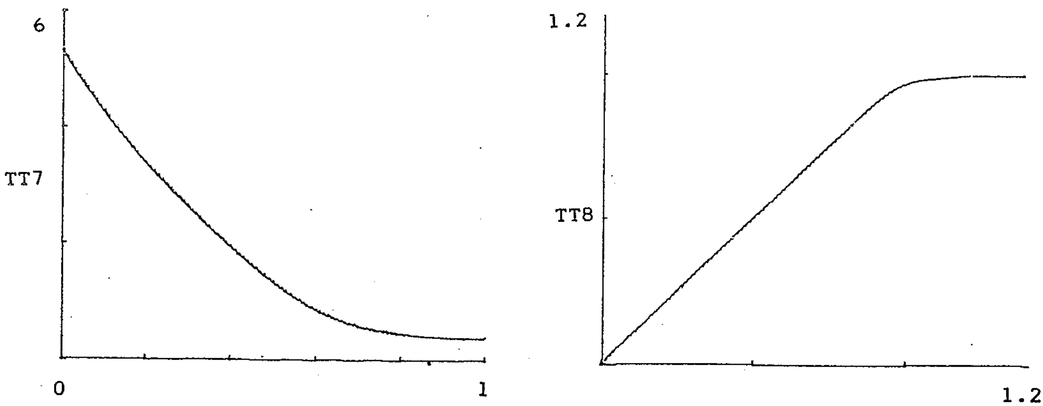

Figure 6. Tables TT7, TT8.

The proposed model is:

$$\dot{x} = x(n_1TT1(RCPx/y)-TT2(RCPx/y)/b_1)-n_2zTT3(x/AREA) \qquad (5.1)$$

$$\dot{z} = b_3z(TT5(\theta_3(x))-\alpha_2TT6(\theta_3(x)) \qquad (5.2)$$

$$\dot{y} = (RM-y)/TT7(y/RM)-yTT8(RCPx/y)k \qquad (5.3)$$

where x and z stand for the prey and predator populations, and y for the food resources supplied by the habitat. Tables TT7 and TT8 are shown in Fig. 6.

A careful analysis of equations (5.1.) and (5.2.) shows that they are analogous to the (2); whereas equations (5.2) and (5.3), considering z constant, are a model of the Kaibab type, analogous to the one in (Goodman 74).

In equations (5) they appear a constant k whose relative value is of a great interest to generate the different behavior modes the model can exhibit. For different values of model parameters the model can show point attractors, limit cycles and strange attractors.

Fig. 7a shows the output of the system for α_2 = 0.25, and Fig. 7b shows the Poincare map for this case. This map has a hump and it is well known that maps with this shape are related with chaotic attractors.

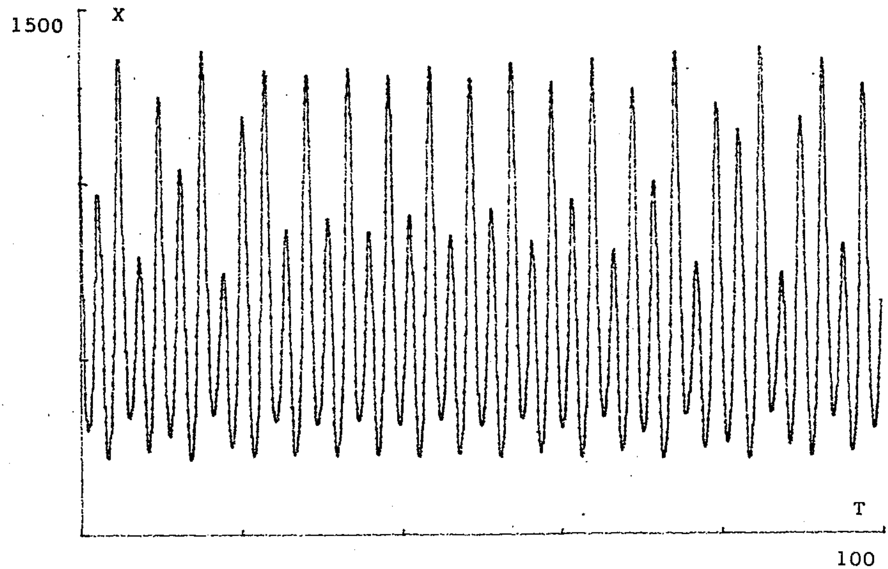

Figure 7a.

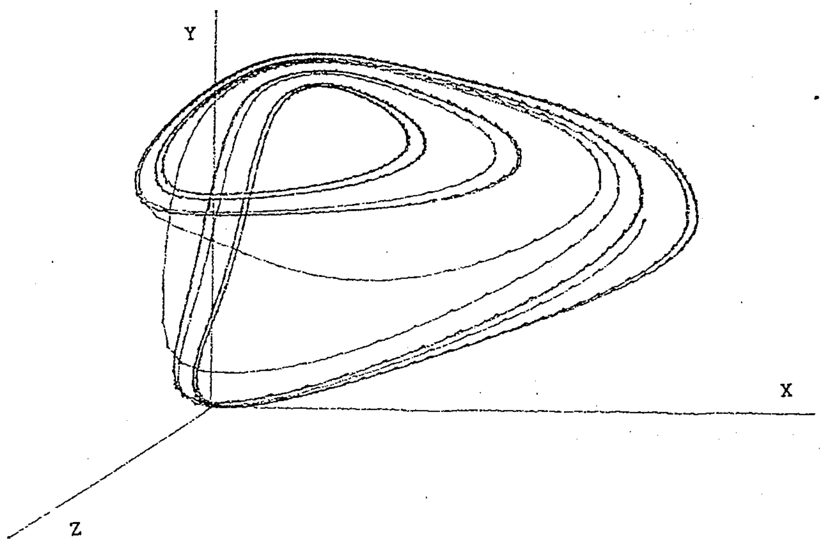

Figure 7b. Output of the system with k = 5.

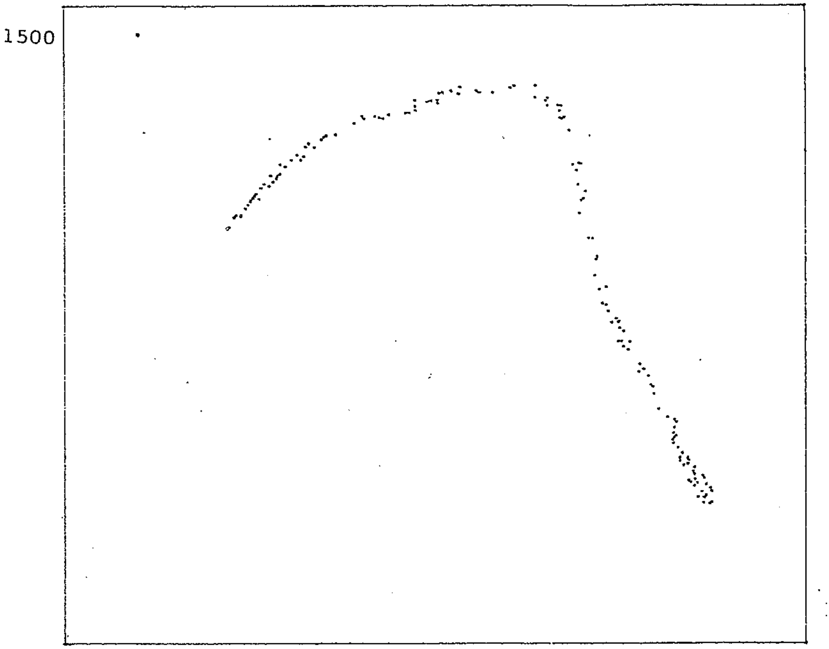

Figure 7c. Poincare map.

5. Conclusions.

Some interesting reflections are sugested by the previous results. We have a single problem (the predator-prey-food system) and two different models (based on different assumptions). Traditionally the system dynamicist programes his model on a computer, takes a plot of the output-model and compares that plot with the real data supplied by the system under consideration. If the two models are in conflict, this will be solved by the "best data fitting" principle. However, if we try to apply that procedure to the system considered above, where strange attractors appear, we are faced with the impossibility of solving it. We could try to fit the trajectories of either of the models to real data. But as far as the attractor is chaotic there are no trajectories which is representative of the model behavior, so we have not got a reference significative of the model behavior. The system dynamicist is faced in such a case with a deep problem regarding his methodology.

Appendix.

R = Rabbits

RBR = Rabbits birth rate

m_1 = 3 Normal birth rate

RDR = Rabbits death rate

b_1 = 1 Rabbit average lifetime

RKL = Rabbits killed by lynx

n_2 = 500 Lynx kill normal

L = Lynx

LBR = Lynx birth rate

b_3 = 1.5 Lynx births normal LDR = Lynx death rate

b_2 = Lynx average lifetime

$\alpha_2 = 1/(b_3 b_2) = 0.25$

REN = 30000

RM = 40000

RCP = 20

Acknowledgment

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