APPLICATION OF MODAL ANALYSIS TO LARGE SYSTEM DYNAMICS MODELS

Amadou Diallo
Departement des sciences administratives
Université du Québec à Montréal
C.P. 8888, Succ. A
Montréal H3C 3P8 CANADA

R.Joel Rahn
Faculté des sciences de l'administration
Université Laval
Ste-Foy Québeç G1K 7P4 CANADA

ABSTRACT

In this paper we present methods developed to aid the application of modal analysis to large system dynamics models. The approach is based on a method of direct linearization which can be achieved using DYNAMO equations derived from the model being studied. The linearization is followed by identification of the sub-systems of the model and a modal analysis of the sub-systems. The sensitivities of the modes to structural and parameter changes are used along with causal loop diagrams to propose changes in the model that will have a desired dynamic effect.

Application is made to Lyneis' model of a firm. As suggested by the analysis, modifications both in the polarity of some relationships and in their strength are introduced to stabilize each sub-system and the results are shown to be positive using the criterion of over-all profitability of the model firm. The paper concludes with a discussion of further developments of the techniques to automate some of the more mechanical elements of the process and to account more completely for the non-linearity of the system.

INTRODUCTION

The analysis of behavior presents a major problem for the use of dynamic models, particularly in the case of the large, non-linear models usually developed in system dynamics studies. The classical approach favors the use of repeated simulations to engage and justify the analyst's intuition as to the sources, importance and sensitivity of behavior modes to changes in parameters. For models that are not too large, the transparency of the model compensates for the lack of rigor, but in large models this approach depends to a disturbing degree on the ability and thoroughness of the analyst and provides no effective means of verifying the correctness and the completeness of the conclusions.

To provide a more rigorous, verifiable basis for behavior mode analysis, several authors (Forrester 1980, Appiah and Cole 1983, Mohapatra and Sharma

1985) have had recourse to some of the tools used for automatic control problems, in particular, modal analysis. In this paper, we propose to build on this work by demonstrating i) a technique for linearizing dynamic models that uses some features of DYNAMO, ii) a management-oriented approach to implementing the results of the modal analysis.

LINEARIZATION TECHNIQUE

The use of modal analysis requires that the system model be linearized about some nominal set-point so that it has the following general form:

$$\dot{X} = AX + PZ + BU \qquad (1)$$

where
- X is the vector of level variables
- U is the vector of external perturbations
- Z is the vector of control variables
- A is the system matrix obtained by derivation of the net rate vector with respect to each level
- P, B are interface matrices obtained by derivation of the net rate vector with respect to each component of Z and U respectively.

The autonomous dynamics of the system are determined, locally, by A. We now develop a method to determine A by means of operations in DYNAMO on the full dynamic model of the system. The approach is based on the observation that the net rate vector can be treated as a function of all of the levels:

$$\dot{X} = f(X) \qquad (2)$$

and the derivative of the net rate with respect to a given level is approximated by:

$$d\dot{X}/dX_i = [\ f(X_i+dX_i) - f(X_i)\ ] / dX_i \qquad (3)$$

We note that the first term of the difference in the numerator in equation 3 is just the net rate evaluated at a perturbed value of the level X_i. Thus the calculation of an approximate value of the linearized system matrix A is achieved by the following steps:

1. create an artificial net rate variable for each level; for example,

   XLEVi.KL = INi.JK - OUTi.JK = the net rate of the level LEVi.K

2. create a second artificial rate variable for each level variable; for example,

YLEVi.KL = XLEVi.JK

3. create an artificial auxiliary variable for each level variable; for example,

ZLEVi.K = XLEVi.JK - YLEVi.JK

4. replace each level equation by an auxiliary equation of the same name in which the right-hand side is of the form:

LEVi.K = LEVi0*(1+STEP(0.01,(2*i-1)*DT)-STEP(0.01,2*i*DT))   (4)

where i indexes the level LEVi in the vector of levels and LEVi0 is the value of the level at the point of linearization.

The forced variation of each level in turn ripples through the set of artificial variables created in steps 1 to 3 in the following fashion (taking some liberties with notation and using only the variation in LEV1 for purposes of illustration):

TIME	LEV1.K	XLEVi.KL	YLEVi.KL	ZLEVi.K
0	LEV10	XLEVi0	YLEVi0	0
DT	LEV10+1%	XLEVi0	YLEVi0	0
2DT	LEV10	XLEVi0+1%	YLEVi0	0
3DT	LEV10	XLEVi0	YLEVi0+1%	+1%
4DT	LEV10	XLEVi0	YLEVi0	-1%
5DT	LEV10	XLEVi0	YLEVi0	0

From the preceding table, it is evident that the numerator of equation 3 is given by the variable ZLEVi at an interval 2*DT after the 1% STEP in LEV1. It is also evident that another level can be forced at a time offset of 2*DT without perturbing the effect of the previous STEP imposed on the variable LEV1.

To complete the calculation of the system matrix A, it suffices to divide the value of ZLEVi.K at each period 3*DT, 5*DT,... by the value of the increment in the corresponding level variable 2*DT time units earlier. We remark that the set of values of ZLEVi.K at a given period (for example, at time = 3*DT), and after division by the value of the variation of the level that was imposed 2*DT time units earlier (for example, 0.01*LEV1.K), is the column vector of the system matrix A corresponding to the same level (for example, LEV1).

MODAL ANALYSIS

The determination of an approximate system matrix A in the neighborhood of a given system state allows us to perform a modal analysis. For purposes of following sections which deal with implementation of the results of such analysis, we review briefly some salient points of this well-known approach to understanding dynamic systems.

First, we remark that the system matrix represents the net effect of the influence of one level on another by summing up all of the influences that are shown in the original, non-linear model by separate causal chains between each pair of levels. Thus the polarity of a link in the system matrix depends on the numerical evaluation of the strengths of each link between each pair of levels and hence depends on the system state about which the linearization is performed.

Modal analysis is based on a decomposition of the behavior of a linear dynamic system in terms of its eigenmodes characterized by: a) constant eigenvectors which span the range of the system matrix and b) complex eigenvalues (roots, k, of the characteristic equation : det $(A - k*I) = 0$) which determine the modes of behavior that the system can exhibit either alone or in arbitrary combination depending on the initial state of the system. The behavior of the autonomous system has the general form (Porter and Crossley 1972):

$$X(t) = \sum_i L_i' X(0) \exp(K_i t) R_i \qquad (5)$$

where

L_i, R_i are left- and right-eigenvectors of the system matrix A and
K_i are the eigenvalues and
$X(0)$ is the initial state.

From this form, we see that the system will be stable if all of the eigenvalues have negative real parts and that stability will be increased if the negative real parts of all eigenvalues increase in absolute value.

As well as providing a complete description of the autonomous behavior of the linearized system, modal analysis provides the means to assess the sensitivity of behavior to changes in parameter values. In particular, the sensitivity of eigenvalues with respect to variations in elements of the system matrix is derived from the following relation (Porter and Crossley 1972):

$$dK_i/da_{kl} = L_{kl} * R_{li} \qquad (6)$$

where L_{kl} and R_{li} are elements of the left and right modal matrices whose column vectors are corresponding left- and right-eigenvectors. We recall that the eigenvalues and eigenvectors are complex so that equation 6 represents the sensitivity of both real and imaginary parts of the eigenvalues.

In the application of these methods to Lyneis' model of a manufacturing firm, the numerical analysis of the eigenvalues and eigenvectors of the system matrix A was performed principally by means of two routines in the IMSL package running on an IBM 4381, namely:

EIGRF 'Eigenvalues and eigenvectors of a Real general matrix in Full storage mode' for the eigenvalues and

LEQ2C 'Linear EQuation - complex matrix - high accuracy solution' for the left and right eigenvectors.

APPLICATION TO LYNEIS' MODEL

In Corporate Planning and Policy Design (Lyneis 1980), the author presents a model including all of the traditional functional areas of a manufacturing firm: production, employment, equipment, raw material supply, marketing and finance. The moderately large size and reasonable completeness of the model make it a good candidate for demonstrating the application of the methods described above.

Linearization

The linearization of the model about the initial state of the base run was performed according to the method described previously after modifying the model equations in the following ways:

SMOOTH functions were replaced by explicit level equations;

DELAY3 functions were replaced by DELAY1 functions with level equations written explicitly;

DELAY3I equations in the equipment sector were written explicitly in terms of levels and rates.

The reduction of the order of the DELAY3 functions is necessary to avoid intractable problems associated with multiple eigenvalues generated by what is in fact merely a convenient simplification of dynamic structures in the system being modeled. This problem has been mentioned by other authors with respect to the evaluation of elasticity coefficients in the presence of the DELAY3 structure which conserves physical flows (Graham and Pugh 1983, Forrester 1983).

Decomposition into sectors

The system matrix A is difficult to analyze because of its large size (71x71 after the simplifications introduced during the linearization phase) and its sparsity. It appeared both desirable and feasible to decompose the matrix into square sub-matrices corresponding to each sector of the firm. A sector was defined, somewhat arbitrarily and loosely, as a group of variables, in a given functional area, having a large number of direct inter-relationships. A direct inter-relationship is identified by a non-zero entry in the system matrix. The decomposition was performed manually and somewhat laboriously; development of a computer-assisted approach would be worthwhile for larger models. The result was a system matrix A with less sparse zones arranged along the main diagonal and corresponding to the Production, Employment, Raw Material Supply, Capacity Acquisition and Finance sectors. The Market sector in this model has no dynamic structure of any significance and is not considered further.

The decomposition into functional sectors provides three advantages for this type of analysis:

- the intrinsic behavior of each sector is emphasized and associated with a functional decision center in the firm;
- a number of numerical convergence problems that arise when dealing with large, sparse matrices are reduced or eliminated;
- the results are more easily communicated to managers who are more aware of those aspects of the problem definition that are relevant to their own sector of activity.

The major disadvantage of decomposition is the possibility that the problematic behavior mode is strongly dependent on direct relationships between sectors. To some extent, this difficulty can be handled by simulation of the whole model to verify the effects of solutions proposed by the sector analyses. As well, the decomposition into sectors can provide a basis for subsequent, successive re-composition of sectors into groups whose behavior can be analyzed up to the limits of available numerical methods.

Modification and adjustment

The goal of modal analysis is to achieve a better understanding of the nature of the modes of behavior that a model can display in a region of the state space. This knowledge can then be used to suggest changes in the model structure that will 'improve' the behavior by making it conform to qualitative and quantitative criteria supplied by the model user.

One method for implementing the results of a modal analysis is by pole-assignment (Porter and Crossley 1972) in which the eigenvalues of the system matrix are assigned specific values and a system matrix corresponding to this assignment is derived. This process, when applied to models of complex organizations, is rather abstract and begs the question of how to re-design the information flows in the firm to generate the assigned behavior modes. To complete the implementation, a similarity transformation must be found that transforms the abstract system matrix into a structure that can be related to the physical and information structure of the firm. There are few guidelines to aid this search.

We propose a two-phase approach that is less structured than pole-assignment but more directly related to the underlying model of the firm. In the first phase, we seek to fix the general characteristics of all modes. In the model under study here, this includes stabilizing unstable modes. Stabilization is necessary because modes are independent components of the behavior so that, in theory, an unstable mode could form part of the system behavior for arbitrary initial conditions. An unstable mode would grow without limit, which is impossible in a real system. In realistic, non-linear systems, the non-linearities induce changes in structure that limit the growth of local instabilities. The process of fixing the characteristics of all modes thus implies a possible change of structure that, in the full model, would be induced by non-linear effects. Thus the first phase seeks to replace the linearized system model, with its problematic behavior, by a linear model, with improved behavior, that will be a viable alternative to a range of non-linear models.

The second phase consists of the adjustment of the modified model to achieve specific dynamic characteristics such as damping and oscillations with specific time constants or ranges of time constants. In both phases, modifications are made to the linearized system matrix A using information from the sensitivity analysis to guide the changes. The implementation of the results of this process requires introducing the suggested modifications into the original model in terms that are significant for the organization.

APPLICATION

Having performed the linearization and the decomposition into sectors, modal analysis is applied to the sectors. For purposes of this paper, we describe the process and results for the production sector.

Production sector

After linearization and modification as described above, the production sector is comprised of the following variables:

DDFI Desired Days Finished Inventory
PI Parts Inventory
WIP Work-In-Process
FI Finished Inventory
UOSD Unfilled Orders to be Shipped Direct
APR Average Production Rate
PC Production Completions

Note that in this form, Production Completions is the name used for the explicit level internal to the delay structure in the original model. The linearized system matrix is shown in Figure 1 as well as the eigenvalues. Since the imaginary part of each eigenvalue is zero, there are no oscillatory modes; however, modes 6 and 7 have positive real parts. To stabilize these two modes by making their real parts negative, we must change elements of the system matrix that affect these two modes. The sensitivity analysis results reveal that a change in any single element has either no effect or opposing effects on the two modes: stabilizing one while de-stabilizing the other.

SYSTEM MATRIX A

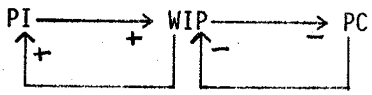
DDFI	PI	DOSD	FI	WIP	APR	PC
-0.20D-04	0.0	0.0	0.50D-07	0.0	0.0	0.0
-0.12D 01	-0.83D-06	-0.29D-02	0.42D-02	0.41D-02	0.0	0.0
0.0	0.0	-0.60D-01	-0.25D-05	0.0	0.0	0.0
0.0	0.0	-0.60D-01	0.0	0.0	0.0	0.10D 01
0.12D 01	0.83D-06	0.29D-02	-0.42D-02	-0.41D-02	0.0	-0.10D 01
0.28D-01	0.0	0.69D-04	-0.69D-04	-0.70D-04	-0.17D-01	0.0
0.57D-01	0.42D-07	0.15D-03	-0.21D-03	-0.21D-03	0.0	-0.50D-01

EIGENVALUES

	K1	K2	K3	K4	K5	K6	K7
Re	-0.17D-01	-0.50D-01	-0.60D-01	-0.42D-02	-0.14D-08	0.81D-05	0.11D-03
Im	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Figure 1 System matrix and corresponding eigenvalues of the linearized Production Sector

Since it is impossible to stabilize these modes on the basis of sensitivity arguments, we must look more closely at the causal relations to track down the positive loops that cause the instability. We find that these loops involve PI, WIP and PC:



Normally, and contrary to the preceding causal diagram derived from the system matrix, an increase in Work-In-Process leads to a decrease in Parts Inventory and an increase in Parts Completions. To correct this error in causality before dealing with the absolute values of the parameters, we reverse the signs of the links from WIP to PI and to PC. Implicitly, we are claiming that the system state used for linearization is not capable of generating reasonable, long-term behavior so that we must modify the system structure to show behavior that is acceptable. Modal analysis helps us to focus on those areas of the model which will repay our efforts to modify system behavior and shows us in which direction to modify parameters in the search for improved behavior.

The results of changing the signs of the two relationships mentioned above are shown in Figure 2 where all eigenvalues have negative real parts and the imaginary parts are still all zero. Note that the numerical analysis routines do not allow us to specify any particular relationship between the previous and the current set of eigenvalues. In Figure 2, we interpret the reciprocal negative real eigenvalues as adjustment times to get a clearer picture of the relationship between the time scales of the behavior modes (adjustment time, AT) and the time horizon of the model which is typicallty several hundreds of days.

EIGENVALUES

	K1	K2	K3	K4	K5	K6	K7
Re	-0.70D-01	-0.60D-01	-0.390D-01	-0.52D-07	-0.61D-05	-0.42D-02	-0.11D-01
Im	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AT	14 d	17 d	25 d	55000 yr	460 yr	1 yr	90 d

Figure 2 Eigenvalues of the linearized, stabilized Production Sector

From Figure 2 we see that the decay times for modes four and five are extremely long. Sensitivity analysis can be used to suggest changes that will shorten these times without unduly lengthening the decay times associated with the other modes. In this way, we can design the model so that it responds reasonably rapidly to changing external conditions while it does not generate intrinsically unstable behavior.

Examination of the sensitivity matrices reveals that the largest sensitivity is associated with the effect on K5 of the link from FI to DDFI represented by the system matrix element a_{14}. Since the sensitivity coefficient is positive, we reduce the strength of this link gradually, verifying the effect on the eigenvalues at each step, until we find that a value of zero gives the result shown in Figure 3.

EIGENVALUES

	K1	K2	K3	K4	K5	K6	K7
Re	-0.17D-01	-0.40D-01	-0.920D-03	-0.94D-02	-0.42D-02	-0.60D-01	-0.20D-04
Im	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AT	60 d	25 d	3 yr	100 d	240 d	17 d	137 yr

Figure 3 Eigenvalues of the linearized, stabilized Production Sector with coefficient $a_{14} = 0.0$

Mode seven has a relatively long adjustment time after this step. The sensitivity matrix for mode 7 indicates that the minor loop around FI is the only determinant of this eigenvalue. We change the coefficient a_{11} from -0.20E-04 to -0.20E-02 and this changes the eigenvalue by the same amount giving an adjustment time of about 500 days.

To sum up, in the production sector we are lead to make the following changes in the system matrix in order to eliminate explosive modes and reduce adjustment times:

1. Change the signs of the links from WIP to PI and from WIP to PC.
2. Eliminate the link from FI to DDFI.
3. Increase the strength of the minor loop around DDFI.

Other sectors

The same procedure is applied to the Employment and Finance sectors (Diallo 1985) and results in the following modifications of the original system matrix:

1. Change the sign of the link from LBR Labor Being Recruited to LHR Labor Hiring Rate.
2. Strengthen the minor loops around each of PRBA Professionals Being Assimilated and PROF Professionals.
3. Strengthen the minor loop around LTD Long Term Debt.

Implementation

The implementation of the results of the modal analysis require that they be translated in terms of the firm without disturbing the system by affecting other links unduly. To guide this phase of the work, we use those parts of the causal loop diagram surrounding the region of each of the modifications suggested previously. Ultimately, the changes to be proposed below are applied to Lyneis' original model so that an evaluation of the changes can be made in the context of the full, non-linear model.

WIP to PI and WIP to PC

In the causal diagram of Figure 4, we see that the effects of WIP on PI are the net result of the positive influence on POLC (due to the sequence of two negative influences on WIPC and ECESPR) and of the negative influence on POLC which is transmitted to PI via PR with a change of sign. We recall that in the original system matrix, the net result was a positive link from WIP to PI.

Similarly, the effects of WIP on PC are the net result of opposing effects on POLC which are transmitted to PC via PR with no change in sign. In the original system matrix, the net result was a negative link from WIP to PC.

To change the signs of each of these links, as suggested by the modal analyis, two possibilities seem evident:

1. Eliminate the positive link from POL to POLC.
2. Change the table function ECEPR to counteract the positive link from POL to POLC.

The first is a structural change; the second is a parameter change. Unfortunately, both of these suggestions result in changes that influence other links in the model very strongly. In particular, links between PI, PC, DSPI and the levels L, ACOR and DSPI are adversely affected.

Since no other change of polarity of a relevant link appears realistic, we are lead to eliminate the link from WIPC to DPR; i.e., the Desired Production Rate will no longer take account of Work-In-Process. The equation for DPR becomes:

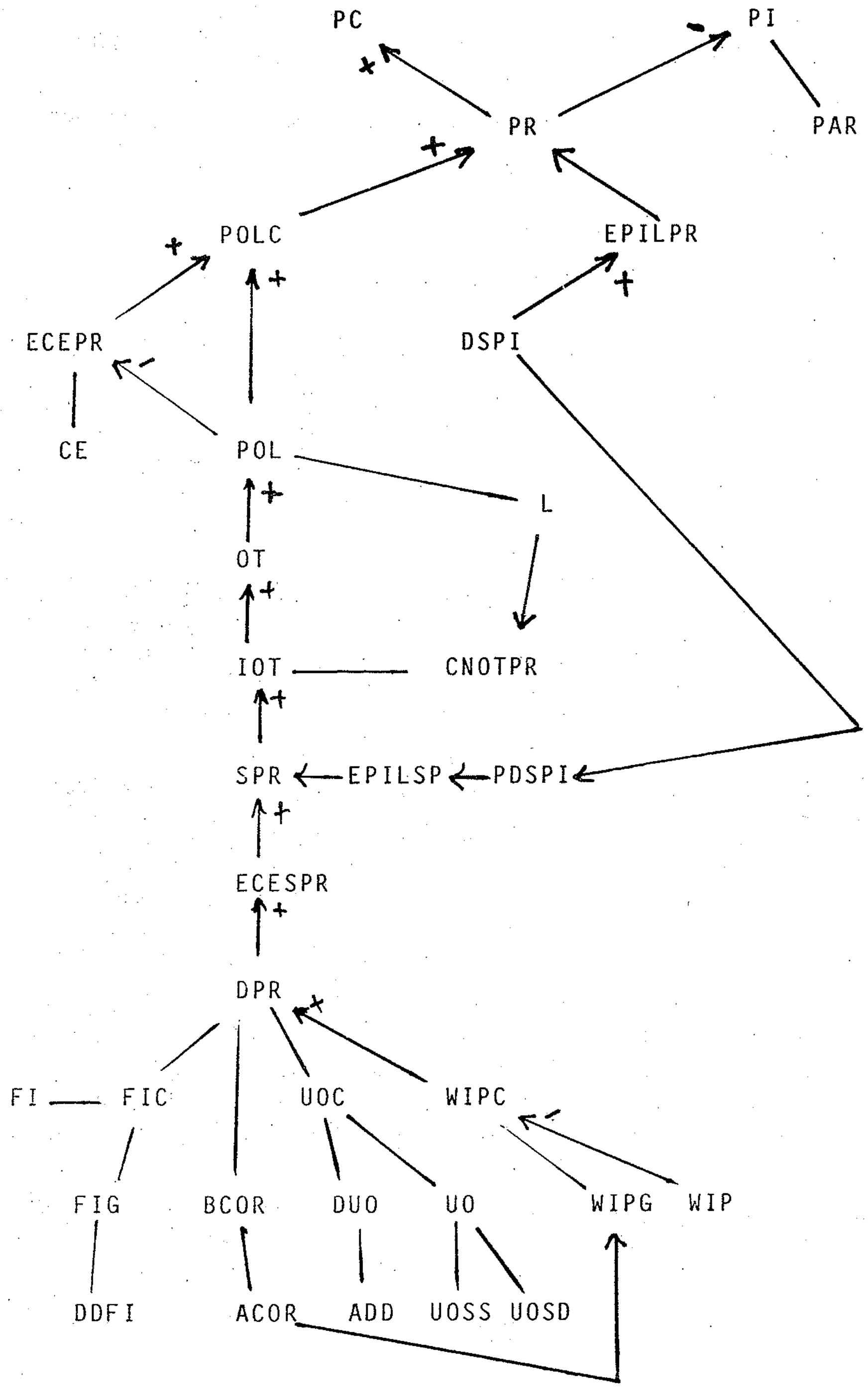


Figure 4 The influences of WIP on PI and PC

A DPR.K=(BCOR.K+FIC.K+UOC.K)*EFPDPR.K

This modification runs counter to the practice in other well-known models (Forrester 1961).

Elimination of the link FI to DDFI

The level DDFI is rewritten as:

L DDFI.K=DDFI.J+(DT)((COR.JK/ACOR.J)*DDFIN-DDFI.J)/TDCT

The link LBR to LHR

Analogously to the case of the links from WIP to PI and PC, examination of the causal loop diagram leads us to eliminate the link from DLBR to LBR in the equation for IHR which is now written as:

A IHR.K=ALAR.K+(DL-L.K)/TAL

The minor loop around PRBA

The goal is to reduce the adjustment time from about 1000 days to about 10 days according to the modal analysis. From a causal loop diagram of the sector, the choice is between reducing TAPROF from 120 to 10 days (which is unrealistic in terms of the system) or changing the table function FFPRBA. To limit undesirable effects on other variables, we replace the table function by a linear function of PRBA which preserves the positive polarity and which approximates numerically the adjustment time of 10 days under normal conditions. A more robust formulation is possible but modal analysis is incapable of suggesting what such a formulation might be. The revised version of FFPRBA is

A FFPRBA.K=PRBA.K/10

The minor loop around PROF

Analogous to the modification of the minor loop around PRBA, we wish to reduce the adjustment time for PROF from its value of infinity (the system matrix coefficient was zero in the initial state of the original model) to a smaller value. Again we replace the relevant table function FFPR by a linear function of PROF which preserves the polarity of the relationship in the original model. The revised version is

A FFPR.K=PROF.K/50

The choice of the numerical factor was somewhat arbitrary; sensitivity analysis of this relationship could be performed to determine a more robust policy.

The minor loop around LTD

Using the causal loop diagram, we are lead to modify the table function for the Per cent Debt Financing in order to strengthen the minor loop in question. The modification that we use is

T TPDF=1/.9/.8/.7/.6/.5/.4/.3/.2

This function reacts more quickly to increases in the debt-equity ratio in the low range but maintains some debt financing for high values of the debt-equity ratio. No elaborate study of the sensitivity of behavior to modifications of this table were performed. It could well be that, at high values of the debt-equity ratio, the original table function values are more justifiable.

Summary of implementation-oriented changes

In summary, we have changed several relationships in the original model which imply

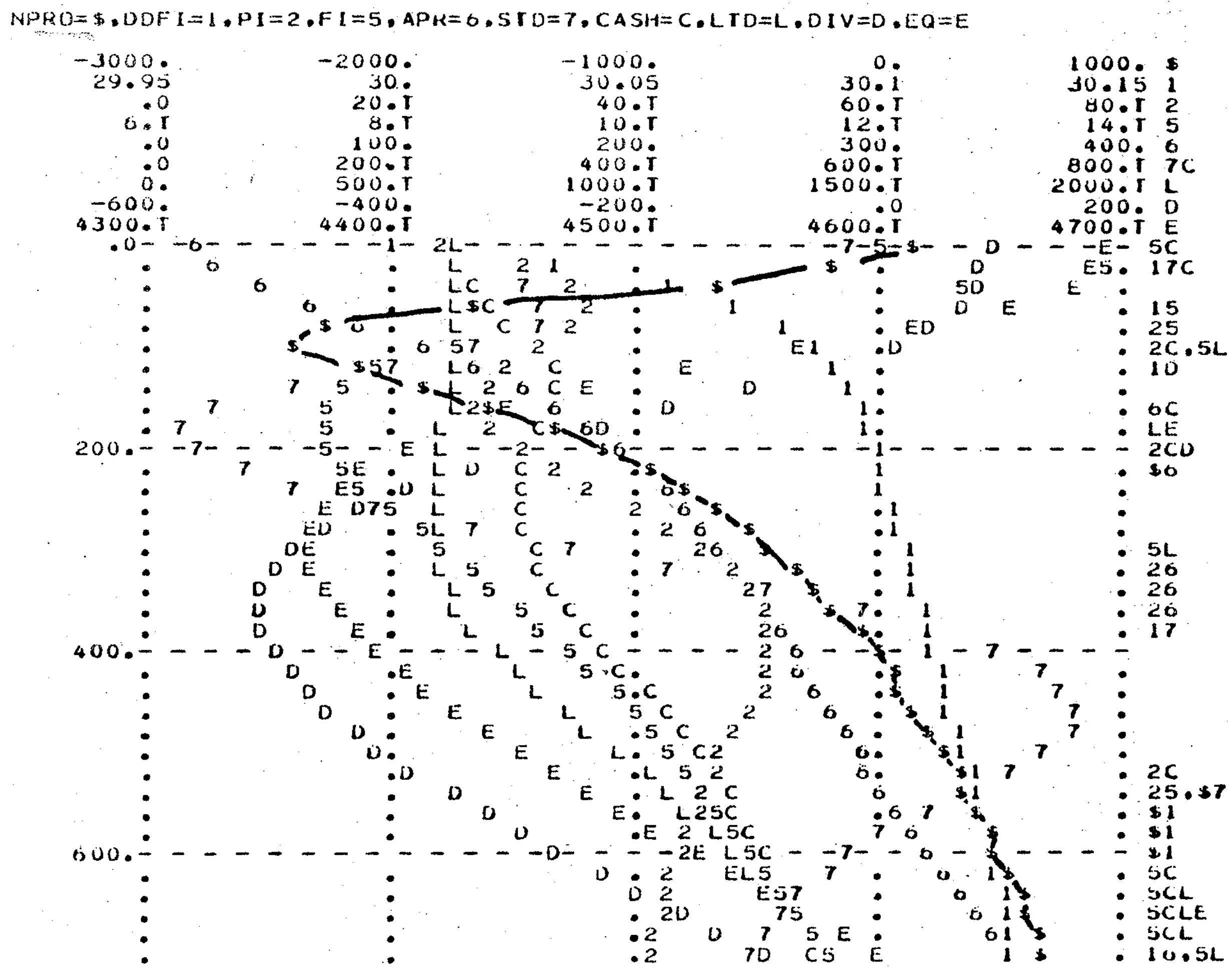
- more emphasis on long-term borrowing
- higher firing rates and turnover for professionals
- a simpler manpower hiring policy that no longer takes account of Labor Being Recruited
- a Desired Production Rate that no longer takes account of Work-In-Process
- a Desired Days of Finished Inventory which depends on the market but no longer on the size of Finished Inventory.

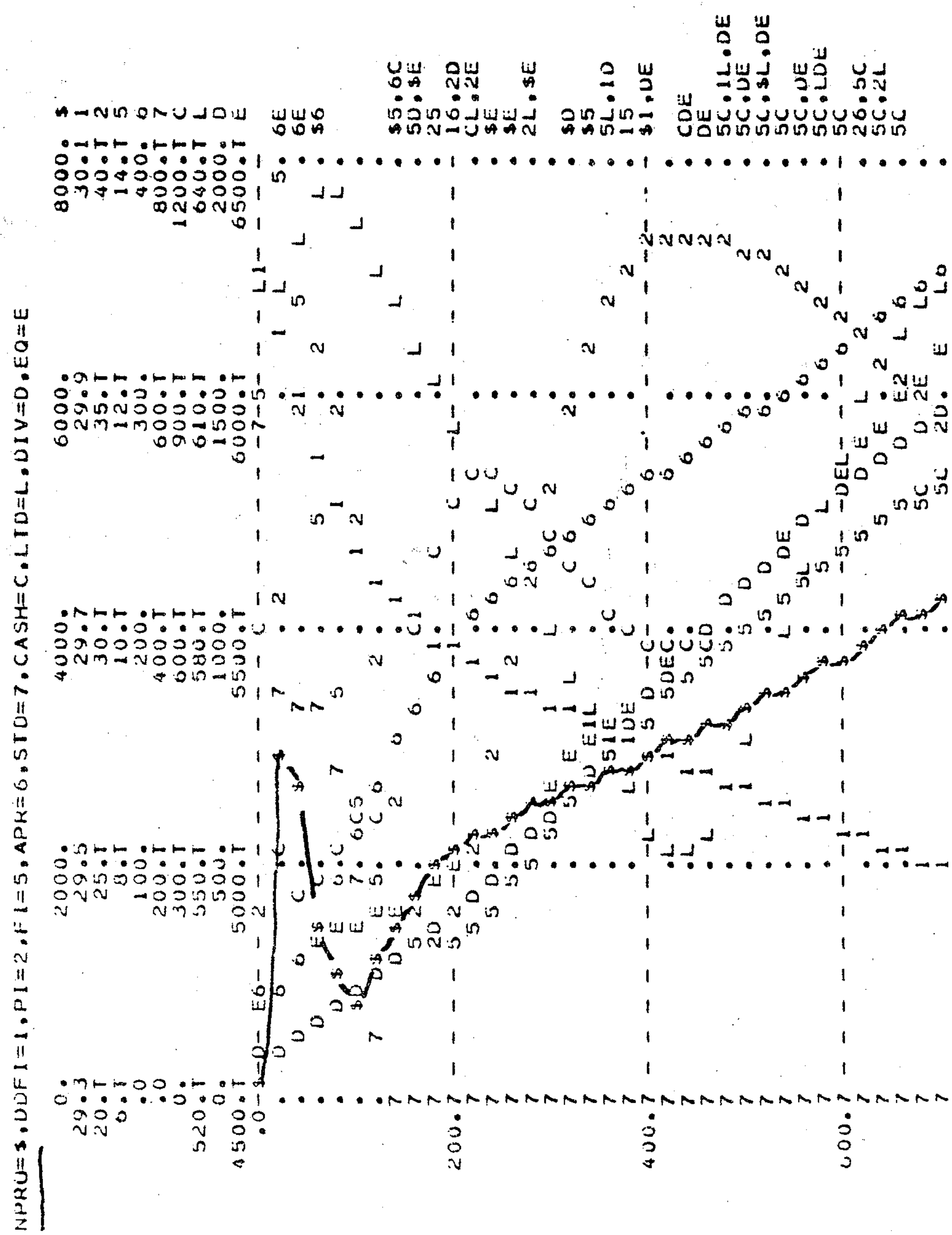
SIMULATION RESULTS

The results of these changes are shown by a comparison of simulation before and after implementing the modifications as seen in Figures 5 and 6 respectively. In Figure 5, the initial model of Lyneis is subject to a STEP in demand of 10% at day 5 and shows a drop in profits out to day 100. The reduction in losses continues out to day 400 where profits finally become positive. In Figure 6, the modified model, starting from the same initial conditions and with the same demand function, has profits that are always positive. As well, a close examination of other variables shows that the model is more stable than the original.

CONCLUSIONS

We have demonstrated in this paper a method for linearizing system dynamics models written in DYNAMO and have used some notions from modal analysis, namely the calculation of eigenvalues and the use of eigenvalue sensitivity to suggest modifications to system structure that are expected to change behavior. We demonstrated the process using Lyneis' model of a firm as a test case. It has been our experience that the limitations on modal


NPRO=$,DDFI=1,PI=2,FI=5,APR=6,STD=7,CASH=C,LTD=L,DIV=D,EQ=E
-3000. -2000. -1000. 0. 1000. $
29.95 30. 30.05 30.1 30.15 1
.0 20.T 40.T 60.T 80.T 2
6.T 8.T 10.T 12.T 14.T 5
.0 100. 200. 300. 400. 6
.0 200.T 400.T 600.T 800.T 7C
0. 500.T 1000.T 1500.T 2000.T L
-600. -400. -200. .0 200. D
4300.T 4400.T 4500.T 4600.T 4700.T E
.0
200.
400.
600.



igure 6 Full model after implementing changes suggested by modal analysis

analysis arise from the numerical difficulties of dealing with large, sparse matrices. We suggest an approach in which decomposition of a model into sectors that pose fewer numerical difficulties is combined with a whole-model evaluation of the results suggested by modal analysis. Modal analysis can help the analyst focus on relationships that are important determinants of behavior and it can be applied in large models that permit decomposition into sectors. As was mentioned in the Implementation section, modal analysis cannot replace or substitute for the analyst's ability to synthesize structures that can be implemented in an organization.

Further developments to aid the active use of modal analysis that can be envisioned are an automatization of the linearization process and of the integration of the linearization with the eigenvalue and sensitivity calculations. Finally, further effort should be devoted to understanding the effects on the analysis of the choice of the system state about which the linearization is to be performed. The approach suggested here, of stabilization and adjustment, may not always be viable particularly in cases in which endogenous growth is to be modelled explicitly as well as the factors that control the growth. The automatization of the mechanical steps of the process could permit exploration of the use of a sequence of models with different dynamic characteristics, representing different growth or environmental phases in the organization's development and linked by a strategy that is determined by the analyst.

REFERENCES

Appiah, R.K. and R.G. Coyle (1983), "A modern control engineering approach to system dynamics", in 1983 International System Dynamics Conference Plenary Session Papers, Morecroft et al (ed.), Chestnut Hill MA, July 1983

Diallo, A. (1985), "Application de l'analyse modale à la dynamique des systèmes pour améliorer la performance des firmes", unpublished doctoral thesis, Faculté des sciences de l'administration, Université Laval

Forrester, J.W. (1961), Industrial Dynamics, MIT Press, Cambridge MA.

Forrester, N.B. (1982), "A Dynamic Synthesis of Basic Macroeconomic Theory: Implications for Stabilization Analysis", System Dynamics Group paper D-3384, Sloan School of Management, MIT, Cambridge MA

Forrester, N.B. (1983), "Eigenvalue Analysis of Dominant Feedback Loops", in 1983 International System Dynamics Conference Plenary Session Papers, Morecroft et al (ed.), Chestnut Hill MA, July 1983

Graham, A.K. and A.L. Pugh III, "Behavior Analysis for Large DYNAMO Models", in 1983 International System Dynamics Conference Plenary Session Papers, Morecroft et al (ed.), Chestnut Hill MA, July 1983

Lyneis, J.M. (1980), Corporate Planning and Policy Design: A System Dynamics Approach, MIT Press, Cambridge MA

Mohapatra, P.K.J. and S.K. Sharma (1985), "Synthetic Design of Policy Decisions in System Dynamics Models - A Modal Control Theoretic Approach", System Dynamics Review, vol.1, no.1, Summer 1985

Porter, B. and R. Crossley (1972), Modal Control:Theory and Applications, Taylor and Francis, London