STUDYING THE RELATIONSHIP AMONG THE WHOLE, PARTS AND ENVIRONMENT OF A SYSTEM DYNAMIC MODEL
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Abstract

From the viewpoint of system dynamics the whole structure and functions of a system do not simply equal the algebraic sum of the parts which the system consists of. There universally exist mutual relationships between the transmition and exchange of information, energy and material within the whole system, its parts and its environment. The aim of this paper is to study the organic ties among the three of the whole, parts and environment of a system and the transmition involved under specific conditions. In this paper, the logical relationships between the internal and external parts of the system are described. Two concepts of the transmition matrix and the relationship matrix are presented along with the definitions of the grades of the grades of variables. Their concepts and definitions, combining the theory of the model reference self-compensation, are used to form a new method by which the functions of the structures and relationship between the whole system and its parts can be identified. Some satisfied results have been obtained from testing this new method on the Boom-town Dynamic Model.

I - INTRODUCTION

It was pointed out by Aristotle that the whole is more than the sum of the parts. The behavior of a system is dependent not only on the structures and functions of the parts and the influence of the environment on the system, but also on the feedback organization of all parts within the system. That is, the whole function and behavior of the system result from the common interactions of the internal parts with the external influence. It is the entirety principle of systems. Such a case may practically be faced in order to discuss the most interesting parts of a system after having built a wider ranged dynamics model. These parts need to be drawn from the model and considered as individual units for analysis (change the parameters, extend or simplify the structure etc.). And besides, when we start to build a system dynamics model, we often have difficulties drawing the system boundary all at once because of lack of experience. Usually a larger scale model included the relative factors is in order and is built first. Then the proper parts are gradually divorced by repeated analyzing and testing. In whichever case above, the relationships between the whole and parts are concerned. If we simply separate the interesting part from the system, we will act contrary to the entirety and the relativity viewpoints of systems, and we will be lost in the morass of mechanism. So it is necessary that we identify the relationships among the whole, parts and environment in both the characteristic and the numerical aspects.

It is not only essential for system analysis and system synthesis, but also good for realizing more profoundly the organization and the regularity of systems' gradation, similarity, relativity etc.

II - THE LOGICAL RELATIONSHIP AMONG THE WHOLE, PARTS AND ENVIRONMENT

The whole, parts and environment are interconnected. They can transform each other under specific conditions. While in them the system boundary plays an important role, it either connects them or isolates them. There are two types of boundaries. One is the system boundary. A contour is imagined between the system and its environment, the two intercompensated parts. The other is the substructure boundary. It plays a double-sided role. On the one hand it determines that the parts are subordinated to the whole. And on the other hand it determines that the substructures internal and external to it are intercompensated. It is the actions of these two types of boundaries that make the whole, parts and environment of a system transform each other under specific conditions. For instance, if the system boundary expands outwards, some parts originally out of the system are now included in and the original system becomes a substructure. And if it contracts inwards, some parts within the system are divorced out and the remainder forms a new system. It should really be noted that in this case there are two parts of environment: one is the original and the other is the part of the substructure divorced out from the system. This is the main point of this paper. In the same way, if the substructure boundary expands outwards sufficiently, it will reach the system boundary. And if it contracts inwards enough, the substructure will be excluded. However no matter what we define the system boundary and substructure boundary as, the total energy and mass are conserved.

III - THE MATHEMATICAL DESCRIPTION FOR SYSTEM DYNAMIC MODEL

Based on the gradation nature of systems, we can divide a system into several interrelated substructures (say P):

$$S = S_1 \oplus S_2 \oplus \ldots\ldots\ldots \oplus Sp$$

Some of these are interesting. In general any substructure is directly related by only a few other ones. Tat is, a substructure possesses comparative individuality. So we can mathematically describe each substructure after resolving the system to simplify the analysis.

We know that there are three kinds of basic variables and equations in the system dynamics model : Level, Rate and auxiliary variables. Their abbreviations are L, R and A respectively. These three kinds of variables and equations can describe the system behavior in various ways where they are static or dynamic, linear or nonlinear, connected in series, parallel or feedback. According to the nature of variables and equations of the system, we give some mathematical descriptions and relative conceptions and definitions as follows to meet the needs of the new method.

First we define the variables:

Auxiliary (A) and Level (L) as the first grade variables

Rate (R) as the second grade variables

Pure Rate ($\dot{L}$) as the third grade variables.

Then we define the equations :

Auxiliary (A) as the first grade equations

Rate (R) as the second grade equations

Level (L) as the third grade equations.

Now a new kind of mathematical description for system dynamics model is given:

$$\dot{\mathbb{L}} = \mathbb{P} \cdot \mathbb{R}$$

$$\begin{pmatrix} \mathbb{R} \\ \mathbb{A} \end{pmatrix} = \begin{pmatrix} \mathbb{W}_1 \\ \mathbb{W}_1 \end{pmatrix} \begin{pmatrix} \mathbb{L} \\ \mathbb{A} \end{pmatrix}$$

$$= \mathbb{W} \begin{pmatrix} \mathbb{L} \\ \mathbb{A} \end{pmatrix}$$

where

$\dot{\mathbb{L}}$ - pure rate variables vector

$\mathbb{R}$ - rate variables vector

$\mathbb{A}$ - auxiliary variables vector

$\mathbb{P}$ - transmition matrix

$\mathbb{W}$ - relationship matrix

The matrix $\mathbb{P}$ is called transmition matrix, because it transfers the rate variables from time t to t+1. Generally the pure rate variables vector $\mathbb{L}$ is only the linear combination of rate variables, so the matrix $\mathbb{P}$ is a constant one. The matrix $\mathbb{W}$ is called the relationship matrix, because it describes the various nonlinear relationships between the first grade and the second grade variables at the same time.

There are several advantages of the mathematical description above as follows:

1) The dynamic parts or the static parts of the system are described individually

2) The linear parts or the nonlinear parts of the system are described individually

3) If the system possesses n variables within which m is the number of level variables and l is the number of rate variables, the sum of the computer storage units occupied by both the matrix $\mathbb{P}$ and $\mathbb{W}$ are

$$m \times l + (n - m)(n - l)$$

$$= n^2 - [\, m(n - l) + l(n - m) \,]$$

Usually there is a larger difference between n and l or between n and m. So the new mathematical description can save m(n-l) + l(n-m) more computer storage units than the common method. Especially when n is larger, this advantage is more remarkable.

In addition the new mathematical description possesses some characteristics as follows:

1) The third grade equations are dynamic. And the third grade variables are usually the linear combinations of the first and the second grade variables.
2) The first and the second grade equations are static. And the first and the second grade variables are onlv the algebraic nonlinear computation of the first grade variables.
3) The variable grades equal the equation grades in the left side of the equations. The variables grades in the right side of the equations are one grade lower than those in the left except the first grade variables, which are of the same grade both in the left and the right.

Now we can discuss the Model Reference Self-Compensation (MRSC) method by the help of the new mathematical description.

IV - THE MRSC METHOD

The problem lying before us is how to construct the system boundary so that the simplified system has almost the same functions and behavior as the original system. By the second paragraph we know in this case there are two parts of the environment. One is the original, which is already known. The other is the part of the substructure divorced out from the system. This is the main point of the problem. We denote the var ables of the interesting substructure by adding "*" upper right, and the remainders by adding "o". Then the system can be rewritten as :

$$\begin{bmatrix} \dot{\mathbb{L}}^* \\ \\ \dot{\mathbb{L}}^o \end{bmatrix} = \mathbb{P} \begin{bmatrix} \mathbb{R}^* \\ \\ \mathbb{R}^o \end{bmatrix}$$

$$\begin{bmatrix} \mathbb{R}^* \\ \mathbb{A}^* \\ \mathbb{R}^o \\ \mathbb{A}^o \end{bmatrix} = \mathbb{W} \begin{bmatrix} \mathbb{L}^* \\ \mathbb{A}^* \\ \mathbb{L}^o \\ \mathbb{A}^o \end{bmatrix}$$

In the special case, if the system is linear, the relationship matrix $\mathbb{W}$ is constant. The flowchart of MRSC is shown in figure 4.1

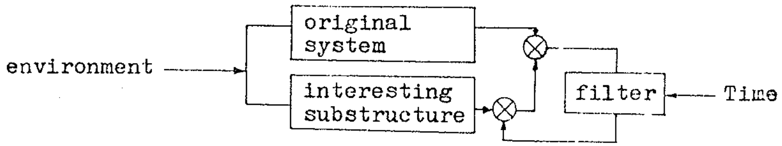

figure 4.1

The principle of MRSC is like these. Take the original system as the reference model and the interesting substructure as a controllable object. Compare the bihavior of these two blocks and get an error series, by which the time variable filter is controlled. And then lead the output of the filter to the output of the interesting substructure to compensate its behavior so that the difference between the original system and the interesting substructure can be made as small as possible.

The time variable filter consists of two parts. The one $Q_1(t)$ is decided by the time response function of the divorced level variables and used for the primary compensation. The other $Q_2(t)$ is a time series model. It is controlled by the error series from the original system and the primarily compensated substructure and is used for the perfect compensation.

The substructure boundary can be drawn according to the actual demands. There are two ways to proceed. The first is the Nature Structure (NS) method. We can tell by its name that the interested substructure is drawn out by the natural structure of the system. The second is the Relationship Decoupling (RD) method. That is, the transmition matrix and the relationship matrix in the system model are decomposed in the order from the higher grade equations and variables to the lower ones. Then the interesting parts remain and the others are eliminated. The practice has proven that the two different ways can provide the same results. The NS method is simple and obvious. It is suitable for a large number of dynamic systems. However sometimes we have to use the RD method because of the vague substructure boundaries or the complex connections of the substructures. RD is not as simple as NS, but it is more reliable.

The detailed procedures of MRSC are concluded as follows:

1) Decompose the interesting variables $\mathbb{L}^*$, $\dot{\mathbb{L}}^*$ in the third grade equations and find the transmition matrix $\mathbb{P}^*$.
2) Eliminate the variables $\mathbb{R}^o$, $\mathbb{A}^o$ in the second grade equations and find $\mathbb{W}_1^*$. Then make a curve $Q_1(t)$ to fit the time response of $\mathbb{L}^o$.
3) Eliminate the variables $\mathbb{A}^o$ in the first grade equations and calculate $\mathbb{W}_2^*$.
4) Set up a time series model $Q_2(t)$ based on the difference series between $\mathbb{L}^o(t)$ and $Q_1(t)$.
5) Add $Q_2(t)$ to $Q_1(t)$.

If we use NS to draw the substructure boundary, all the procedures of matrices decomposition above can be omitted. Because the grades of the left side variables are higher than those of the right side in the procedure 1) and 2), they can go easily.

While in the procedure 3) both side variables are the first grade. Some algebraic operations are needed for calculating the matrix W_2^*, but the calculations are not very complex since the dimension of W_2^* is much lower than the dimension of the system.

Now we can realize the advantages of the graded equations and variables.

V - SIMULATION EXAMPLE

We take the example of the Boon-town Dynamic Model to test the MRSC method. This model consists of three substructures: Population, Business construction and Housing sector. Suppose the Business construction sector is most interesting. We should therefore divorce it out from the system for special analysis. Both the NS method and the RD method are used in determining the substructure boundary. The some results are derived. There are sixty four variables altogether in the system and twenty eight in the substructure. Two TABLE functions are used for compensating the effect of the Population sector and the Housing sector separately. We found the effect of the housing sector to be trivial, thereby precluding compensation, and the effect of the population sector to be obvious, thereby necessitating compensation with a suitable time series model ARIMA (1,1,o). Finally, we obtained satisfactory results of course, we have to compensate perfectly the effects of each eliminated sector if necessary.

The variables of BSNSS (Business construction) and JPW (Job per worker) are specially studied in the simulation of the dynamic model, which ran from 1920 to 2020 in one year length, one hundred data for each in all. The following two TABLE functions are used to compensate the effects of the Population sector and Housing sector separately:

1) A POP.K = TABHL (TPOP, TIME.K, 1935, 2015, 8)
   T TPOP = 200/377.9/824.3/1793.7/3234.3/3562.5/
   X 3516.7/3462.2/3419.7/3387.5/3363.6

2) A HOUSES.K = TABHL (THOUSES, TIME.K, 1935, 2015.8)
   T THOUSES = 1/1.277/1.277/1.279/1.216/1.175/
   X 1.21/1.242/1.268/1.288/1.302

where POP and HOUSES are Level variables of the population and the housing separately. The data in the TABLE functions can directly be obtained from the time response curves of the relative variables in the system.

The compensation of the effect of the population sector is realized by the time series model ARIMA (1,1,o) with the principle of the least squares:

$$\nabla Z_t = (0.771 \pm 0.237) \nabla Z_{t-1} + a_t$$

$$a_t \in N(0, 6.7)$$

$$t \in 1920, 1921, \ldots\ldots\ldots 2020$$

The time response curves of the variables BSNSS and JPW both in the original system and in the compensated substructure are shown in figure 5.1. The statistical analysis results of the compensation accuracy are shown in table 5.1.

In order to test the functions of the compensated substructure, we changed the same parameters either in the original system or in the compensated substructure, and then ran the model. Two experiments were made in the first one the parameter ALB (average life of business, internal variable) was increased by 20%, from 50 to 60 units. The accuracy analysis of the simulation results are shown in table 5.2. And in the second one the parameter PBBCF (predicted boom business construction factor, external variable) was also increased by 20%, from 0.2 to 0.24 unit. The accuracy analysis of the simulation results are shown in table 5.3

These two experiments above prove that the compensated substructure by MRSC maintains higher accuracy even when some of parameter in it changes by about 20%.

TABLE 5.1 Compensation Accuracy Analysis

Tested variables	BSNSS	JPW
Max.absolute error	0.768	0.019
Mean absolute error	0.327	0.006
Possibility of relative error >1%	8%	12%
Possibility of relative error >2%	2%	0%
Possibility of relative error >3%	0%	0%

TABLE 5.2 Accuracy Analysis ALB = 60

Tested variables	BSNSS	JPW
Max.absolute error	2.148	0.041
Mean absolute error	0.524	0.014

Possibility of relative error > 1%	38%	62%
Possibility of relative error > 2%	12%	24%
Possibility of relative error > 3%	2%	6%

TABLE 5.3 Accuracy Analysis PBBCF=0.24

Tested variables	BNSS	JPW
Max. absolute error	5.212	0.039
Mean absolut error	0.377	0.009
Possibility of relative error > 1%	18%	22%
Possibility of relative error > 2%	12%	6%
Possibility of relative error > 3%	2%	2%

VI - CONCLUSION

In this paper, the logical relationships among the whole, parts and environment of a system are described. Two conceptions of the transmition matrix and relationship matrix are presented and the definitions of the grades of variables and equations are given. These conceptions and definitions combined with the theory of the model reference self-compensation are used to form a new method by which the relationships of the structures and functions between the system and its parts can be identified. MRSC is a high speed and low cost method, since it does not need repeated calculations and all the procedures can be orderly completed at once. The computer storage units can be greatly saved because of its reasonable mathematicál descriptions. MRSC demonstrates its clear physical conceptions, simple mathematical methods, controllable compensation accuracy and flexible usage. It is a new contribution to system analysis.

REFERENCES

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