

DYNAMIC MODELING OF ADOPTION, REJECTION AND LIFE CYCLES OF INNOVATIONS

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ABSTRACT

The well established theory and methodology of the assurance sciences, especially the probabilistic reliability and availability theory, is applied to the modeling of the dynamics of innovation adoption processes. In particular, this approach allows one to model the dynamics of the diffusion of innovations through complex organizational decision networks. The approach taken here is an analytic one. However, it provides a logical framework for dynamic computer aided approaches. As the management and control of the dynamics of the innovation adoption is becoming increasingly important, obvious extensions of this approach are in the direction of optimal control systems concepts and applications. A number of empirical examples from the American automobile and steel industries are discussed.

INTRODUCTION

In the modern world the management of the dynamics of the adoptions of innovations is becoming increasingly important. There seems to be an intensifying replacement of old products, processes and service systems, respectively, by new products, processes and service systems. Numerous technological substitution effects, e.g. replacing steel by aluminum, ceramics and plastic composites or replacing copper by fiber optics, are creating basic changes in the structure of commodity markets. The various political, social, economic and business implications of such dynamics of innovation adoptions are profoundly important to any society and generate serious private and public sector management concerns. In particular, there is a need to develop effective or pragmatic approaches to the relevant accounting of the dynamics of innovation adoptions and their substitution effects.

The literature on the models and case studies of the diffusion of innovations is vast. The survey of this literature is not within the scope of this presentation. The objective of this investigation is pragmatic: How can one apply the well established theory and methodology of the assurance sciences, e.g. probabilistic reliability and availability theory, to the relevant modeling of the dynamics of adoption and diffusion of innovations even in rather complex organizational decision networks. If this approach is an effective one, it could become an aid for planning and decision making processes. It should be noted that the emphasis is on the dynamic transient behavior as distinct from the steady state analysis.

The mathematical foundations of the probabilistic reliability and availability theory required for this presentation are adequately summarized, for example, by Shooman (Shooman, 1968). The application of the availability theory to the institutional and product life cycles has also been investigated previously (Jutila, 1972). The application of the availability theory to the saturation trajectories over time for adoption, rejection and life cycles of innovations is only an obvious and natural extension (S. Jutila and M. Jutila, 1986). The purpose of this presentation is to indicate how this is done in a systematic way and how it can be applied to a number of special cases.

ONE-WAY PROCESSES OF ADOPTION OR REJECTION OF INNOVATIONS

The adoption or rejection of an innovation can be examined in a two-state framework: In State #1 the innovation is adopted and in State #2 it is rejected. Let  $S(t)$  be the probability that the innovation is in State #1. The probability that the innovation is in State #2 is then  $S'(t) = 1 - S(t)$  as a function of time  $t$ . Using the conventional terminology for diffusion of innovations,  $S(t)$  is the saturation function and  $S'(t)$  is the desaturation function. Let  $r(t)$  be the rate of adoption and let  $h(t)$  be the rate of rejection as functions of time  $t$ . These rates are assumed to be empirically measurable and exogenously specified (in a very similar manner as failure rates and repair rates are measurable in reliability and availability theory).

One-Way Rejection Process

Figure 1 illustrates the setting for a one-way rejection process. In this case the adoption rate  $r(t)$  is zero. The system is initially in State #1 at  $t = 0$ .

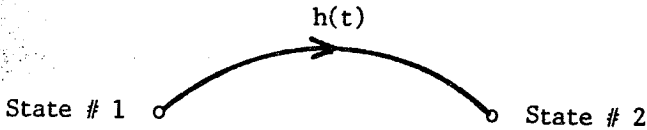


Figure 1. The rate flow graph for one-way rejection process.

The differential equation for the saturation function  $S(t)$  is as follows:

$$dS(t)/dt = - h(t)S(t); \quad S(0) = 1. \quad (1)$$

The solution to this differential equation is as follows:

$$S(t|r(t)=0) = \exp(-H(t)) = A(t). \quad (2)$$

$H(t)$  is the cumulative rejection function and is equal to the definite integral of  $h(t)$  from zero to  $t$ . It is assumed that  $h(t)$  is a non-negative function of time  $t$ .

One-Way Adoption Process

Figure 2 illustrates the setting for a one-way adoption process. In this case the rate of rejection  $h(t)$  is assumed to be zero and the system initially in State #2. The force of adoption  $r(t)$  is positive. The respective differential equation for the saturation function  $S(t)$  is as follows:

$$dS(t)/dt = + r(t)S'(t) = r(t)(1-S(t)); \quad S(0) = 0. \quad (3)$$

The solution to this differential equation with  $R(t)$  being the cumulative adoption (i.e definite integral of  $r(t)$  from zero to  $t$ ) is as follows:

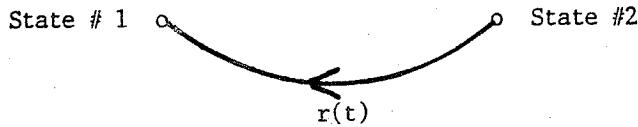


Figure 2. The rate flow graph for one-way adoption process.

$$S(t|h(t)=0) = 1 - \exp(-R(t)) = 1 - B(t); \quad S(0) = 0. \quad (4)$$

In a summary, one should note the following definitions:

$$A(t) = S(t|r(t)=0 \text{ and } S(0)=1) = \exp(-H(t)) \quad \text{and} \quad (5)$$

$$B(t) = 1 - S(t|h(t)=0 \text{ and } S(0)=0) = \exp(-R(t))$$

AN EXAMPLE OF A ONE-WAY PROCESS: PURE TECHNOLOGICAL SUBSTITUTION

Technological substitution is a common phenomenon. Sometimes a new innovation replaces several older innovations. In many cases a single old product is replaced by a new product. An excellent example of this is the replacement of the old ply tires by the radial tire in North American automobile markets (S. Jutila and M. Jutila, 1986). This kind of a pure substitution process is frequently observed. Let  $y$  be the new innovation and let  $x$  be the old innovation. The adoption saturation trajectory  $S(t)_y$  of the new innovation is then

$$S(t)_y = 1 - \exp(-R(t)); \quad S(0)_y = 0 \text{ initially at } t = 0.$$

where  $R(t)$  is the cumulative adoption function of the new innovation. But it is also the cumulative rejection function of the old innovation. The saturation trajectory of the old innovation will then collapse as follows:

$$S(t)_x = \exp(-R(t)); \quad S(0)_x = 1 \text{ initially at } t = 0.$$

Figure 3 is an illustration of a typical symmetry obtained in such cases.

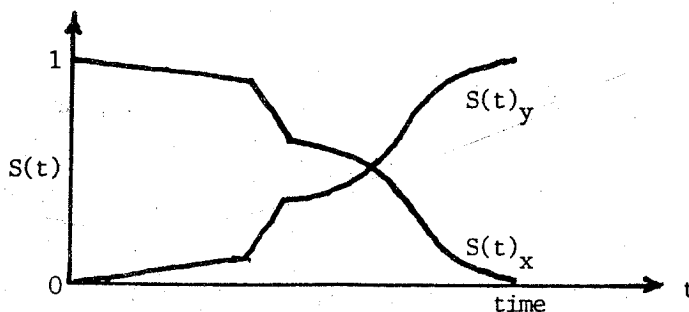


Figure 3. An illustration of the symmetry of the pure substitution.

In the pure substitution processes the cumulative adoption function of the new innovation is equal to the cumulative rejection function of the old innovation.

#### THE FORCE OF ADOPTION AND THE FORCE OF REJECTION

The forces of adoption and rejection are defined respectively as follows:

Force of adoption has the **magnitude**  $r(t)$  ( a non-negative function of time  $t$  ) and a **direction from** State #2 of rejection to State # 1 of adoption.

Force of rejection has the **magnitude**  $h(t)$  ( a non-negative function of time  $t$  ) and a **direction from** State #1 of adoption to State # 2 of rejection.

For the purposes of a proper empirical identification of these forces, the identification of the appropriate direction of the force is the first step. Then one identifies the behavior of the magnitude over time. Figure 4 illustrates a possible behavior of the magnitude of a force.

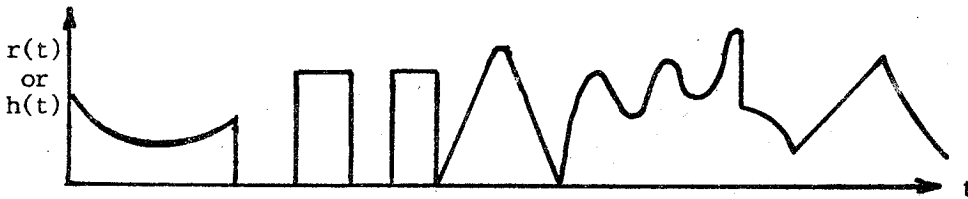


Figure 4. Possible behavior of the magnitude of a force.

The magnitude of the force may take several different characteristics era by era over time. It may go to infinity in a finite time  $t$ , an so on.

In general, for the practical modeling purposes, one should note the following:

If the saturation function  $S(t)$  is monotonic increasing and if it goes to one as the time  $t$  goes to infinity, then this process can be modeled as a one-way adoption process.

If the saturation function  $S(t)$  is monotonic decreasing and if it goes to zero as the time  $t$  approaches infinity, then this process can be modeled as a one-way rejection process.

It should be noted that the trajectories of  $S(t)$  may not be and, in fact, often are not S-shaped. Nor are they necessarily monotonic.

If the saturation function  $S(t)$  is not monotonic or if it does go neither to one nor to zero, or both, then this process can be modeled by a two-way adoption-rejection process.

The two-way process will be discussed subsequently. Before this, some added concepts for one-way processes are introduced.

## ADDED CONCEPTS FOR ONE-WAY PROCESSES

Per Figure 1 for a rejection process or Figure 2 for an adoption process one can define the probability density functions, respectively, as  $g(t)$  and  $f(t)$  in the following manner:

$$g(t) = d(1-S(t))/dt = h(t)\exp(-H(t)) = h(t)A(t) . \quad (6)$$

$$f(t) = dS(t)/dt = r(t)\exp(-R(t)) = r(t)B(t) .$$

The mean time to rejection (MTTR) and the mean time to adoption (MTTA) are, respectively, as follows:

$$\text{MTTR} = \int_0^{\infty} t g(t)dt = \int_0^{\infty} A(t)dt , \text{ and} \quad (7)$$

$$\text{MTTA} = \int_0^{\infty} t f(t)dt = \int_0^{\infty} B(t)dt .$$

The respective variances are:

$$\text{Var}_A(t) = \int_0^{\infty} (t - \text{MTTR})^2 g(t)dt \text{ and} \quad (8)$$

$$\text{Var}_B(t) = \int_0^{\infty} (t - \text{MTTA})^2 f(t)dt .$$

These definitions will be referred to in later discussions.

## The Constant Force Case

Consider a one-way adoption process with a constant force  $r(t) = b$ . Then

$$f(t) = b \exp(-bt) \quad (\text{exponential distribution}),$$

$$\text{MTTA} = 1/b , \text{ and}$$

$$\text{Var}_B(t) = 1/b^2$$

## Linearly Increasing Force

Let  $r(t) = b^2 t$ . Then

$$S(t) = 1 - \exp(-0.5 b^2 t),$$

$$f(t) = b^2 t \exp(-0.5 b^2 t), \quad (\text{Rayleigh distribution}),$$

$$\text{MTTA} = \sqrt{\pi/2} (1/b), \text{ and}$$

$$\text{Var}_B(t) = (1 - \pi^2/4)(2/b^2) .$$

## A Case of Saturation in a Finite Time

In many cases innovation adoption saturates to a hundred percent level in a finite time  $T$ . A simple model is one where

$r(t) = K/(T-t)$  where  $K$  is a dimensionless constant and  $0 \leq t \leq T$ . Then

$$f(t) = (K/T^K)(T-t)^{K-1},$$

$$R(t) = \ln [T/(T-t)]^K,$$

$$S(t) = 1 - [(T-t)/T]^K, \text{ and}$$

$$MTA = T/(K + 1).$$

The parameter  $K$  controls the shape of the adoption trajectory. If  $K = 1$ , the adoption trajectory is linear. For  $K > 1$  the trajectory is concave from below, the more so the larger  $K$  is. For  $0 < K < 1$  the trajectory becomes increasingly convex from below as  $K$  approaches zero.

The above two parameter model can be expanded to a three parameter model as follows:

$$r(t) = (K/T)[1 - (t/T)]^{-n}; \quad n > 1, K > 0, T > 0, 0 \leq t \leq T,$$

$$f(t) = (K/T)[1 - (t/T)]^{-n} \exp[-(KT^{n-1}/(n-1))((T-t)^{1-n} - T^{1-n})],$$

$$R(t) = (K/(n-1))T^{n-1}[(T-t)^{1-n} - T^{1-n}], \text{ and}$$

$$S(t) = 1 - \exp[-(KT^{n-1}/(n-1))((T-t)^{1-n} - T^{1-n})].$$

## TWO-WAY PROCESSES OF ADOPTION, REJECTION AND LIFE CYCLES OF INNOVATIONS

In many cases the trajectories of adoption, rejection or life cycles of innovations are not monotonic nor do they reach necessarily a level of one hundred or zero percent. The trajectories may exhibit ups and downs indicating, respectively, a predominance of a force of adoption or a force of rejection. For example, the trajectories of adoption, rejection or life cycles of innovations in American automobile industry often exhibit strong ups and downs corresponding to business cycles and dislocations such as the 1973-75 and 1979-82 oil crises. They are also affected by changes in consumer tastes and the penetration of Japanese and European cars into North American markets (S. Jutila and M. Jutila, 1986). Even after considerable smoothing these trajectories still exhibit cyclical ups and downs and deviations from the "classical" S-shaped forms. Similar comments apply also to the adoption, rejection and life cycles in American steel industry. The interplay between technical substitutions of open hearth, basic oxygen and electric melting are complex involving regional variations in sources of and types of raw materials, types and proximity of markets, nature of competition, and a number of economic considerations relating to capital investments (Kie, 1986). Thus the forces of adoption and rejection tend to be complex rather than accountable by a simple explanatory process or model.

In view of the above comments, it is rather clear that many adoption, rejection or life cycle processes result from an interplay of a force of adoption with a force of rejection. This is a two-way process. The rate flow

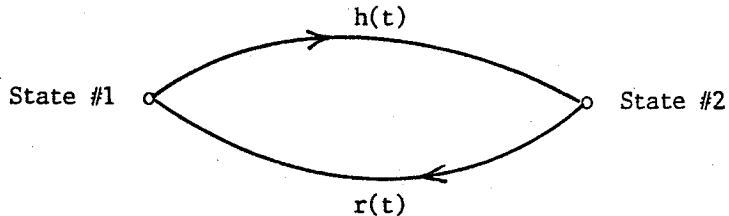


Figure 5. The rate flow graph for two-way process.

graph for the two-way process is given in Figure 5. The respective differential equation for the saturation or life cycle of the innovation is as follows:

$$dS(t)/dt = -h(t)S(t) + r(t)(1 - S(t)) \quad \text{or} \quad (9)$$

$$dS(t)/dt + [h(t) + r(t)]S(t) = r(t)$$

with the initial condition  $S(0) = S_0$ ,  $0 \leq S_0 \leq 1$ .

This is a first order linear differential equation with non-constant coefficients. It can be readily solved by the introduction of the appropriate integrating factor. Noting definitions of  $A(t)$  and  $B(t)$  in Equations 5, the solution to the above differential equation is

$$S(t) = A(t)B(t) \left[ \int_0^t \frac{r(x)}{A(x)B(x)} dx + S_0 \right] \quad (10)$$

If  $r(t) = 0$  and  $S_0 = 1$ , then the solution in Equation 10 reduces to that given by Equation 2. If  $h(t) = 0$  and  $S_0 = 0$ , then the solution in Equation 10 is that specified by Equation 4. If the forces of adoption and rejection are constant, i.e. if  $h(t) = a$  and  $r(t) = b$ , and if  $S_0 = 0$ , then

$$S(t) = [b/(a+b)] [1 - \exp(-(a+b)t)] \quad (11)$$

If  $a > 0$  and  $b > 0$  the saturation function  $S(t)$  will level off at  $b/(a+b)$  which is less than 100 %.

Equation 10 provides the basis for a computer aided simulator that generates trajectories  $S(t)$  for various pairs of forces of adoption and rejection. Figure 6 gives the flow diagram of the simulator. This allows one to experiment what kinds of forces would generate trajectories, for example, to match a "real world" situation or trajectory. One learns, with some trials and errors, rather quickly how to control the shapes of  $S(t)$  by some appropriate choices of  $r(t)$  and  $h(t)$ . In many cases the "classical" S-shaped adoption or life cycle trajectories can be generated by assuming some fairly simple forms for the forces of adoption or rejection. For example, one could try the

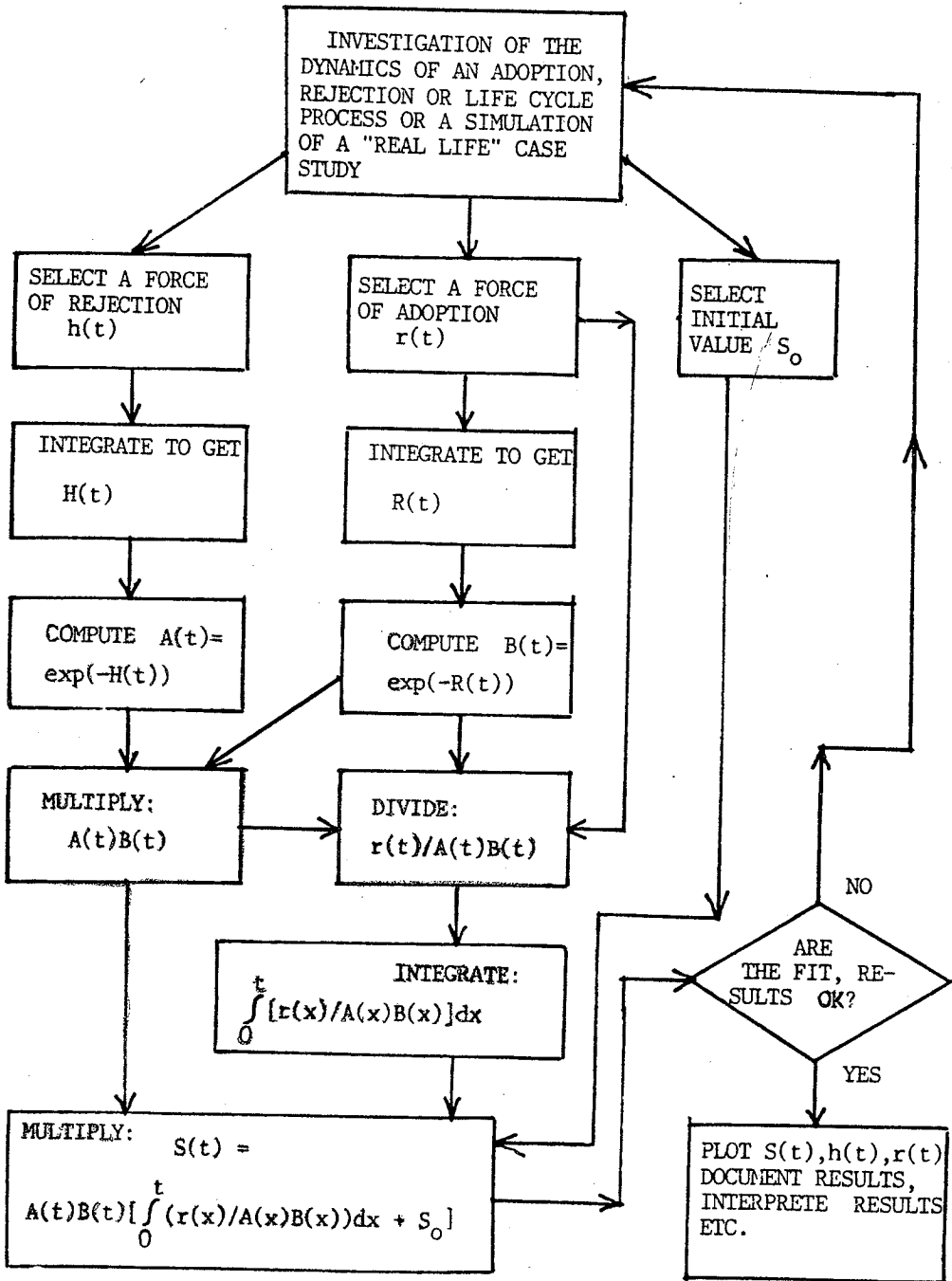
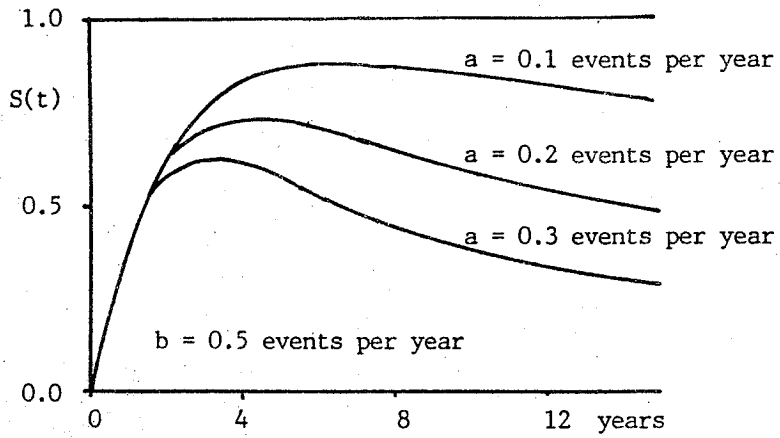
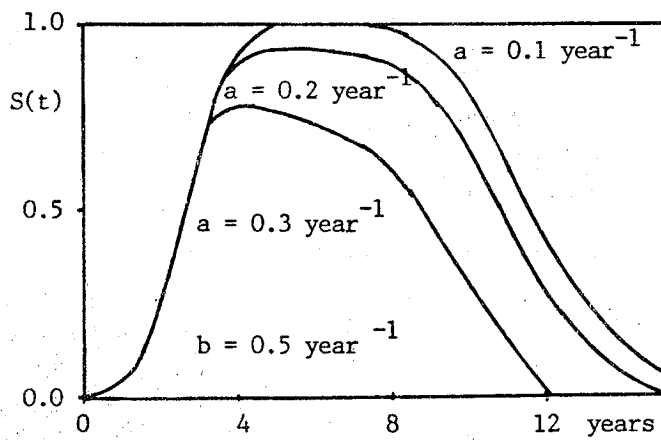


Figure 6. The flow diagram of the trajectory simulator.





CASE A:  $h(t) = a^2 t$  and  $r(t) = b$



CASE B:  $h(t) = a^4 t^3$  and  $r(t) = b^3 t^2$

Figure 7. Examples of simulator outputs for life cycles.

following relatively simple types of forces for rejection and adoption:

$$h(t) = a^n t^{n-1} \quad \text{and} \quad r(t) = b^m t^{m-1} \quad (12)$$

These forces are associated with the Weibull probability density functions of the respective one-way processes:

$$f(t) = a^n t^{n-1} \exp[-(a^n/n)t^n] \quad \text{and} \quad g(t) = b^m t^{m-1} \exp[-(b^m/m)t^m] \quad (13)$$

$a$  and  $b$  are characteristic frequencies of the rejection and adoption processes, respectively, given in units of events per year or year<sup>-1</sup>.  $n$  and  $m$  are dimensionless power constants. Figure 7 provides examples of simulator outputs for these types of forces. In Case A a constant force of adoption is overtaken by a linearly increasing force of rejection. As its characteristic frequency  $a$  increases, the life cycle is respectively increasingly suppressed. In Case B both forces increase over time, but again the force of rejection is overtaking the force of adoption. Figure 8 illustrates the dynamic modeling of the adoption trajectories of the delayed wiper controls for the General Motors luxury car Corvette, medium-line car Camaro, and the low-cost line Chevette.

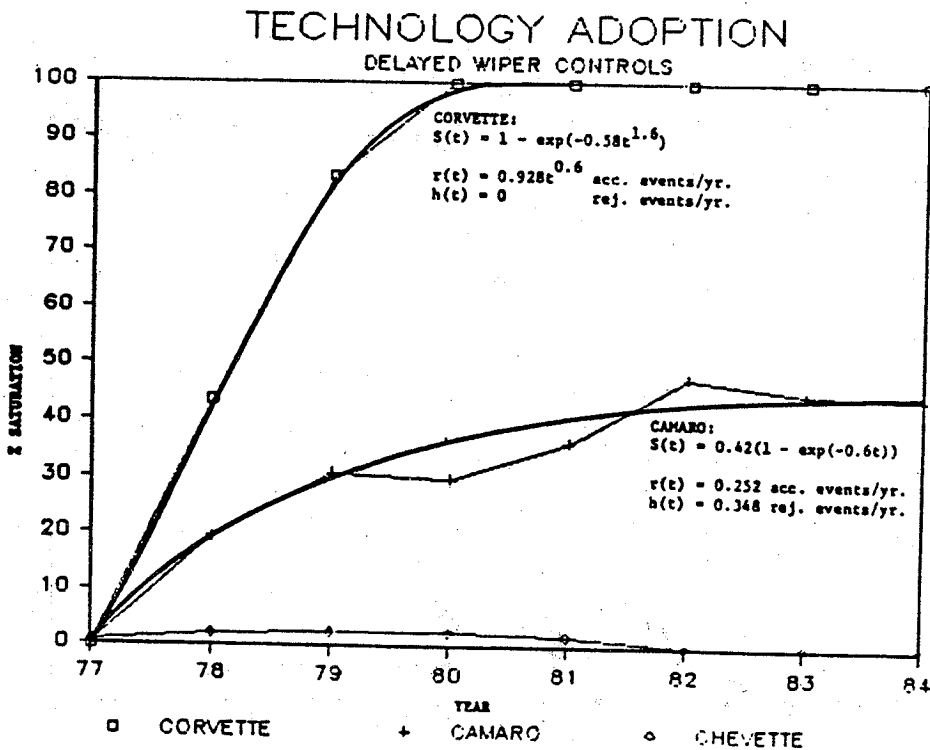


Figure 8. Examples of modeling adoption trajectories for General Motors cars.

For Corvette the adoption is a one-way process modeled with  $r(t) = 0.928 t^{0.6}$  and reaching near saturation in about three years. For Camaro the adoption levels off at little over forty percent of the product line. This case can be modeled with Equation 11 where  $r(t) = 0.252$  acceptance or adoption events per year and  $h(t) = 0.348$  rejection events per year. For Chevette the adoption experiences a life cycle reaching a maximum of only few percent of the product line segment. Typically in the traditional American automobile markets the luxury models lead in the adoption of new innovations as options. Then medium line cars follow. The last and least adoption takes place thereafter in the low cost automobile product lines. It should be noted here that the decision to adopt wiper controls is product line specific and does not involve complex corporate level hierarchical decision making procedures. This is reflected by the lack of the "classical" S-shaped form of the adoption trajectories. In a contrast, the decisions to adopt automatic transmission, power steering, air conditioning, disc brakes and even radial tires across product lines would involve more complex hierarchical corporate level decision making procedures. This would increase the S-shaped form of the adoption trajectories. It would also increase the mean time MTTA to adoption as well as the variance  $\text{Var}p(t)$  of the time  $t$  to adoption. Figure 9 illustrates these tendencies.

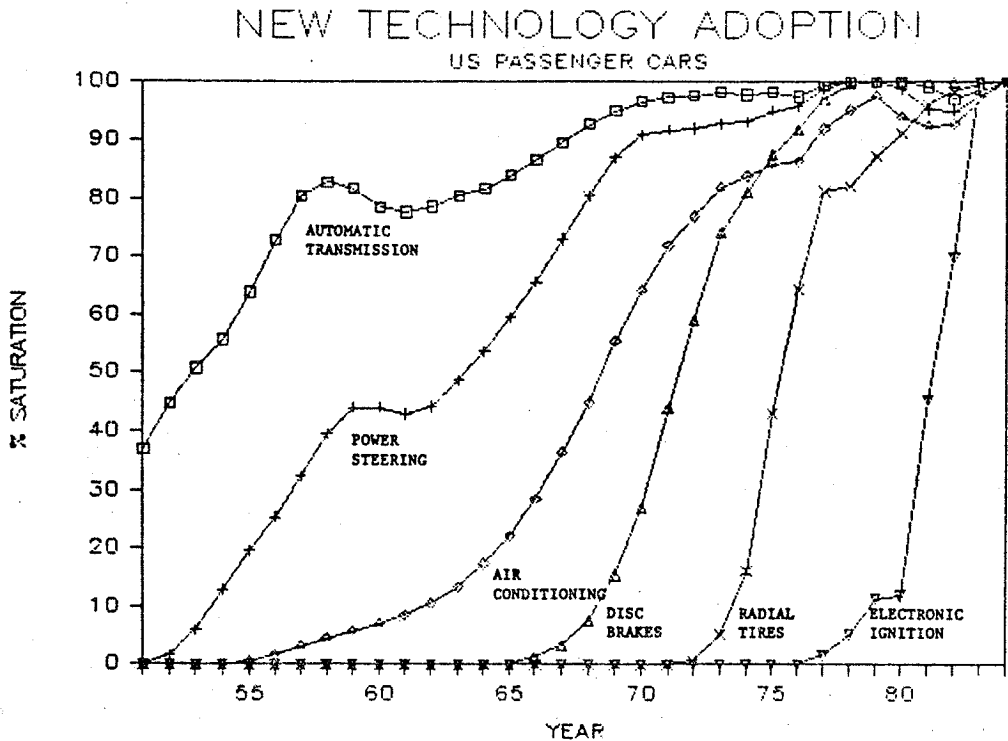


Figure 9. Examples of adoption trajectories for U.S. passenger cars.

## INNOVATION DIFFUSION PROCESSES IN INSTITUTIONAL AND ORGANIZATIONAL NETWORKS

Innovation diffusion processes take place in complex institutional and organizational decision networks. A typical illustration of a decision and implementation network is the so-called PERT flow graph associated with a project e.g. a development program for a new product or an implementation program for a new process or a service system. Such a network involves typically several parallel and sequential tasks and stages. Each task or stage requires certain amount of time for implementation. The usual practice is to identify the so-called critical path and the tasks or stages along it. As the number of tasks along the critical path increases, so do typically the expected time for the completion of the project and its variance. The probability density function of the time for the completion of the project becomes also, respectively, increasingly bell-shaped and the associated cumulative probability function becomes increasingly S-shaped.

In real world situations innovation diffusion processes may involve several institutional actors and organizations. For example in American automobile industry an implementation of a new innovation may involve a complex national and international supplier network. It may involve governmental institutions in regards to approvals relating to highway safety, pollution standards, and energy efficiency. It might involve interruptions generated by labor-management disputes and strikes. On the demand side there can be also complex decision stages, e.g. stages associated with wholesaling, retailing, financial services for automobile buyers, insurance, licensing, and, of course, the stages involved in buyer behavior. All these are influenced by rational expectations for either optimistic or pessimistic future trends. For example, the buyer behavior is sensitive to employment and disposable income expectations, and, respectively, the sellers must react by not being caught with an undesirable inventory situation. The U.S. automobile industry is very sensitive to business cycles. The slumps of 53-54, 57-58, 60-61, 69-70, 73-75, and 79-81 show up strongly in car sales and influence the adoption trajectories of all major innovations, as seen in Figure 9 even after the data used in figure was smoothed by a three year moving average method.

It is rather obvious that the dynamics of an innovation diffusion process, and specifically the shape of the trajectory  $S(t)$ , is strongly influenced by two major factors: 1) the number of decision stages along the "critical path" of the relevant decision network, and 2) the magnitudes of the reaction times associated, respectively, with each and all of these decision stages. Figure 9 illustrates a tendency of the compression of the time to adoption and its variance from 1950 to 1984 in U.S. automobile industry. This may reflect, at least in part, improvement in organization (e.g. reduction in the number of stages in the decision making process), improvement in information flows and communication (e.g. reduction in the response time at each stage of decision making), and improved reliability of the decision making processes. Thus there is a possible trend toward more accurate and responsive decision systems. Part of this tendency is forced by foreign competition (S.Jutila and M. Jutila, 1986).

The points made above, i.e. the influence of the number of decision stages and their respective response times upon the shape of an adoption trajectory, can be illustrated in terms of a simple and well-known one-way Markov process.

Figure 10 illustrates the rate flow graph of an  $n+1$  state one-way Markov process. For a one-way two-state adoption process described by Figure 2 and Equation 4 the state 0 corresponds to the initial state of rejection, State #2 with the respective state probability  $S_0(t)$  having an initial condition  $S_0(0) = 1$ , i.e. the system is initially in the state of rejection. It is now assumed that there are  $k = 1, 2, 3, \dots, n-1$  tasks (states) to be realized sequentially before the final task  $n$ , i.e. the full adoption of an innovation, is accomplished. The state  $n$  is then State #1 of adoption in Figure 2, and the respective state probability  $S_n(t)$  is then the adoption trajectory or saturation function of the innovation adoption process. The decision  $k$  is defined as a move from the state  $k-1$  to the state  $k$ ,  $k = 1, 2, 3, \dots, n$ . Thus there are  $n$  sequential decisions to be made corresponding, respectively, to the  $n$  tasks to be realized. It is further assumed that the characteristic transfer rate  $b$ , or the characteristic reaction time  $T = 1/b$ , for all the  $n$  decisions has the same value.

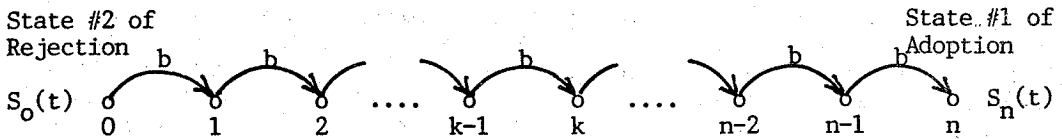


Figure 10. The rate flow graph for a simple case of  $n$  decisions.

The differential equations for the state probabilities  $S_k(t)$  are as follows:

$$dS_0(t)/dt = -bS_0(t) ; S_0(0) = 1,$$

$$dS_k(t)/dt = bS_{k-1}(t) - bS_k(t) ; S_k(0) = 0 \text{ for } k = 1, 2, 3, \dots, n-1,$$

$$dS_n(t)/dt = bS_{n-1}(t) ; S_n(0) = 0 .$$

The solutions for the state probabilities are readily obtained by standard Laplace transform techniques. The results for Figure 2, Equation 4 and Equations 6, 7 and 8 are as follows:

$$S(t) = S_n(t) = 1 - \sum_{k=0}^{n-1} (1/k!)(t/T)^k \exp(-t/T) , \tag{14}$$

$$B(t) = \sum_{k=0}^{n-1} (1/k!)(t/T)^k \exp(-t/T) ,$$

$$f(t) = [(t/T)^{n-1}/(n-1)!](1/T) \exp(-t/T) \quad (\text{Gamma function}) ,$$

$$MTIA = nT ,$$

$$\text{Var}_B(t) = nT^2 , \text{ and}$$

$$r(t) = f(t)/B(t) = (1/T)(t/T)^{n-1} / [(n-1)! \sum_{k=0}^{n-1} (1/k!)(t/T)^k] .$$

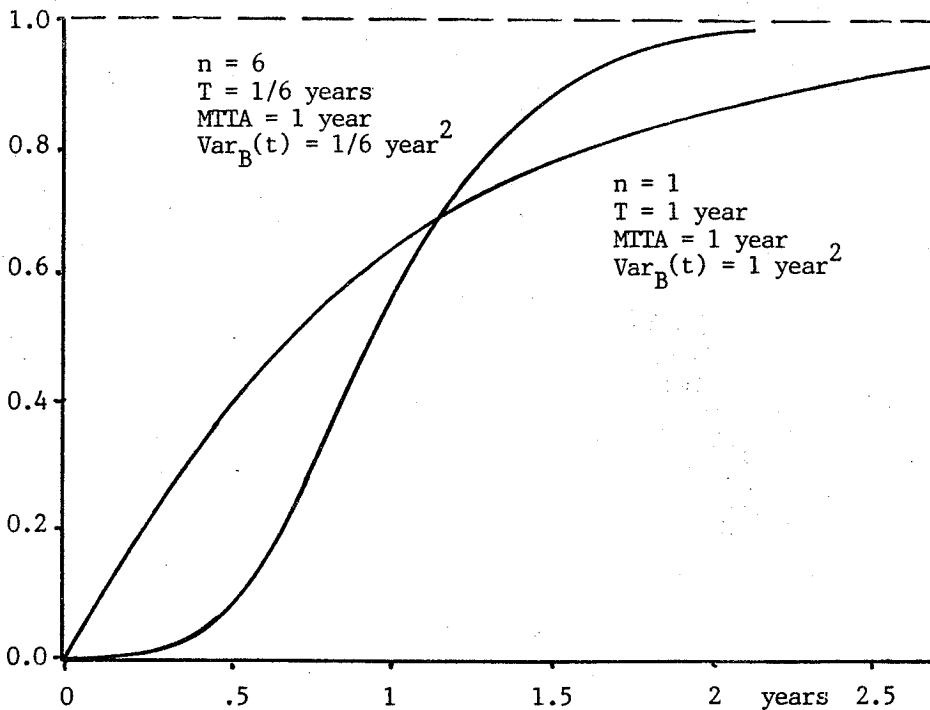


Figure 11. The effect of reduced response time and increased number of decision levels on the shape of the adoption trajectory.

The Markov model used here has the property that the probability of transition from the state  $k-1$  to the state  $k$ ,  $k = 1, 2, 3, \dots, n$ , is independent of the transition probabilities of the previous moves. Not only is MTTA the sum of the response times of all the decisions. The variance is also the sum of the variances of all the decisions in the chain. As the number  $n$  of decisions is increased, the probability density function  $f(t)$  (Gamma function) will go to a normal distribution. Thus the adoption trajectory  $S(t)$  will become increasingly S-shaped as the number  $n$  of decisions is increased.

The characteristic reaction time  $T$  of a decision plays a very important role. This can be illustrated by a simple example. Assume MTTA is kept fixed at one year while the number  $n$  of decision levels is increased to six. Let  $T = 1$  year be the characteristic response time for  $n = 1$  and let  $T = 1/6$  years for  $n = 6$ . The variance for  $n = 1$  is one year squared. For  $n = 6$  it is  $1/6$  years squared. Figure 11 illustrates this comparison. The risk associated with the time to adoption as measured by variance is less for  $n = 6$  with  $T = 1/6$  years than for  $n = 1$  with  $T = 1$  year. For  $n = 6$  the adoption trajectory is already quite S-shaped. The number of decision levels and the response time of each decision level are important control parameters in the design of adoption trajectories from the point of view of planning and management of adoption or rejection of innovations.

For a one-way two-state rejection process (Figure 1 and Equations 2,6,7 and 8) the state 0 in Figure 11 is State #1 of adoption with an initial condition given by  $S_0(0) = 1$ . In Equations 14  $S(t)$ ,  $B(t)$ ,  $f(t)$ ,  $MTR_A$ ,  $Var_B(t)$  and  $r(t)$  are replaced for the rejection process, respectively, by  $1-S(t)$ ,  $A(t)$ ,  $g(t)$ ,  $MTR$ ,  $Var_A(t)$  and  $h(t)$ .

The above model can be generalized. Consider a Markov process with an initial (starting) state 0 with a state probability  $S_0(t)$ ,  $S_0(0) = 1$ , with only outgoing rate flows connecting to various states in a Markov network of a finite number of transient states (tasks) interconnected by various rate flows (decisions). There is a final state  $n$  representing the final adoption of the innovation. Its state probability is  $S_n(t)$ . The initial conditions for all the states, except the initial state 0, are zero. The final state  $n$  is the only absorbing state in the system, receiving rate flows from various other states in the system. One can always find a solution for the state probabilities of such a system, including  $S_n(t)$ . Once  $S_n(t)$  is determined, then also the respective probability density function and the respective force can be determined for either a rejection (Figure 1) or adoption (Figure 2) process. If a rejection process or an adoption process ends up, respectively, in a state of complete rejection or adoption, it is necessary that the cumulative rejection function  $H(t)$  and the cumulative adoption function  $R(t)$  go to infinity in finite time  $t$  or as  $t$  approaches infinity. This condition is satisfied by the above type of a Markov process.

One can further assign one Markov process of the above type for the task-decision network of an adoption process specifying thereby an adoption force  $r(t)$ . Another Markov process could be assigned for a rejection task-decision network specifying thereby a force of rejection  $h(t)$ . The two Markov processes with the opposing forces  $r(t)$  and  $h(t)$  can be then combined to a two-way two-state process illustrated in Figure 5 and specified by Equation 10. This way the dynamic modeling approach via network modeling of adoption and rejection is extended to the Markov network modeling of life cycles. Thus it is possible to structure out for planning and control purposes how the adoption and rejection task-decision networks should be designed in order to generate some desired trajectory characteristics of an innovation life cycle.

While the Markov models of the above type can be always "mapped" into the one-way or two-way processes illustrated in Figures 1, 2 and 5, the converse is not necessarily true. That is, there may exist forces  $r(t)$  and/or  $h(t)$  (see Figure 4) to which there are no corresponding Markov network models. External effects often cause dislocations in otherwise planned and controlled adoption and rejection processes. In U.S. markets investment goods and consumer durable goods are typically sensitive to business cycles. Such external impacts can cause changes in the parameter values of a task-decision network or can even cause minor or major restructuring of such networks. These kinds of reaction and adaptation effects cannot be captured by assuming an everlasting fixed Markov network structure. While the Markov network approach helps to structure out how particular task-decision networks influence the trajectories of adoption, rejection and life cycles of innovations, it may not be sufficient for taking into account, both, the internal firm or industry level as well as broader overall economy level structural changes. In the general context of the rejection, adoption and life cycle processes per Figures 1, 2 and 5 it is possible to introduce a variety of dislocating forces.

COMMENTS ON DISLOCATIONS OF INNOVATION TRAJECTORIES

It is not unusual that innovation trajectories deviate from the planned or expected ones. For gradual and mild deviations corrective adaptations may provide sufficient management control. However, often internal events, e.g., strikes or sudden management changes, and/or external events, e.g., recessions, political upheavals or natural disasters, can generate rather rapid and uncontrollable dislocations of the innovation trajectories. It is important to identify what kinds of forces or combinations of forces are associated with a dislocation phenomenon. Here an interruption is defined as an event that quite suddenly and unexpectedly changes the force operative in a one-way two-state process. An interference is defined as an event that brings the opposing force suddenly and unexpectedly to a play in an initially assumed one-way process. A dislocation process may involve a combination of an interruption and an interference. Figure 12 provides a simple illustration for these three cases.

Consider a simple planned or expected adoption trajectory  $S(t) = 1 - \exp(-t)$  with a force of adoption  $r(t) = 1$  event per year. In Figure 12 Case A illustrates an interruption where  $r(t) = 0$  from  $t = 1$  year to  $t = 1.5$  years. This causes a horizontal dislocation of the adoption trajectory. Case B illustrates an interference where an otherwise absent force of rejection appears from  $t = 1$  year to  $t = 1.5$  years with a magnitude  $h(t) = 1$  event per year. In this particular case, the dislocation of the trajectory is down and right. In Case C there is the combination of the above interruption and interference causing an even deeper dislocation to the down and right.

The identification of the forces present in a dislocation phenomenon is not without problems. The simulator (Figure 6) can be used to experiment what kinds of forces may be associated with an observed trajectory and its dislocations. The question arises, is the situation unique, or can the same shape of a trajectory be generated by different combinations of forces. This is a potential identification problem. For the existence of this problem a special case of non-uniqueness is sufficient: Consider Cases A and B. Case A is clear. An interruption in this case generates a horizontal dislocation of the adoption trajectory. Can an interference in Case B also generate a horizontal dislocation similar to Case A? The answer is yes. Such a special case exists. Let  $r(t) = b$  and  $h(t) = a$  with an initial condition  $S_0$  in Equation 10. This is the situation of Case B:

$$S(t') = [b/(a+b)] + [S_0 - (b/(a+b))] \exp(-(a+b)t') \text{ where } t' = 0 \text{ at } t = t_0.$$

$t_0$  is the moment of time at which an interference with a force  $h(t) = a$  events per year is introduced. In the special case where  $S_0 = b/(a+b)$  the trajectory remains horizontal with the magnitude  $S_0$ . For this case one cannot deduct from the shape of the trajectory uniquely whether there has been an interruption or an interference.

In reliability and maintainability engineering failure and repair rates are obtained experimentally as the prime source of information. In actuarial and insurance business similarly the forces of mortality for people in various living environments are obtained experimentally. In both cases this information is essential for achieving an optimal performance. The same must be done for an optimal management of innovations.



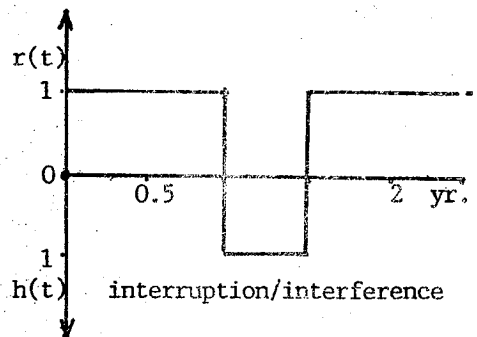
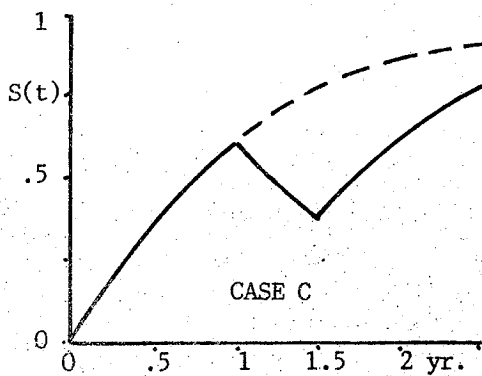
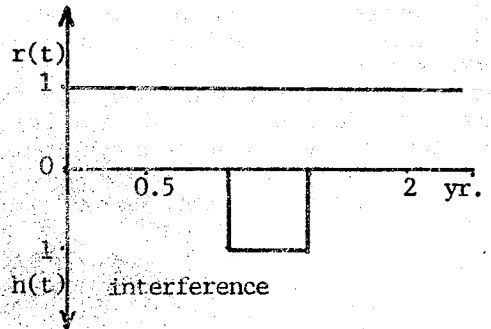
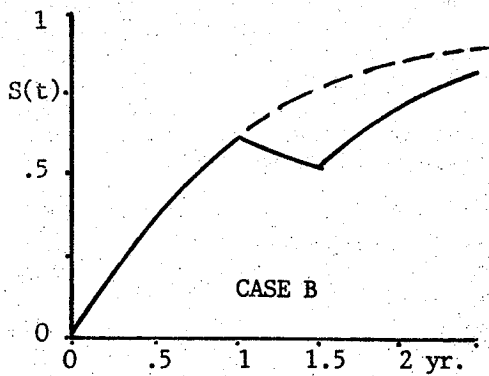
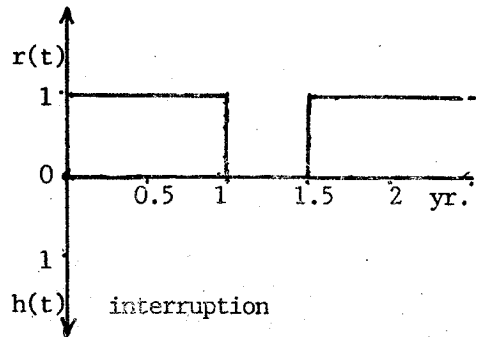
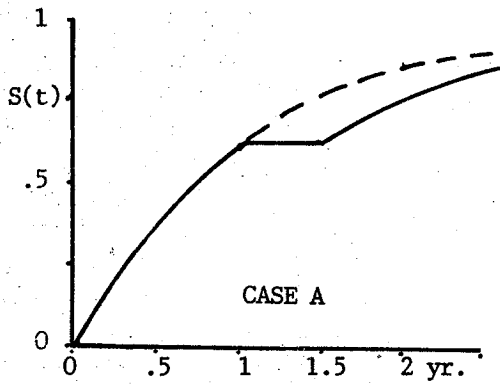


Figure 12. Simple examples of dislocation effects on adoption trajectories.

CHANNELING OF INNOVATIONS: APPLICATIONS OF COMBINATORIAL PROBABILITY THEORY

Typically an adoption or a rejection goes through a network of channels before a final adoption is realized. For example, a new product may have to be channeled first from the manufacturer to the wholesaler, then from the wholesaler to the retailer, and finally from the retailer to the customer. This would be a serial or sequential channeling network. Another example is one where a new product is channeled to the target market segment through a channeling network with several parallel channels, e.g. via channel of direct mail marketing in parallel with a number of independent agents channeling the product, and perhaps, the manufacturer's own sales force doing the job also in parallel. For the subsequent discussion it is assumed that the trajectories  $S_k(t)$  associated with the channel  $k$ ,  $k = 1, 2, 3, \dots, n$ , are mutually independent probabilities for the  $n$  distinct channels.

Parallel Channeling

The trajectory  $S(t)$  for a network of  $n$  parallel channels, noting Equations 5, is as follows:

$$S(t) = 1 - \prod_{k=1}^n (1-S_k(t)) = 1 - \prod_{k=1}^n B_k(t) \text{ for adoption, and} \quad (15)$$

$$= 1 - \prod_{k=1}^n (1-A_k(t)) \text{ for rejection.}$$

As an example, let  $S_k(t) = 1 - \exp(-bt)$  be the adoption trajectory for all the  $n$  channels in the parallel channeling network. Then  $S(t) = 1 - \exp(-nbt)$  with  $f(t) = nb \exp(-nbt)$  and  $r(t) = nb$ . Thus the force of adoption is reinforced  $n$ -fold over that of a single channel.

Serial Channeling

The trajectory  $S(t)$  a network of  $n$  channels in series, noting Equations 5, is as follows:

$$S(t) = \prod_{k=1}^n S_k(t) = \prod_{k=1}^n (1-B_k(t)) \text{ for adoption, and} \quad (16)$$

$$= \prod_{k=1}^n A_k(t) \text{ for rejection.}$$

As an example, let  $S_k(t) = \exp(-at)$  be the rejection trajectory for all the  $n$  channels. Then the force of rejection,  $a$ , for a single channel is reinforced to a force of rejection  $na$  for the serial channeling network with  $n$  channels.

Composite Channeling

Equations 15 and 16 can be used to find trajectories for composite channeling networks, i.e. networks with various parallel and serial sub-structures. Figure 13 illustrates the probability flow network for a simple composite case. In this case the parallel rule is applied to  $S_1(t)$  and  $S_2(t)$ . Then serial rule is applied with  $S_3(t)$ , and finally parallel rule with  $S_4(t)$ . This

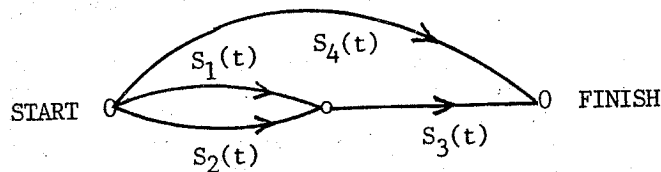


Figure 13. An example of a composite channeling network.

generates the following trajectory  $S(t)$  for this composite channeling network:

$$S(t) = 1 - (1-S_4(t))[1-S_3(t)(1-(1-S_2(t))(1-S_1(t)))]$$

If the adoption trajectory would be  $S_k(t) = 1 - \exp(-bt)$  for the four channels, then

$$S(t) = 1 - [\exp(-2bt) + \exp(-3bt) - \exp(-4bt)]$$

and the force of adoption would be

$$r(t) = b[2 + 3\exp(-bt) - 4\exp(-2bt)] / [1 + \exp(-bt) - \exp(-2bt)] .$$

The combinatorial probability can be expanded to the cases where the channel trajectories are not necessarily mutually independent probabilities (Shoeman, 1968).

#### AN OVERVIEW

The previously mentioned concepts and methods can be combined in a variety of ways for dynamic modeling of adoption, rejection and life cycles of innovations. For example, the modeling of an adoption process may start with the modeling of a set of channels with a respective set of Markov task-decision networks. Then the channels are combined into an appropriate channeling network. Then its trajectory and the respective force of adoption are computed. At this point one might introduce a set of interruptions or interferences treated as exogenous effects superimposing forces of adoption and/or rejection upon the previous force of adoption. Then one could, for example, by using the simulator illustrated in Figure 6, compute the overall trajectory involving all the above structural considerations and effects. It is quite obvious that this type of a modeling effort could benefit greatly from computer aided approaches, such as the utilization of discrete and continuous system modeling and simulation programs, e.g. IBM CSMP, DYNAMO, SIMSCRIPT, GASP, and SIMULA. It should be noted that, although the previous discussions have been in the framework of continuous systems, they could be also formulated in the framework of discrete systems.

#### CONCLUDING REMARKS

The main goal of this presentation was to investigate how to apply the concepts and methods of the well-established assurance and actuarial sciences, especially probabilistic reliability and availability theory and practice, to the dynamic modeling of adoption, rejection and life cycles of innovations. This is a first step leading to several other directions of investigations.

For example, there remains the crucial question how to empirically identify and measure the forces of adoption and rejection associated with an innovation process. In an other direction, the economics of innovation processes must be brought into the modeling process. This is necessary in order to introduce the concepts of optimality and optimal control so necessary for proper management of innovation processes. Yet in another direction, innovation processes take place in structurally and institutionally changing social environments. Thus one should investigate various adaptive approaches to the modeling and management of innovation processes. The impacts of new innovations that also may replace old innovations may have complex impacts upon society, e.g., the trade-off between automation and employment. Thus the dimension of "Technology Assessment" needs to be investigated as it does not only involve the economic but also social and political factors.

#### REFERENCES

- Jutila, S.T. (1972), "Generalizations of Availability Models", Industrial Mathematics, Vol. 22 Part 2, pp 87-98
- Jutila, S.T. and Jutila, J.M. (1986), Diffusion of Innovation in American Automobile Industry, Business Research Center Working Paper Series 86-7, The University of Toledo, Toledo, Ohio, U.S.A.
- Kie, C. (1986), Regional Variation of Process Innovation in the U.S. Steel Industry, Masters thesis, Department of Geography and Planning, The University of Toledo, Toledo, Ohio, U.S.A.
- Shooman, M. (1968), Probabilistic Reliability, McGraw-Hill, New York