Aggregate Dynamics of a Population of Commodity Cycle Models

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Introduction

A fundamental problem in the understanding of dynamic behavior of complex systems is to determine how the aggregate behavior of the system arises from the behavior of its components. System dynamics models show the importance of representing explicitly the interactions between multiple sectors and policy variables (rates) for understanding behavior in the aggregate. But these models beg the question of explaining the behavior, even of the sub-sectors, by representing in an aggregate way the information flows and the transformations used by decision-makers. For example, in a model of a manufacturing organization, the decisions of individual product-line managers may be represented by a single aggregate rate variable. A more basic problem, and the subject of this paper, is the relationship between the structure and behavior of the aggregate (the organization) and of its components (the individual decision-makers).

In previous work (Rahn, 1985; 1987), study of the aggregate behavior of ensembles of similar dynamic structures were based on an asymptotic expansion of the Master Equation for stochastic birth and death processes. The resulting equations describe the evolution of various moments of the probability distribution of the levels. The inspiration for this approach came from recent work on self-organizing systems (Nicolis and Prigogine, 1977) in which non-linear interactions between components supports novel, non-equilibrium behavior. The expected source of novel behavior in this approach is the set of stochastic events associated with each term in the net rate of a level. Novel behavior arises when aggregate effects such as diffusion (due entirely to stochasticity) compete successfully with other, non-stochastic effects such as drift to support the new behavior such as self-organizing oscillations.

The theoretical results are mixed at best. Linear systems do not show novel behavior. Some non-linear models may show new growth modes as revealed by formulas for the eigenvalues of the linearized versions of such models. The new growth modes shown by a reduced-order version of the Commodity Cycle model (Meadows, 1970) suggest that this non-linear model may support novel behavior in some regimes of operation. This paper partially lays to rest that expectation.

The Experiments

The analysis of general, non-linear models by the Master Equation approach is mathematically excessively complicated. The asymptotic series approach used in earlier papers deals only with a linearized version of a model. An attempt to simulate the stochastic processes implied by each rate term was unfruitful. For these reasons, it was decided to shift the basis of the aggregation and to proceed by directly simulating the interactions between component submodels to detect if possible the generation of novel behavior due to these interactions. In this study, the source of novel behavior is no longer presumed to come from the stochasticity and non-linearity of each term in the rate equations but rather from the interactions imposed on a population of component submodels. The submodels comprise the elements of the population and the interaction mechanism affects the behavior of the aggregate.

For the purposes of this study, a population of interacting submodels based on the Commodity Cycle model was established with the following characteristics:

- -all submodels have the same basic structure as a global reference model
- -delay and smoothing interval parameters are distributed over the population.

The use of the same structure in the submodels as in the global model is based on the approach proposed in (Rahn, 1985) in which the global model is interpreted as representing the policy style of individual actors in the system. By this hypothesis, a similar style (sources and use of information in the rate equations) is attributed to the components of the aggregate system. The distribution of parameters over the population provides variability that is dynamically significant and may support the generation of novel behavior in the aggregated system. For four of the five distributed delay parameters, Beta distributions were used with means equal to the value of the same parameter in the global model but with ranges from a lower limit of a minimum delay time to an upper limit of 10 times the minimum. The fifth parameter, Average Life of Production Capacity, a symmetric Beta distribution about the mean of 200 months was used.

Two different global models were used as the basis of the submodels. The first was the standard, 12-level, non-linear Commodity Cycle model. The second was a quasi-linear version of the same model in which the non-linear TABLE functions linking Inventory to Consumption Rate and to Desired Production Capacity were replaced by approximations making Consumption Rate and Desired Production Capacity piecewise linear functions of Inventory. The purpose of this non-trivial linearization was to provide a means to test the usefulness of eigenvalue analysis to detect changes in behavior. In the event, the linearization proved useful in another, unforeseen way.

Besides the structure of the submodels, the aggregate behavior is determined by the interaction mechanisms between the submodels. In these experiments, a two-step approach was implemented. First a market-link mechanism was imposed to determine the Consumption Rate and Desired Production Capacity. To maintain the structural similarity mentioned above, the same TABLE functions as in the non-linear global model were used. In general, this provided estimates of Consumption Rate and Desired Production Capacity on the scale of the system. The second step, then, was to distribute the resulting values among the component submodels in proportion to some measure of the size of the submodel.

Two market-link or co-ordinating mechanisms were used in these experiments. A ´uniform market´ was characterized by an aggregate Inventory and Consumption Rate which gave an aggregate (in fact, average) Coverage and Price which then determined Consumption Rate and Desired Production Capacity. In the linearized model, aggregate Inventory was used in the separate linearizations for Consumption Rate and Desired Production Capacity. A ´parallel market´ was characterized by having each submodel determine its own ´equivalent market´ Price based on its own Coverage.

Three distribution mechanisms were used to allocate the estimates of Consumption Rate and Desired Production Capacity among the component submodels. The first was simply to assign the same value to each submodel, i.e. a uniform distribution with a weighting factor equal to the reciprocal of the number of component submodels. The other two methods used the ratio of the Inventory or Production Capacity of a component submodel to the aggregate value of the respective variable. We considered each submodel to represent the behavior of individual producers; these distribution measures were used as proxies for market share and relative size of each producer.

From this brief description, we see that the parallel markets-uniform distribution model gives the largest degree of independence to each submodel; co-ordination is relatively weak. The uniform market-proportional distribution models imposed a higher degree of co-ordination on the component submodels. The effects of these aggregate structural differences will be seen in the results.

To perform the experiments, Professional DYNAMO+, v. 3.1c, was used to simulate the component submodels by indexing a standard model and supplying the distributed values of the delay parameters described above. Initial conditions for the component submodels were based on the fixed, near-equilibrium values for the global model and distributed using the reciprocal of the number of component submodels. Practical limits imposed by the software necessitated using small populations (10 or 20) of component submodels. In the case described here, this limitation does not pose a statistical problem for this study since the only use of the distribution of parameters is to generate a reasonably broad variation in parameters. Alternative choices of parameters are possible but have not yet been tested, e.g. a sample of two submodels with one submodel having the maximum value and one the minimum value of each distributed parameter. The randomized choice of parameters used here seeks only to establish with some generality the nature of the behavior of such populations. Specifically chosen populations may indeed show other behavior.

The Results

Figures 1, 2 and 3 show typical results of these experiments. In the (a) exhibit of each Figure, the range of each variable: Inventory, Production Capacity and Coverage is shown with a solid line for the Inventory range variable. In the (b) exhibits, the maximum, minimum and mean values of the Inventory are shown with the mean value traced by a solid line. Figure 1 shows results for the non-linear, uniform market with the distribution mechanism proportional to Production Capacity. Figure 2 shows results for the non-linear, parallel markets with the distribution mechanism being the reciprocal of the number of component submodels. Figure 3 shows results for the linear, parallel markets with the same distribution mechanism as in Figure 2. In all cases shown here, the models were perturbed by a sinusoidal multiplier of the Consumption Rate of constant amplitude (50% of the mean rate) and period of 48 months.

In general, the behavior of the aggregate variables is similar to the behavior of the global model and this is equally true for the linearized as for the non-linear versions. The aggregate oscillations of level variables are closer to sinusoidal, even in the non-linear models, but have larger amplitude than global variables. It is only in the behavior of the ranges that some interesting effects appear. The uniform-market and the non-uniform-distribution mechanisms act to limit the amplitude of variation of the range variable whereas these same variables grow, albeit slowly, when the parallel market is combined with the uniform distribution mechanism to co-ordinate the component submodels.

The reasons for the lack of novel behavior may be explained by an analysis of a reduced-order version of the linearized model. After eliminating all delays and smoothing elements, the model is of second-order. The equilibrium points of the model are determined by the parameters of the piecewise-linear approximations of Consumption Rate and Desired Production Capacity for each of the intervals of Inventory. Calculation reveals that there is only one <u>feasible</u> equilibrium point. A feasible equilibrium point is a value of Inventory that falls <u>within</u> the interval whose parameters determine the equilibrium point. Further, the equilibrium point of each interval below the 'equilibrium interval' is larger than the upper bound of the interval and in fact is increasing up to the interval neighboring the equilibrium interval on the left. Finally, each interval above the equilibrium interval has an equilibrium point that is below its lower bound and, except for the neighboring interval on the right, beloweven the lower bound of the equilibrium interval and decreases, becoming negative for intervals far from the equilibrium interval. Thus the model tends to 'focus' on the equilibrium interval and the market mechanisms replicate this focusing for each submodel while the distribution mechanisms are too weak to counter the focusing.

The existence of the focusing characteristic with its unique system-wide equilibrium suggests that these models are unlikely to support bifurcations or chaotic behavior. Early results from experiments similar to those reported here suggested that some frequency-doubling effects might appear but subsequent work revealed that only range variables sometimes showed such an effect and it could be explained by phase differences between the submodels generating the maximum and minimum values that make up the range measure.

An explanation for the growth of the ranges under parallel markets nd uniform distribution is found by recalling that in this case, each component submodel sees a Consumption Rate based on its own Inventory but distributed uniformly. Thus a high Inventory submodel would have a high Consumption Rate but for the uniform distribution which gives it a lower Consumption Rate and thus does not draw down the high Inventory. In the other models, the 'focusing' effect is felt uniformly by all submodels and is re-inforced by the non-uniform distribution mechanism so that high Inventory submodels have higher Consumption Rate which reduces or restrains the growth of Inventory and thus limits the growth of the range variable.

Conclusions and Further Work

With the gamut of co-ordination mechanisms used in this study, we conclude that the interpretation of the global model as a model of a representative decision-maker and the use ofthe global model to trace the average behavior, even for non-linear models, is adequate when the objective is to represent a population of similarly structured decision-makers. New behavior of the aggregate measures is more likely to come from new decision-making structure (which begs the question of the similarity of the submodels to the global model), or from additional micro-structure to deal with extreme cases or 'boundary states' of the submodels. For example, mechanisms that generalize the distribution method by eliminating 'small' submodels or by emphasizing the strength of 'large' submodels could lead to changes in the composition of the population and hence in aggregate behavior.

Extensions of the current work to incorporate distributions of the TABLE function parameters would generalize the idea of individual component submodels whose structure remains fixed while providing an extra source of variability. Although chaotic behavior and bifurcation effects do not seem likely to arise from the current form of the Commodity Cycle model, the linearization approach suggests an alternative way to predict the possibility of such behavior by verifying the existence of multiple feasible equilibrium points. Such linearized models may be simpler to analyze for the aggregate effects of their stochastic behavior than the fully non-linear models. The non-trivial cost of such simplification is the derivation of a consistent linearization of the model.

References

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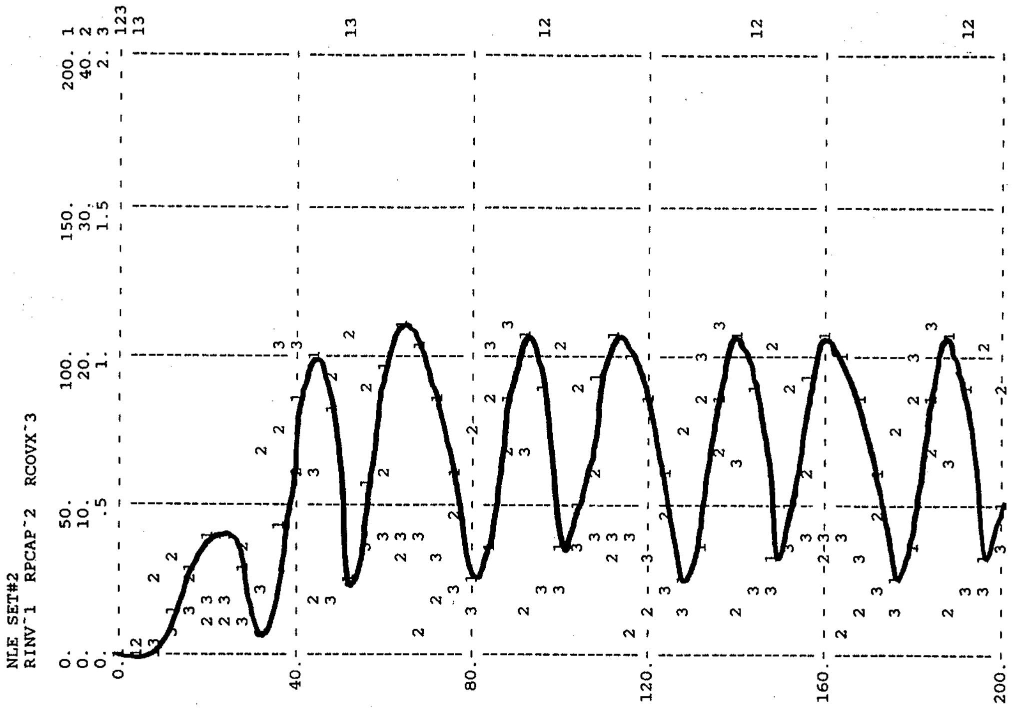

Figure 1a. Range (Max-Min) of Inventory, Capacity, Coverage from a set of 10 Commodity Cycle Submodels; non-linear; uniform market

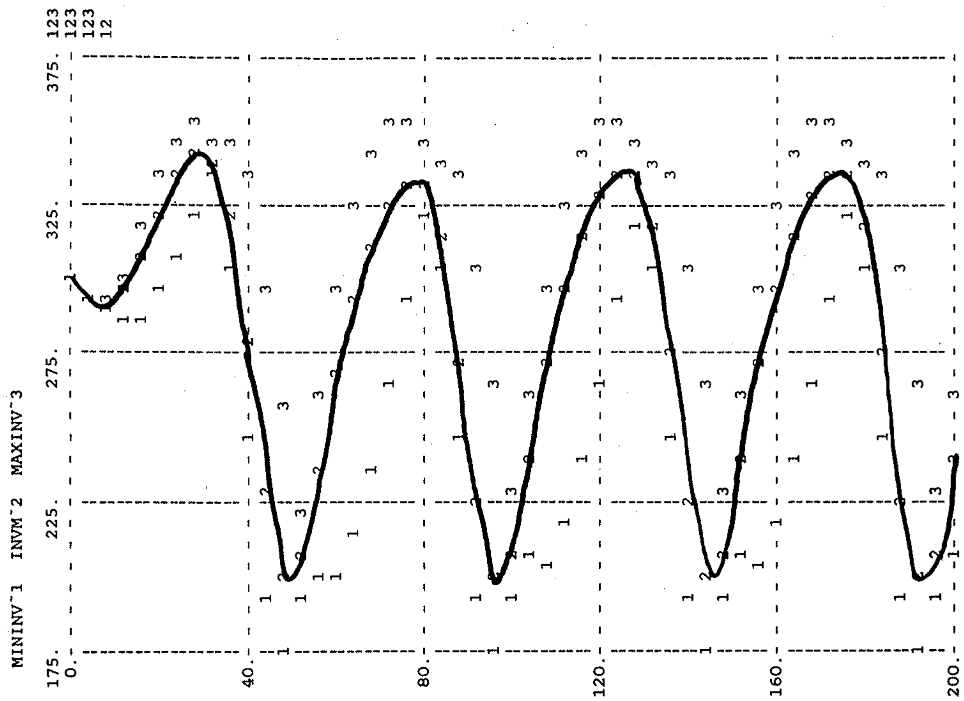

Figure 1b. Max, Min, Mean Inventory from a set of 10 Commodity Cycle Submodels; non-linear; uniform market

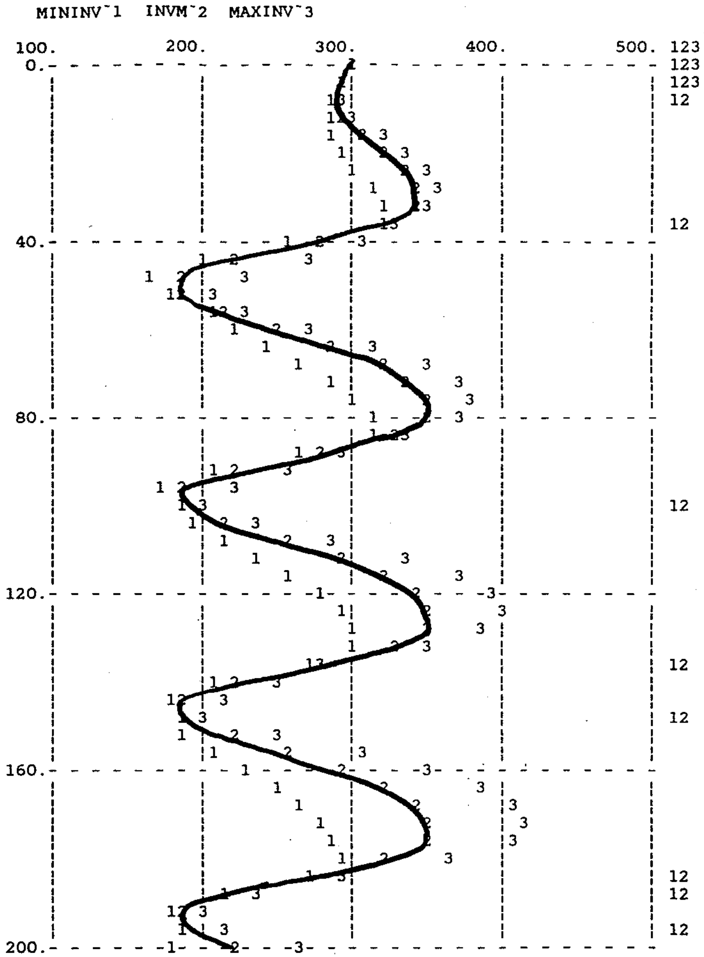

Figure 2b. Max, Min, Mean Inventory from a set of 10 Commodity Cycle Submodels: non-linear; parallel markets.

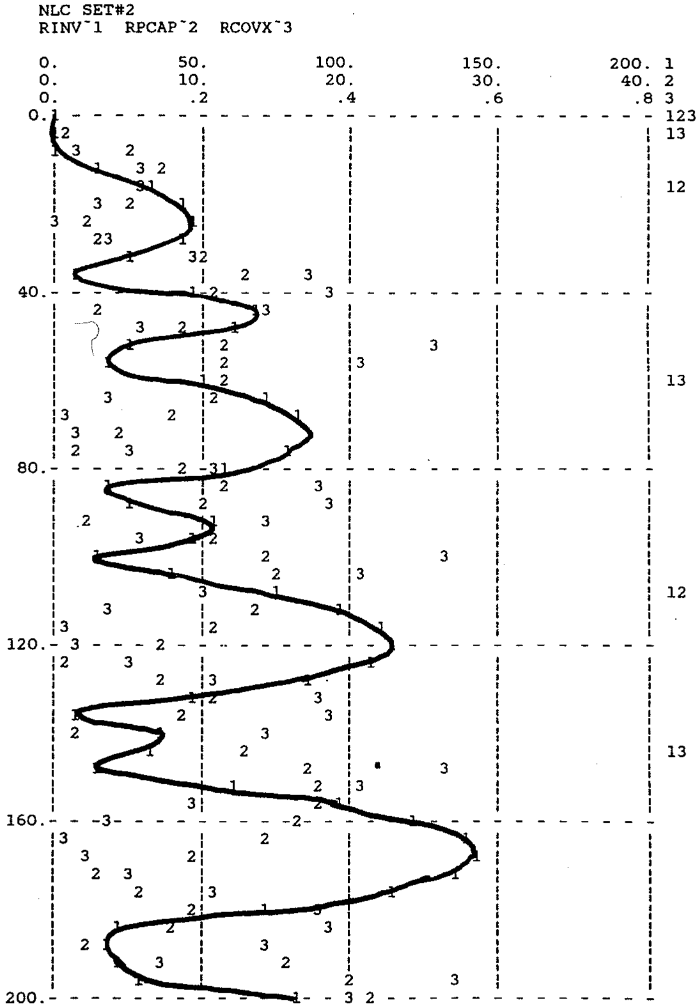

Figure 2a. Range (Max-Min) of Inventory, Capacity, Coverage from a set of 10 Commodity Cycle Submodels: non-linear; parallel markets

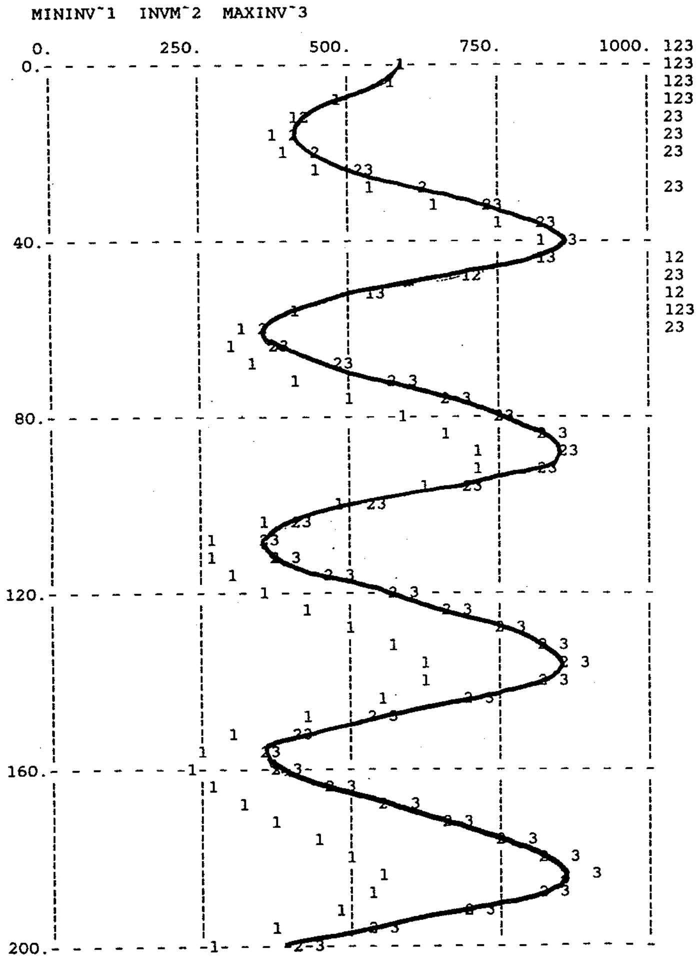

Figure 3b. Max, MIN, Mean Inventory from a set of 10 Commodity Cycle Submodels: linear; parallel markets

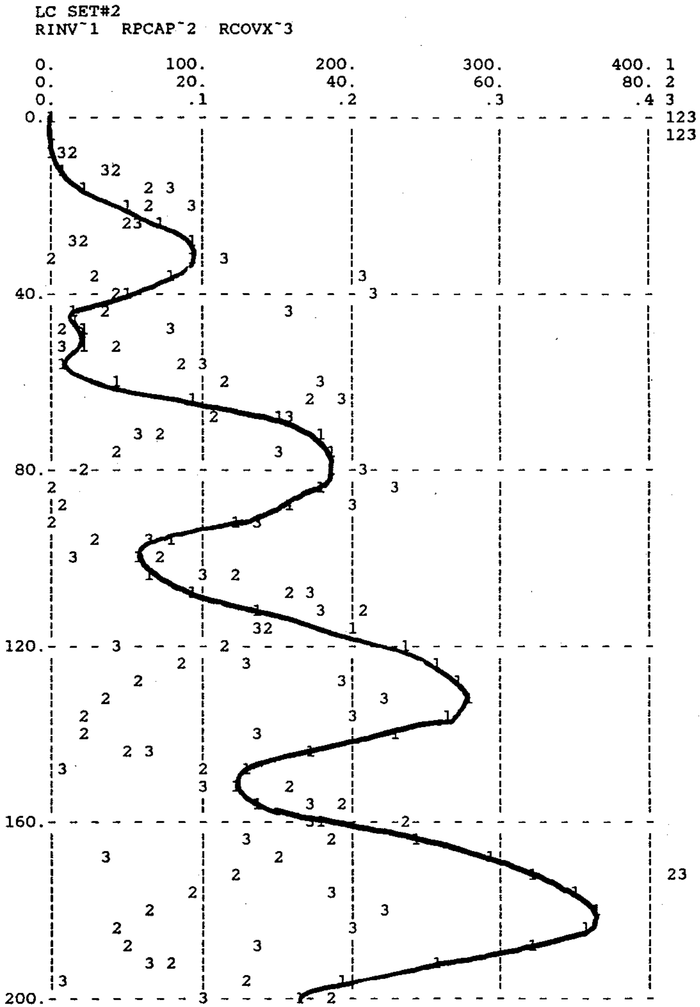

Figure 3a. Range (Max - Min) of Inventory, Capacity, Coverage from a set of 10 Commodity Cycle Submodels: linear; parallel markets