# SYNERGETICS AND SOCIAL SCIENCE

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**Abstract**

General concepts for the quantitative description of the dynamics of social processes are introduced. They allow for embedding social science into the conceptual framework of synergetics. Equations of motion for the socioconfiguration are derived on the stochastic and quasideterministic level. As an application the migration of interacting human populations is treated. The solutions of the nonlinear migratory equations include limit cycles and strange attractors. The empiric evaluation of interregional migratory dynamics is exemplified in the case of Germany.

## 1. INTRODUCTION

In the last 20 years a new level of understanding of complex systems in physics, chemistry and biology has been reached. The new concepts find their general expression in synergetics, the science of macroscopic space-time structures of multi component systems composed of interacting units.

Let us first summarize some of these central concepts: The macroscopic space-time behaviour of multi-component systems is governed by the dynamics of a few order parameters only. The systematic reason for this fact is the "slaving principle" set up by H. Haken /1/: He could show in a rather general form that the dynamic behaviour of the huge number of microscopic degrees of freedom is already determined (slaved) by a small number of order parameters. Therefore the micro-variables can be eliminated, so that a self-contained dynamics of order parameters alone arises. This dynamics is formulated in terms of nonlinear equations of motion containing several control parameters which may be adjusted experimentally. Such equations exhibit universal structures which are in many respects independent of the nature of the units composing the system.

This fact is the origin of the interdisciplinary relevance of synergetics.

The global dynamics of order parameters described by such equations may change and new space-time patterns may arise, if the control parameters pass critical domains. Such changes of the global nature of solutions are denoted as phase transitions.

Since the specific structure of the macroscopic dynamic patterns described by the set of nonlinear equations is not predetermined by any specific form of boundary conditions or control-parameters, the use of the word "selforganization" is justified in this context.

The main objective of the present article is to show, how social science can in principle be embedded into the conceptional framework of synergetics (for a more comprehensive treatment see /2/).

Indeed, it is intuitively clear, that also the human society as being composed of many individuals, is a system to which synergetic laws should apply:

a) The society is governed by relatively few political, economical, cultural, religious and social order-parameters only.
b) The decisions and actions of the individuals are "slaved", that means influenced, orientated, biased, even predetermined, by the prevailing social, economic, cultural and political climate established by the order-parameters of the social systems.
c) The global evolution of a society may primarily be seen as a selfcontained dynamics of the endogenous order-parameters which however is controlled by exogenous influences like environment, resources, economic constraints, foreign relations etc.
d) The internal structure of a society is only partially predetermined by external influences and evolves in a selforganizing way. The outcome however is not unique. Instead, different modifications of social systems may establish under the same external circumstances.
e) In particular in critical situations the evolution may destabilize so that a phase transition (revolution) into a new state and another dynamic mode may take place.

The notions about the society mentioned so far seem to be more or less wellknown and commonplace. The main problem is however to cast them into a quantitative form such that the powerful methods of mathematical analysis can be applied to sectors of the society as well as to systems of natural science.

Let us anticipate some achievements to be provided by a quantiative theory of social systems:

a) The connection between the microlevel of decisions and actions of individuals and the macrolevel of equations of motion for collective order-parameters of the society must be established.
b) The possible stationary states, evolutions and revolutions of a society must be reflected in the global structure of the dynamics of quantitative model systems.
c) Under sufficiently welldefined conditions the quantitative models should be amenable to comparison with concrete empirical systems through regression analysis and forecasting.

## 2. GENERAL CONCEPTS OF QUANTIFICATION

Trying to achieve a quantification of social processes we proceed from the microlevel of individuals with their attitudes and decisions to the macrolevel of quantitative collective variables and their dynamics. Finally, the interdependence of micro- and macro-level must be discussed.

### 2.1 The Microlevel

Aspect Space The space $\mathfrak{A}$ comprises all (independent) aspects which the social system may provide for the individual. Examples are: politics, religion, education, habitation, economic supply and demand. Each aspect is considered as a dimension of the aspect space.

Attitude Vectors. The dimensions of $\mathcal{O}\!\mathcal{l}$ are further defined in such a manner, that the individual can have different attitudes with respect to each aspect. In the case of the above mentioned aspects the attitudes are: political opinions, religious denominations, education levels, place of residence, production activity, consuming habits.

If $A$ apects $a = 1, 2, \ldots A$ are considered and $d_a$ attitudes $i_a = 1, 2, \ldots d_a$ belong to aspect $a$, the attitude vector $\underset{\sim}{i} \in \mathcal{O}\!\mathcal{l}$ of an individual is defined by $\underset{\sim}{i} = \{ i_1, i_2, \ldots i_A \}$ The space $\mathcal{O}\!\mathcal{l}$ can now be defined as the $A$ dimensional lattice space with $L = \prod_{a=1}^{A} d_a$ lattice points, whose coordinates are given by the attitude vectors.

Homogeneous Subpopulations. The society is composed of subpopulations $\mathcal{P}_\alpha$ A homogeneous subpopulation is defined by the (in principle verifyable) assumption that its members exhibit the same probabilistic decision behaviour with respect to the aspects. In general the classes of equal socio-economic background are candidates for homogeneous subpopulations. It should be clear, that a fine-grained description of the society requires its decomposition into more subpopulations than a coarse-grained description.

Individual Probability Transition Rates. The dynamics on the microlevel of a society is generated, if individuals decide to change their attitude. Such decisions can only be described in probabilitstic manner. In this sense we introduce individual probability transition rates:

$p_{ji}^{\alpha}$ = probability transition rate per unit of time for a member of subpopulation $\mathcal{P}_\alpha$ to change from attitude $i$ to attitude $j$ (2,1)

Transition Rates and Utilities. Let us now ask: What is the "driving force" for the transition of an individual between different attitudes? It is assumed that there exists a measure $\check{u}_i^\alpha$ and $\check{u}_j^\alpha$ of the subjective "utility" to a member of subpopulation $P_\alpha$ of the "old" attitude $i$ and the "new" attitude $j$, respectively. The individual compares both utilities and for increasing difference $(\check{u}_j^\alpha - \check{u}_i^\alpha)$ the transition rate in population $P_\alpha$ from $i$ to $j$ will more and more prevail over the reverse rate from $j$ to $i$. Since $p_{ji}^\alpha$ must be a positive definite quantity by definition, its most plausible form in terms of the utilities of the old and new attitude is

$$p_{ji}^\alpha = \nu \exp\{\check{u}_j^\alpha - \check{u}_i^\alpha\} \tag{2,2}$$

Arguing reversely, the formula (2,2) can be subject to empirical verification leading to the determination of the utilities, if the transition rates $p_{ji}^\alpha$ are known (see section 3.4).

Summarizing we may say that the individual attitude vectors are the microvariables and that the individual probability transition rates describe the microdynamics of the system.

## 2.2 The Macrolevel

The microvariables and microdynamics now give rise in a natural way to macrovariables and macrodynamic concepts:

The Socioconfiguration. Let $n_i^\alpha$ members of subpopulation $P_\alpha$ take the attitude $\underset{\sim}{i} = \{i_1, i_2, \ldots i_A\}$. Then the socioconfiguration

$$\underset{\sim}{n} = \{n_1^1 \ldots n_1^P, n_2^1 \ldots n_2^P, \ldots n_i^\alpha \ldots, n_L^1 \ldots n_L^P\} \tag{2,3}$$

characterizes in a given moment the distribution of attitudes within the total population of the society. In general it can be assumed that $n_i^\alpha \gg 1$, and $n_i^\alpha$ can be treated as a quasi-continuous variable. The socioconfiguration is considered as a central (multicomponent) macrovariable of the society.

The Material Situation. Whereas the socioconfiguration quantifies the distribution of abstract attitudes in the society, it is also necessary (and is conventionally done in economics) to describe the material situation within the society. This is accomplished by introducing the situation vector

$$\underset{\sim}{y} = (y_1, y_2 \cdots\cdots y_s) \tag{2,4}$$

consisting of quantified measures and indicators $y_m$ of material variables. The $y_m$ include prizes, capital, commodity stock, inventory, investment and production-rates, housing, infrastructure, etc.

The Configurational Probability Transition Rate. If one individual of $P_\alpha$ changes its attitude from $i$ to $j$ it induces a change of the socioconfiguration

$$\begin{aligned} \underset{\sim}{n} &= \{ n_1^1 \cdots n_i^\alpha \cdots n_j^\alpha \cdots\cdots n_L^P \} \\ \rightarrow \underset{\sim}{n}_{ji}^\alpha &= \{ n_1^1 \cdots (n_i^\alpha - 1) \cdots (n_j^\alpha + 1) \cdots n_L^P \} \end{aligned} \tag{2,5}$$

Since each of the $n_i^\alpha$ members of $P_\alpha$ with attitude $i$ can make this transition to $j$ statistically independently, the configurational probability transition rate from $\underset{\sim}{n}$ to $\underset{\sim}{n}_{ji}^\alpha$ is given by

$$\begin{aligned} w(\underset{\sim}{n}_{ji}^\alpha \mid \underset{\sim}{n}) &\equiv w_{ji}^\alpha(\underset{\sim}{n}) = n_i^\alpha p_{ji}^\alpha \\ &= n_i^\alpha \nu \exp\{ \tilde{u}_j^\alpha - \tilde{u}_i^\alpha \} \end{aligned} \tag{2,6}$$

whereas all transition rates from $\underset{\sim}{n}$ to $\underset{\sim}{n}' \neq n_{ji}^\alpha$ vanish:

$$w(\underset{\sim}{n}' \mid \underset{\sim}{n}) = 0 \quad \text{for} \quad \underset{\sim}{n}' \neq \underset{\sim}{n}_{ji}^\alpha \tag{2,7}$$

## 2.3 Equations of Motion

We shall now see that the introduction of configurational probability transition rates leads in a natural way to equations of motion for the socioconfiguration.

Because of the probabilistic description of the microprocesses we must expect, that the exact description of the dynamics of the socioconfiguration can only be a probabilistic one, too. For this purpose we introduce the probability distribution over socioconfigurations:

$$P(\underset{\sim}{n},t) = \text{probability, that the socioconfiguration } \underset{\sim}{n} \text{ exists at time } t \qquad (2,8)$$

This probability is normalized according to

$$\sum_{\underset{\sim}{n}} P(\underset{\sim}{n},t) = 1 \qquad (2,9)$$

<u>Master Equation</u>. The probability distribution obeys a fundamental equation, the <u>master equation</u> which derives from a dynamic probability balance consideration:

$$\frac{dP(\underset{\sim}{n},t)}{dt} = \sum_{\substack{\alpha,j,i \\ (j\neq i)}} \left\{ w_{ji}^{\alpha}(\underset{\sim}{n}_{ij}^{\alpha}) P(\underset{\sim}{n}_{ij}^{\alpha},t) - w_{ji}^{\alpha}(\underset{\sim}{n}) P(\underset{\sim}{n},t) \right\} \qquad (2,10)$$

The meaning of (2,10) is intuitively clear: The change with time of the probability of configuration $\underset{\sim}{n}$ is due to two counteractive terms on the r.h.s. of (2,10): The first term describes the probability flow per unit of time from all neighbouring configurations into the configuration $\underset{\sim}{n}$, and the last term (with negative sign) subtracts the probability flow <u>from</u> $\underset{\sim}{n}$ into all neighbouring configurations.

The solution of (2,10) not only yields the evolution of the most probable configurations but also the width and form of the probabilistic fluctuations around them. For most applications in social science it contains too much information, which cannot be compared with the poor empirical data. Therefore it is often sufficient to go over to equations for the meanvalues $\bar{n}_i^\alpha(t)$ of the variables $n_i^\alpha$

Meanvalue Equations. The meanvalues of $n_i^\alpha$ are defined by

$$\bar{n}_i^\alpha(t) = \sum_{\underset{\sim}{n}} n_i^\alpha P(\underset{\sim}{n},t) \tag{2,11}$$

In a straightforward manner there can be derived exact equations of motion for the meanvalues from the master equation (2,10) (see /2/):

$$\frac{d\bar{n}_i^\alpha(t)}{dt} = \sum_{j(\neq i)} \left\{ \overline{w_{ij}^\alpha(\underset{\sim}{n})} - \overline{w_{ji}^\alpha(t)} \right\} \tag{2,12}$$

These equations are not yet selfcontained, since the calculation of the right hand side requires the knowledge of the probability distribution $P(\underset{\sim}{n},t)$ For sharply peaked unimodal distributions, however, it is justified to assume

$$\overline{w_{ij}^\alpha(\underset{\sim}{n})} \approx w_{ij}^\alpha(\bar{\underset{\sim}{n}}(t)) \tag{2,13}$$

so that the equations (2,12) take the approximate but selfcontained form:

$$\frac{d\bar{n}_i^\alpha(t)}{dt} = \sum_{j(\neq i)} \left\{ w_{ij}^\alpha(\bar{\underset{\sim}{n}}(t)) - w_{ji}^\alpha(\bar{\underset{\sim}{n}}(t)) \right\} \tag{2,14}$$

Equtions for Material Variables. In order to complete the dynamic description of the society on the macrolevel, it is necessary to set up equations of motion for the material situation vector too. We shall not go in details here, but make a remark which seems essential to the structure of the theory:

Conventional macro-economic models start from phenomenological equations for the macrovariables. This procedure, however, disguises the relation between the micro- and macro-economic level.

An alternative way of finding equations for material variables is suggested by our conceptual framework: The evolution of all material variables - investment, production, prizes and consumption of commodities etc. - is without exception tied to the decisions and actions of subgroups of the population - managers, producers, merchants, consumers etc. - who may be partners or opponents. Therefore the dynamics of material variables can be traced back to the attitudes (including modes of action) and transitions between attitudes of members of subpopulations. Simultaneously this way of setting up macro-economic equations provides the link to micro-economics!

### 2.4 The Interdependence of Micro- and Macro-Level

At a first glance it might seem that the microlevel (individual attitudes, actions and decisions) determines the dynamics of the macrovariables, whereas no feedback takes place from macro- to microlevel. This is however not the case! Instead there exists a cyclic coupling between both levels: The individual activities merge into the collective state and dynamics of the society, and, vice versa, the latter leads to partial adjustment of individual behaviour, so that micro-behaviour and macro-dynamics are coupled selfconsistently.

Formally, the dependence on the macrostate of individual decisions is expressed by the fact, that the utility $\tilde{u}_i^\alpha$ of attitude $i$ to a member of population $P_\alpha$ in general depends on the existing macrostate:

$$\tilde{u}_i^\alpha = \tilde{u}_i^\alpha(\underset{\sim}{n}, \underset{\sim}{y}) \qquad (2,15)$$

Hence, also the individual probability transition rates become a function of the macrostate

$$p_{ji}^\alpha(\underset{\sim}{n}, \underset{\sim}{y}) = \nu \exp\{\tilde{u}_j^\alpha(\underset{\sim}{n}, \underset{\sim}{y}) - \tilde{u}_i^\alpha(\underset{\sim}{n}, \underset{\sim}{y})\} \qquad (2,16)$$

As a consequence the meanvalue equations (2,14) become nonlinear equations. The same holds in general for equations of the material situation.

The nonlinearity of the constitutive equations reflects the complexity of social processes. The "slaving principle" mentioned above is implied by eq. (2,16), since this equation directly ties the micro-decisions to the macrostate, that means to the order parameters. The nonlinear dynamics now leads - depending on initial conditions and on exogeneous control parameters - in general to a complex variety of possibilities for selforganizing dynamic patterns within a society.

## 3. AN EXAMPLE: THE MIGRATION OF HUMAN POPULATIONS

Migration of human populations is an appropriate example of dynamic social processes because of the following reasons:

a) Each individual takes a welldefined decision in a given interval of time either to stay or to change the area of its residence.

b) The interaction of populations on various psychological, social and economic levels leads to nontrivial mgiratory effects, for instance homogeneous intermixture of populations, ghetto formation, growth of metropoles etc.

c) The theory is amenable to comparison with the empiric situation, because migration data are available in many countries.

### 3.1 The General Migratory System

The system of $P$ subpopulations $\alpha = 1,2, \dots P$ migrating between $L$ areas $i = 1,2, \dots L$ is easily seen to be an application of the general concepts developed above, if the following identifications are made:

a) attitude $i$ $\widehat{=}$ "to live in region $i$ "

b) utility $\breve{u}_i^\alpha$ $\widehat{=}$ "measure of attractivity of region $i$ to a member of subpopulation $\mathcal{P}_\alpha$

c) individual probability transition rate $p_{ji}^\alpha = \nu \exp\{\breve{u}_j^\alpha - \breve{u}_i^\alpha\}$

$\widehat{=}$ probability per unit of time of a member of population $\mathcal{P}_\alpha$ to move from $i$ to $j$.

d) socioconfiguration $\underset{\sim}{n} = \{n_i^\alpha\}$

$\widehat{=}$ regional population distribution, such that $n_i^\alpha$ members of subpopulation $\mathcal{P}_\alpha$ live in region $i$.

A simple but nontrivial form of the utilities is found by considering them as functions of the socioconfiguration (2ee (2,15)), which may be expanded in a Taylor series up to first order:

$$\breve{u}_i^\alpha(\underset{\sim}{n}) = \delta_i^\alpha + \sum_{\beta=1}^{P} \sum_{j=1}^{L} \varkappa_{ij}^{\alpha\beta} n_j^\beta \tag{3,1}$$

Here we consider the case, that the attractivity of region $i$ only depends on the population in that region. That means, eq. (3,1) is simplified to

$$\breve{u}_i^\alpha(\underset{\sim}{n}) = \delta_i^\alpha + \sum_{\beta=1}^{P} \varkappa^{\alpha\beta} n_i^\beta \tag{3,2}$$

According to their meaning, the $\delta_i^\alpha$ are denoted as preference parameters, and the $\varkappa^{\alpha\beta}$ as agglomeration parameters. After inserting (3,2) into (2,2) and (2,6) one obtains the explicit form of the master equation (2,10) and the meanvalue equations (2,14). In particular, the meanvalue equations now assume the form

$$\frac{d\bar{n}_i^\alpha}{dt} = \sum_{j(\neq i)} \nu\, \bar{n}_j^\alpha \exp\left[(\delta_i^\alpha - \delta_j^\alpha) + \sum_\gamma \varkappa^{\alpha\gamma}(\bar{n}_i^\gamma - \bar{n}_j^\gamma)\right] - \sum_{j(\neq i)} \nu\, \bar{n}_i^\alpha \exp\left[(\delta_j^\alpha - \delta_i^\alpha) + \sum_\gamma \varkappa^{\alpha\gamma}(\bar{n}_j^\gamma - \bar{n}_i^\gamma)\right] \quad (3,3)$$

In the next sections we shall discuss the results obtained for special cases.

### 3.2 Two Populations Migrating Between Two Areas

One of the simplest cases is that of two interacting populations $\mathcal{P}_\mu, \mathcal{P}_\nu$ migrating between two areas (for instance within a city). The socio-configuration is $\underset{\sim}{n} = \{n_1^\mu, n_2^\mu, n_1^\nu, n_2^\nu\}$. Since the total population numbers of $\mathcal{P}_\mu$ and $\mathcal{P}_\nu$ are constant,

$$n_1^\mu + n_2^\mu = 2\bar{m} \quad ; \quad n_1^\nu + n_2^\nu = 2\bar{n} \quad (3,4)$$

there exist only two dynamic variables

$$x = \frac{n_1^\mu - n_2^\mu}{2\bar{m}} \quad ; \quad -1 \leq x \leq +1$$
$$y = \frac{n_1^\nu - n_2^\nu}{2\bar{n}} \quad ; \quad -1 \leq y \leq +1 \quad (3,5)$$

and the transition rates (2,2) may be cast into the form

$$p_{12}^{\mu} = \nu \exp[\Delta \tilde{u}^{\mu}(x,y)]$$
$$p_{21}^{\mu} = \nu \exp[-\Delta \tilde{u}^{\mu}(x,y)]$$
$$p_{12}^{\nu} = \nu \exp[\Delta \tilde{u}^{\nu}(x,y)]$$
$$p_{21}^{\nu} = \nu \exp[-\Delta \tilde{u}^{\nu}(x,y)] \quad (3,6)$$

with

$$\Delta \tilde{u}^{\mu}(x,y) = \pi^{\mu} + \varkappa^{\mu} x + \sigma^{\mu} y$$
$$\Delta \tilde{u}^{\nu}(x,y) = \pi^{\nu} + \varkappa^{\nu} y + \sigma^{\nu} x \quad (3,7)$$

where $\varkappa^{\mu}, \varkappa^{\nu}$ describes the internal agglomeration trend within populations $\mathcal{P}_{\mu}, \mathcal{P}_{\nu}$, respectively, and $\sigma^{\mu}, \sigma^{\nu}$ the "sympathy" trends to live together with the other population.

Three qualitatively different classes of migratory dynamics can now be described within the model by different choices of the trend parameters $\varkappa^{\mu}, \varkappa^{\nu}, \sigma^{\mu}, \sigma^{\nu}$. (For simplicity we only consider cases with $\pi^{\mu} = \pi^{\nu} = 0$.) Characteristic cases of each class are depicted in figures 1a,b to 3a,b. The figures a) show the fluxlines and figures b) the stationary probability distributions belonging to the same trendparameters. For illustrative purposes small numbers $\bar{m} = \bar{n} = 20$ have been chosen in figures b), leading to a considerable width of the distribution.

Here: Figures 1a,b - 3a,b

Case 1, with moderate values of agglomeration trends $\varkappa^{\mu}, \varkappa^{\nu}$ and sympathy trends $\sigma^{\mu}, \sigma^{\nu}$ leads to one stable fixed point, the homogeneous population mixture (see figures 1a,b).

Case 2, with high values of agglomeration trends $\varkappa^{\mu}, \varkappa^{\nu}$ and positive sympathy trends leads to two stable fixed points in the first and third quadrant describing that both populations tend to agglomerate in the same region (see figures 2a,b). For negative sign of $\sigma^{\mu}$ and $\sigma^{\nu}$ that is for "mutual aversion", the fixed points would be in the second and fouth quadrant. This case describes ghetto formation, that means separate agglomeration of each of the populations in a separate area.

<u>Case 3</u> ith high values of agglomeration trends $\varkappa^{\nu}, \varkappa^{\mu}$ and asymmetric sympathy trends $\sigma^{\nu} = -\sigma^{\mu}$ lead to solutions of the meanvalue equations approaching a limit cycle and to a quadrumodal stationary probability distribution (see figures 3a,b).

An interpretation of this restless migration process can be given as follows: Let us start from an initial state with both populations $\mathcal{P}_{\mu}$ and $\mathcal{P}_{\nu}$ living in region 1. Because of $\sigma^{\mu} = -\sigma^{\nu} = 1,0$, $\mathcal{P}_{\mu}$ tends to live together with $\mathcal{P}_{\nu}$, whereas $\mathcal{P}_{\nu}$ tends to evade $\mathcal{P}_{\mu}$. Therefore $\mathcal{P}_{\nu}$ emigrates from region 1, settling in region 2. Thereupon $\mathcal{P}_{\mu}$ follows $\mathcal{P}_{\nu}$, settling in region2, too, and so on. In our simple model this process of $\mathcal{P}_{\mu}$ chasing $\mathcal{P}_{\nu}$ is continuing forever. More realistically the process describes the sequential erosion of suburbs of some big cities by migration of asymmetrically interacting populations of different social standards or different races.

### 3.3 Three Populations Migrating Between Three Areas

Everyday experience with politics teaches that chaos is a constitutive part of social processes. It must therefore be expected that the equations of motion for social dynamics also reflect this fact. We shall now demonstate that already the migratory meanvalue equations with constant trend parameters comprise cases of deterministic chaos, as soon as the number of independent dynamic variables exceeds the minimal value 3 (for a more explicit discussion of the following results see /3/).

The analysis starts from eq. (3,3) speicalized to three populations $P_\alpha$, $\alpha = 1,2,3$ migrating between three regions $i = 1,2,3$ Because of the constraints

$$\sum_{i=1}^{3} n_i^\alpha = N^\alpha = const \tag{3,8}$$

six of the nine variables $n_i^\alpha(t)$ are independent, so that strange attractors could exist.

A numerical analysis with a set of representative values of the agglomeration matrix $\mathcal{K}^{\alpha\beta}$ yields the following results:

Chaotic trajectories appear, if

a) the intra-group agglomeration trends $\mathcal{K}^{\alpha\alpha}$ are positive above a critical value

b) the sign of inter-group sympathy trends $\mathcal{K}^{\alpha\beta}$, $\alpha \neq \beta$ between at least two groups is asymmetric,

c) the interaction matrix $\mathcal{K}^{\alpha\beta}$ is unsymmetric. and not antisymmetric

Only fixed points or limit cycles appear on the other hand for a fully antisymmetric agglomeration matrix $\mathcal{K}^{\alpha\beta}$

The route to chaos is now demonstrated in figures 4a,b until 8a,b ; for which the concrete agglomeration matrix

$$(\mathcal{K}^{\alpha\beta}) = \begin{pmatrix} 1.7 & 1.5 & -1.5 \\ -1.5 & 1.7 & 1.5 \\ \mathcal{K}^{13} & -1.5 & 1.7 \end{pmatrix} \tag{3,9}$$

has been chosen, where the parameter $\mathcal{K}^{13}$ traverses the values $1.5$, $-0.5$, $-0.55$, $-1.5$.

Here: Figures 4a,b - 8a,b

The figures 4a until 8a represent for different values of $\mathcal{K}^{13}$ the projections of the asymptotic trajectories (attractors) into the

$n_1^1 / n_1^2$ -plane, and figures 4b until 8b exhibit the corresponding Fourier spectra of $\ln [n_1^1(f)]$.

Figure 4a,b and 5a,b show two limit cycles (with corresponding Fourier spectra) belonging to the same value $\varkappa^{13} = 1.5$. Both limit cycles merge into one for $\varkappa^{13} = -0.5$, as shown in figure 6a,b. Period doubling (or better period multiplying) of the limit cycle appears in figure 7a,b after the value of the trend parameter has slightly been shifted to $\varkappa^{13} = -0.55$. Finally the limit cycle has evolved into a strange attractor with continuous Fourier spectrum for the value $\varkappa^{13} = -1.5$, as shown in figure 8a,b.

The structure of the strange attractor can be further investigated by making a Poincaré map of the trajectory: The hyperplane $n_1^1 = 1/3$ is chosen and all points $\{ n_1^1 = 1/3, n_2^1, \ldots n_3^3 \}$ where the strange attractor pierces this hyperplance are registered. The projections of these points into the planes

$(n_2^1, n_1^3)$, $(n_2^2, n_2^3)$, $(n_1^3, n_2^3)$, $(n_2^1, n_2^3)$

are depicted in figure 9. They indicate, that the trajectory of the strange attractor lies almost on a two-dimensional surface. Indeed, a detailed investigation shows, that the fractal correlation dimension of this strange attractor is $D = 2.14$ /3/.

Here:
Figure 9

### 3.4 One Population in L Areas: Empiric Evaluation

Whereas the preceding sections focussed on theoretical investigations demonstrating how the model description comprises various migratory phenomena, we shall now turn to concrete applications /4/.

In federally organized countries with, say, $L$ states or regions usually the following data for interregional migration are available year by year:

| Region | Population | Number of transitions per year from i to j | | | | |
|---|---|---|---|---|---|---|
| 1 | $n_1$ | * | - - - - | $w^e_{j1}$ | - - - - - - | $w^e_{L1}$ |
| 2 | $n_2$ | $w^e_{12}$ | - - - - - | $w^e_{j2}$ | - - - - - - - | $w^e_{L2}$ |
| ⋮ | ⋮ | ⋮ | | ⋮ | | ⋮ |
| $i$ | $n_i$ | $w^e_{1i}$ | - - - - | $w^e_{ji}$ | - - - - - - - | $w^e_{Li}$ |
| ⋮ | ⋮ | ⋮ | | ⋮ | | ⋮ |
| $L$ | $n_L$ | $w^e_{1L}$ | - - - - | $w^e_{jL}$ | - - - - - - - | * |

In the migratory model there correspond the configurational transition rates

$$w^{th}_{ji}(t) = \nu_{ji}(t)\exp(\check{u}_j(t) - \check{u}_i(t))\, n_i(t) \tag{3,10}$$

to the empirical transition matrix elements $w^e_{ji}(t)$ listed above. Formula (3,10) is a generalization of (2,6) including not only regional utilities $\check{u}_i(t)$ but also a mobility factor

$$\nu_{ji}(t) = \nu_{ij}(t) = \nu_0(t)\, e^{-D_{ij}} \tag{3,11}$$

where $\nu_0(t)$ is a global timedependent mobility and $D_{ij} = D_{ji}$ and effective distance between regions $i$ and $j$.

The utilities and mobilities in formula (3,10) now have to be chosen in a manner that leads to optimal agreement between $w^{th}_{ji}(t)$ and the empiric $w^e_{ji}(t)$. A standard procedure for optimal fitting is the least square method: The $\check{u}_i(t)$ and $\nu_{ji}(t)$ are determined by the requirement, that

$$\sum_{j,i=1}^{L} \sum_{t=1}^{T} \left[ w^{th}_{ji}(t) - w^e_{ji}(t) \right]^2 \overset{!}{=} \text{Minimum} \tag{3,12}$$

Figure 10 shows as a result of this regression analysis the timedependent "regional utilities" of the 11 states of the Federal Republic of Germany

Here:
Figure 10

In the further analysis one tries to represent the regional utilities $\check{u}_i(t)$ in terms of the population numbers $n_i(t)$ and further socio-economic key-factors. The following form can be substantiated:

$$\check{u}_i(t) = \varkappa\, n_i(t) - \sigma\, n_i^2(t) + \delta_i(t) \qquad (3,13)$$

In this formula the dependence of $\check{u}_i(t)$ on the size of the region is taken into account by the agglomeration term $\varkappa n_i(t)$ and the saturation term $-\sigma n_i^2(t)$ whereas the "preference" $\delta_i(t)$ is a measure of the size-independent attractivity of the region. The preference can be fitted by appropriately selected socio-economic key-factors $\Omega_i^\alpha(t)$ as follows

$$\delta_i(t) = \sum_\alpha a_\alpha \Omega_i^\alpha(t) \qquad (3,14)$$

Figure 11 depicts the preferences that means the size-independent part of the regional utilities. The final figure 12 shows, how all utilities found in fig. 10 can simultaneously be fitted by using six regional socio-economic variables only: The population number $n_i(t)$ and its square $n_i^2(t)$ (see (3,13)), while $\delta_i(t)$ is fitted according to (3,14) by four key-factors: number of overnight accommodations, export index, rate of unemployment and rate of employment in service sector.

Here:
Figs.11+12

The attempt to represent the regional utilities in terms of socio-economic key factors is of course not made for its own sake but for deeper reasons: In complex selforganizing systems like the society it is difficult to find cause-effect relations in a direct manner. Therefore the fact that the total space-time-dependence of utilities can be represented as linear combination of a few selected socio-economic factors only is taken as an indirect indicator of their causative nature for the migratory dynamics.

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**Figure Captions**

Figure 1a — Fluxlines of meanvalues for agglomeration trends $\varkappa^{\mu} = \varkappa^{\nu} = 0.2$ and sympathy trends $\sigma^{\mu} = \sigma^{\nu} = 0.5$.

Figure 1b — Stationary solution of the master equation for $\bar{m} = \bar{n} = 20$ and trend parameters as in fig. 1a.

Figure 2a — Fluxlines of meanvalues for agglomeration trends $\varkappa^{\mu} = \varkappa^{\nu} = 0.5$ and sympathy trends $\sigma^{\mu} = \sigma^{\nu} = 1.0$

Figure 2b — Stationary solution of the master equation for $\bar{m} = \bar{n} = 20$ and trend parameters as in fig. 2a.

Figure 3a — Fluxlines of meanvalues for agglomeration trends $\varkappa^{\mu} = \varkappa^{\nu} = 1.2$ and asymmetric sympathy trends $\sigma^{\mu} = -\sigma^{\nu} = 1.0$.

Figure 3b — Stationary solution of the master equation for $\bar{m} = \bar{n} = 20$ and trend parameters as in fig. 3a.

Figure 4a and b — The $(n_1^1, n_1^2)$ projection of a first limit cycle and the Fourier spectrum of $\ln[n_1^1(f)]$ for $\varkappa^{13} = 1.5$.

Figure 5a and b — The $(n_1^1, n_1^2)$ projection of a second limit cycle and the Fourier spectrum of $\ln[n_1^1(f)]$ for $\varkappa^{13} = 1.5$

Figure 6a and b — The $(n_1^1, n_1^2)$ projection of one limit cycle with the Fourier spectrum of $\ln[n_1^1(f)]$ for $\varkappa^{13} = -0.5$

| | |
|---|---|
| Figure 7a and b | The $(n_1^1, n_1^2)$ projection of a limit cycle with period multiplicity and the Fourier spectrum of $\ln[n_1^1(f)]$ for $\varkappa^{13} = -0.55$ |
| Figure 8a and b | The $(n_1^1, n_1^2)$ projection of a strange attractor with continuous Fourier spectrum of $\ln[n_1^1(f)]$ for $\varkappa^{13} = -1.5$. |
| Figure 9 | Projection of the traversing points of the phase trajectory of a strange attractor $(\varkappa^{13} = -1.5)$ through the hyperplane $n_1^1 = 1/3$ onto different planes. |
| Figure 10 | Regional utilities $\check{u}_i(t)$ for the 11 countries of the Federal Republic of Germany. Notations: + Schleswig-Holstein, o Hamburg, △ Niedersachsen, □ Bremen, ◇ Nordrhein-Westfalen, • Hessen, * Rheinland-Pfalz, x Baden-Württemberg, = Bayern, > Saarland, < Westberlin. |
| Figure 11 | Regional preferences $\delta_i(t)$ for the 11 countries of the Federal Republic of Germany. Notations as in figure 10. |
| Figure 12 | Representation of regional utilities by socio-economic variables. Symbols: regional utilities from migratory data as in figure 10. Straight lines: Representation of utilities according to (3,13) and (3,14) using $n_i(t)$, $n_i^2(t)$ and four key-factors |

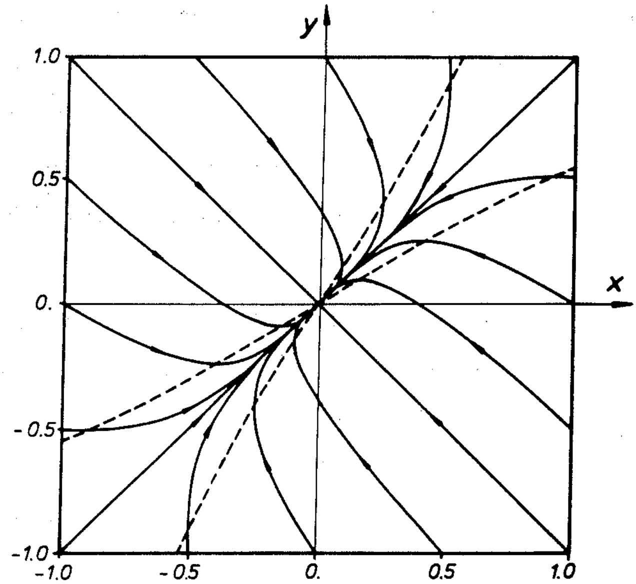


Fig. 1a

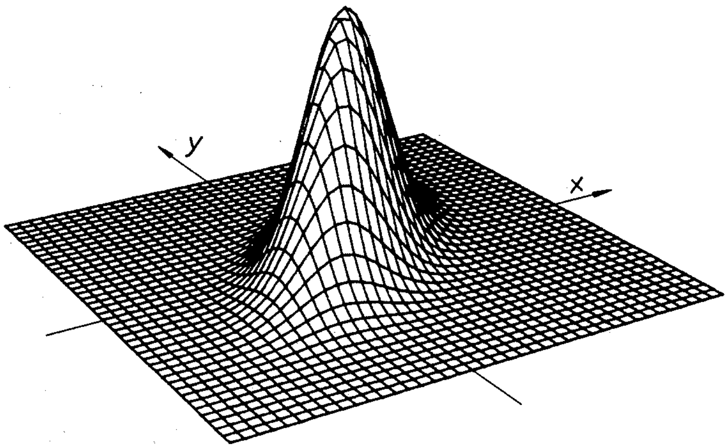


Fig. 1b

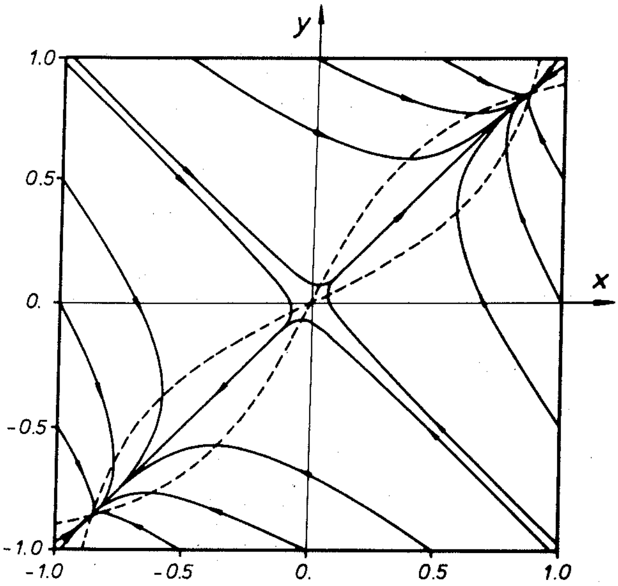


Fig. 2a

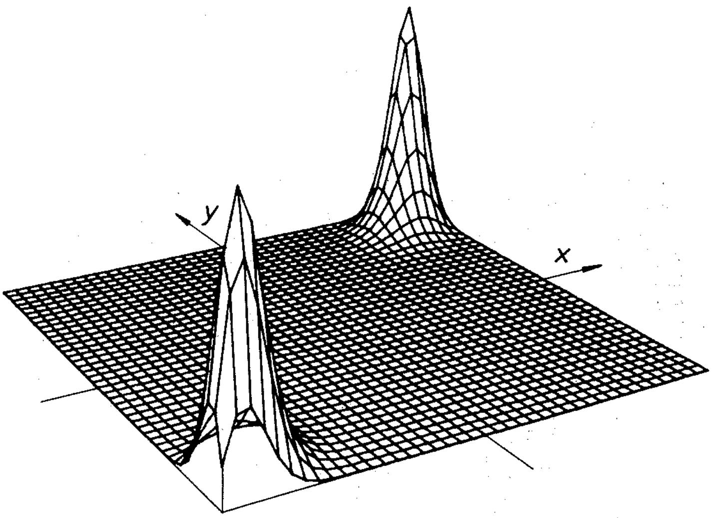


Fig. 2b

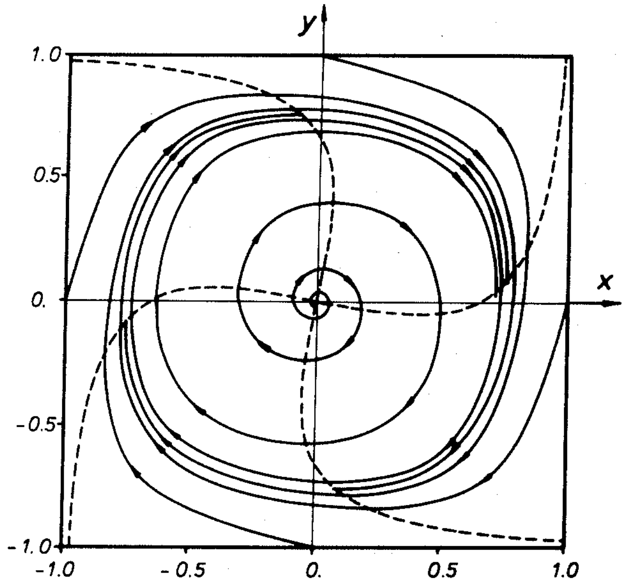


Fig. 3a

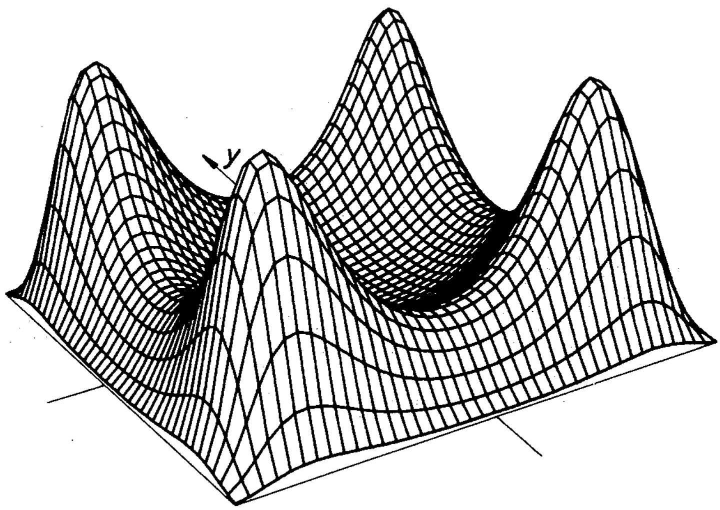


Fig. 3b

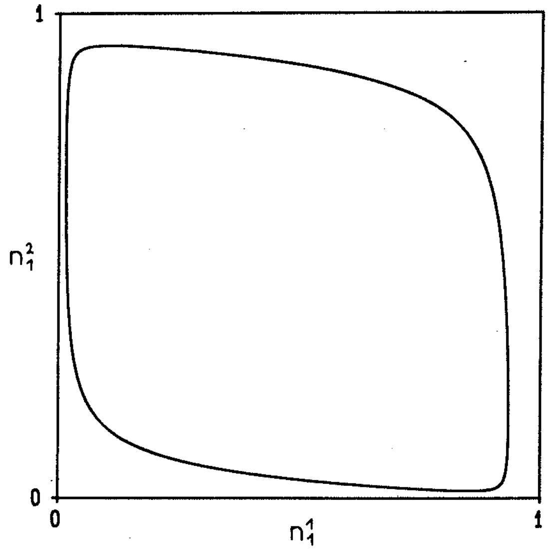


Fig. 4a

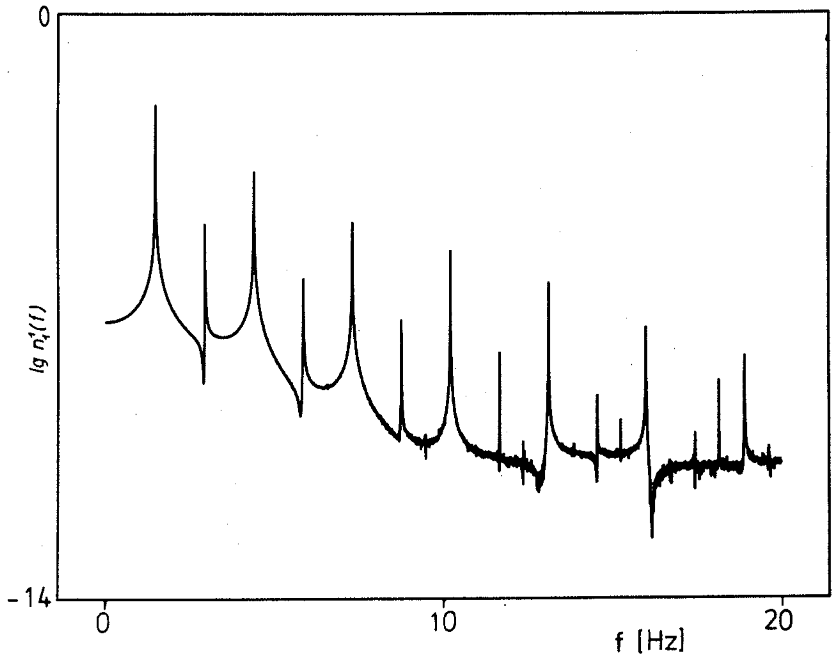


Fig. 4b

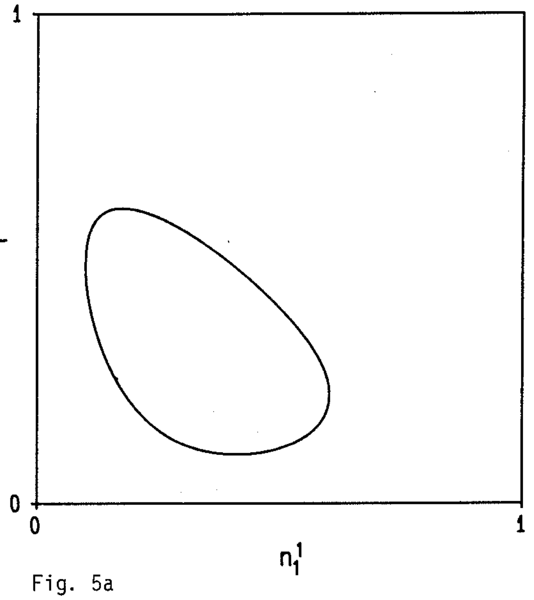


Fig. 5a

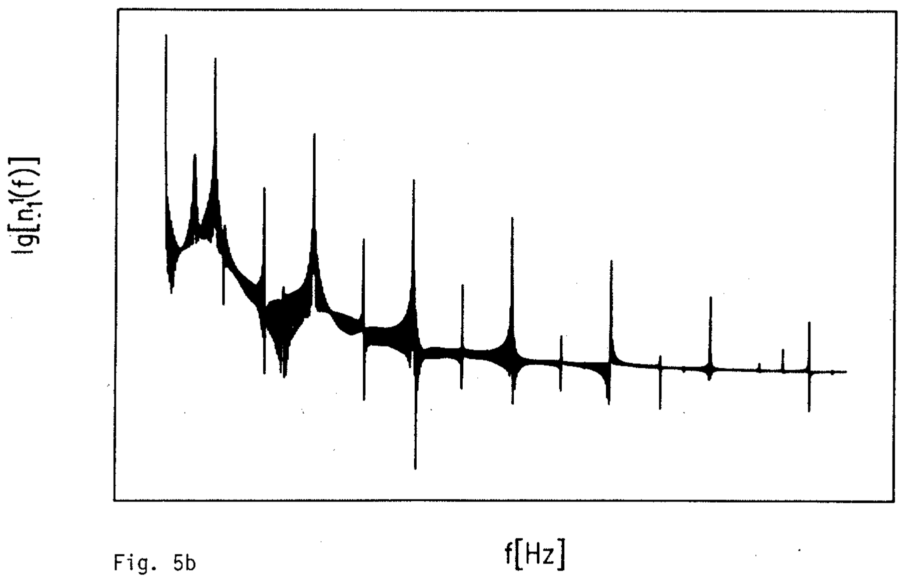


Fig. 5b

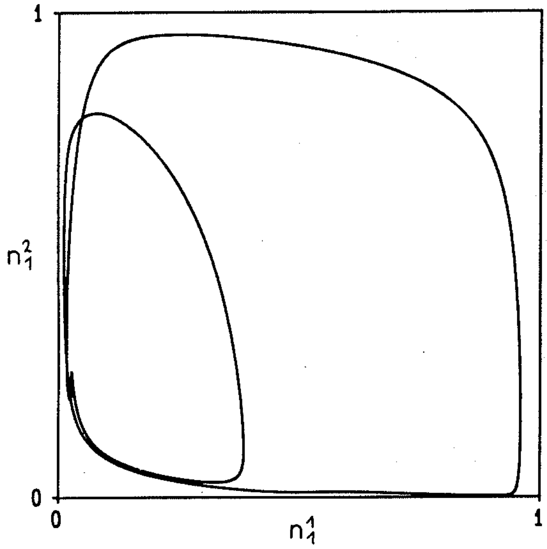


Fig. 6a

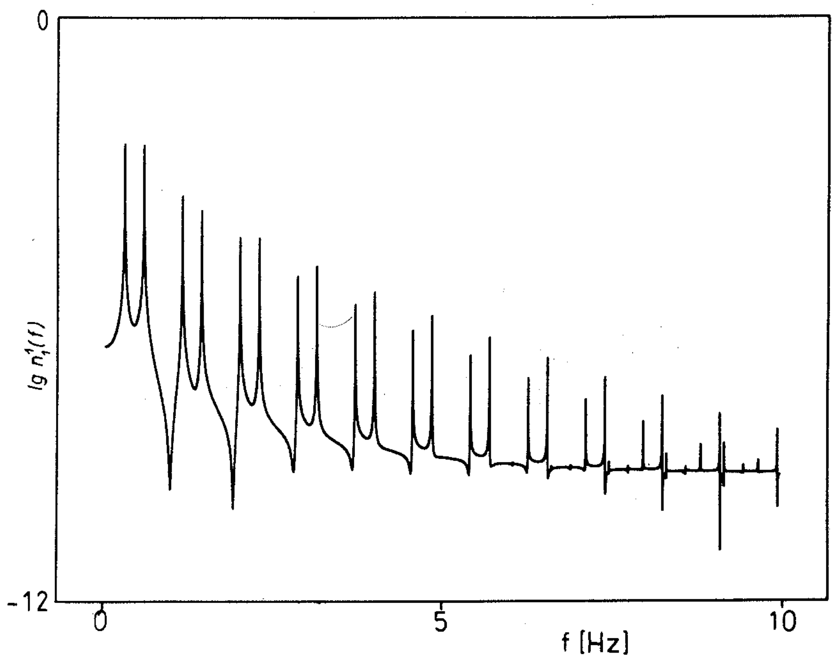


Fig. 6b

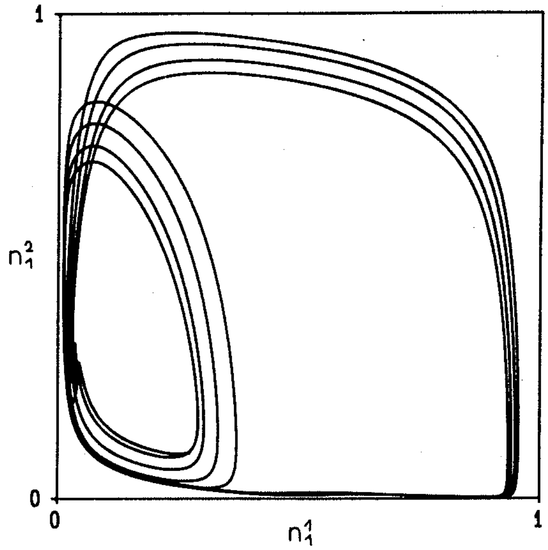


Fig. 7a

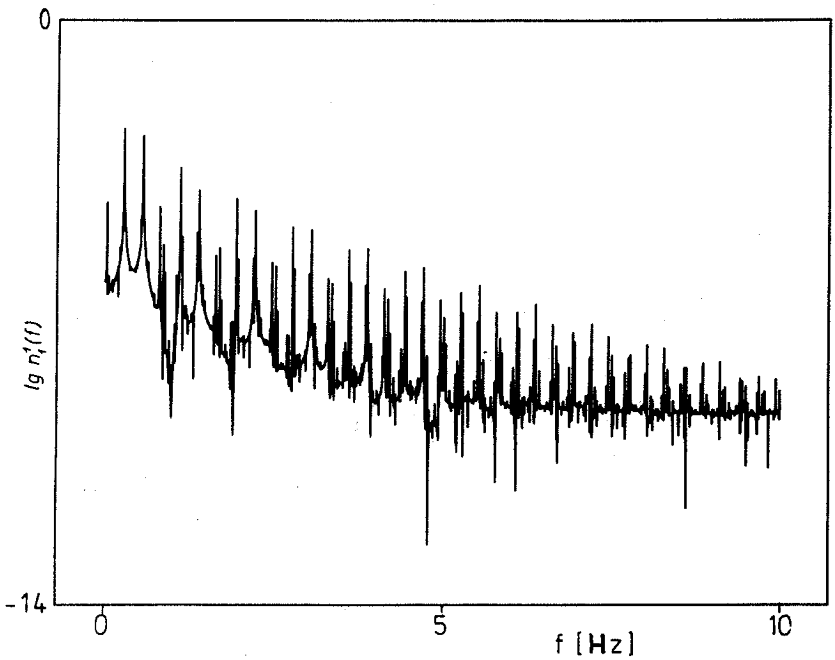


Fig. 7b

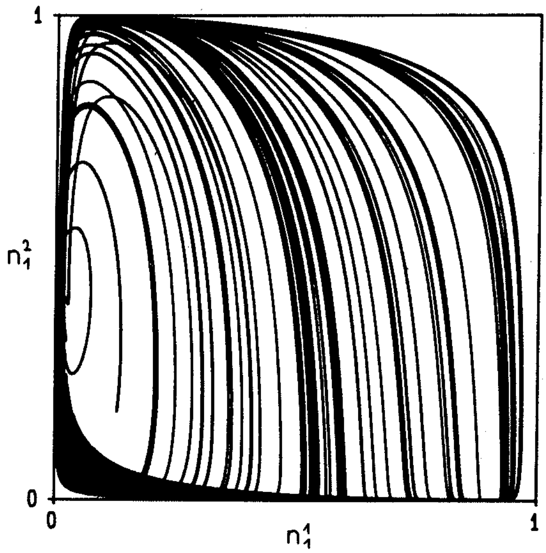


Fig. 8a

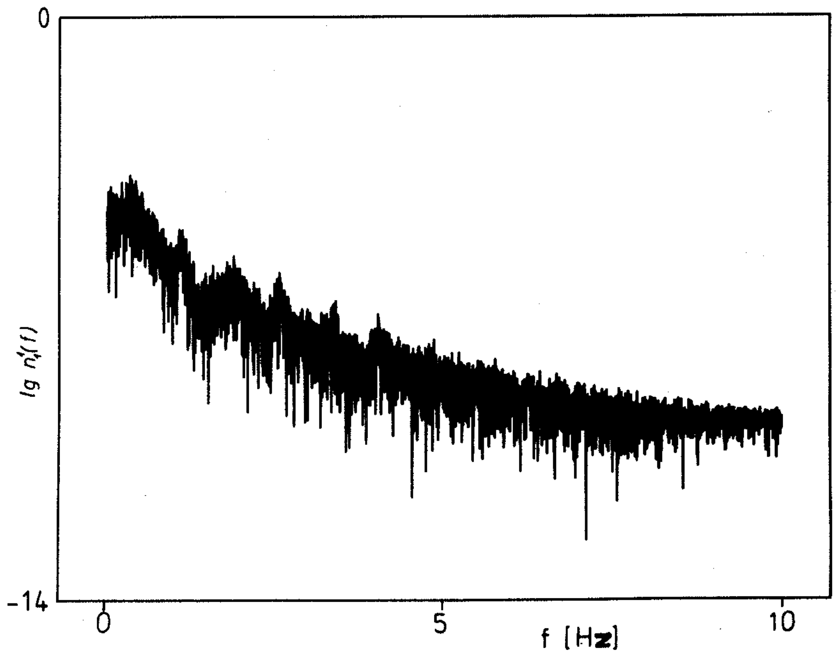


Fig. 8b

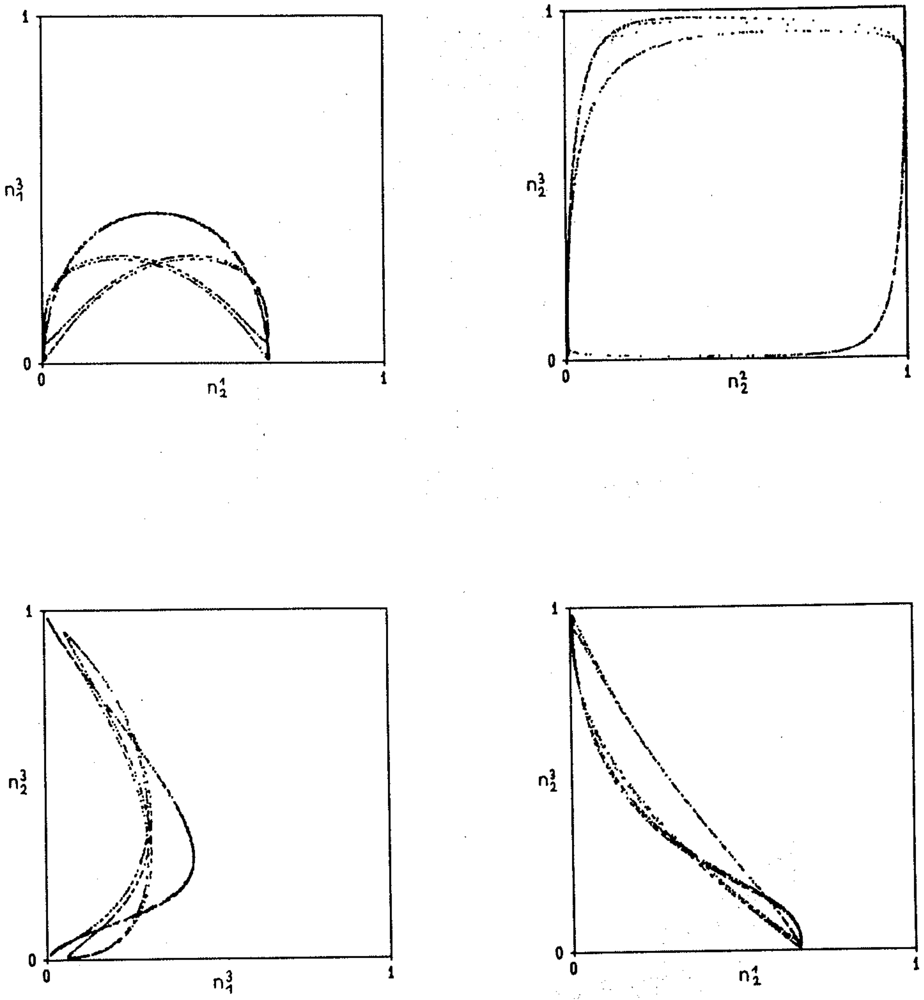


Fig. 9

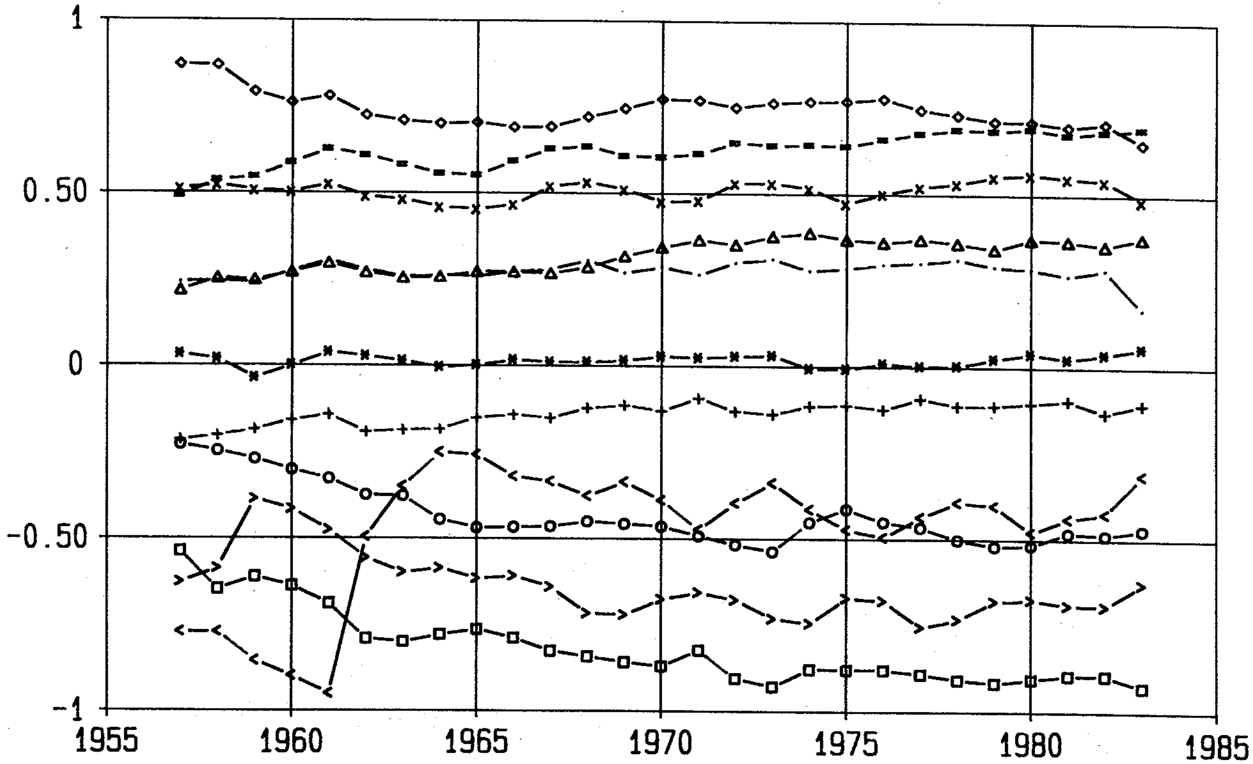


Fig. 10

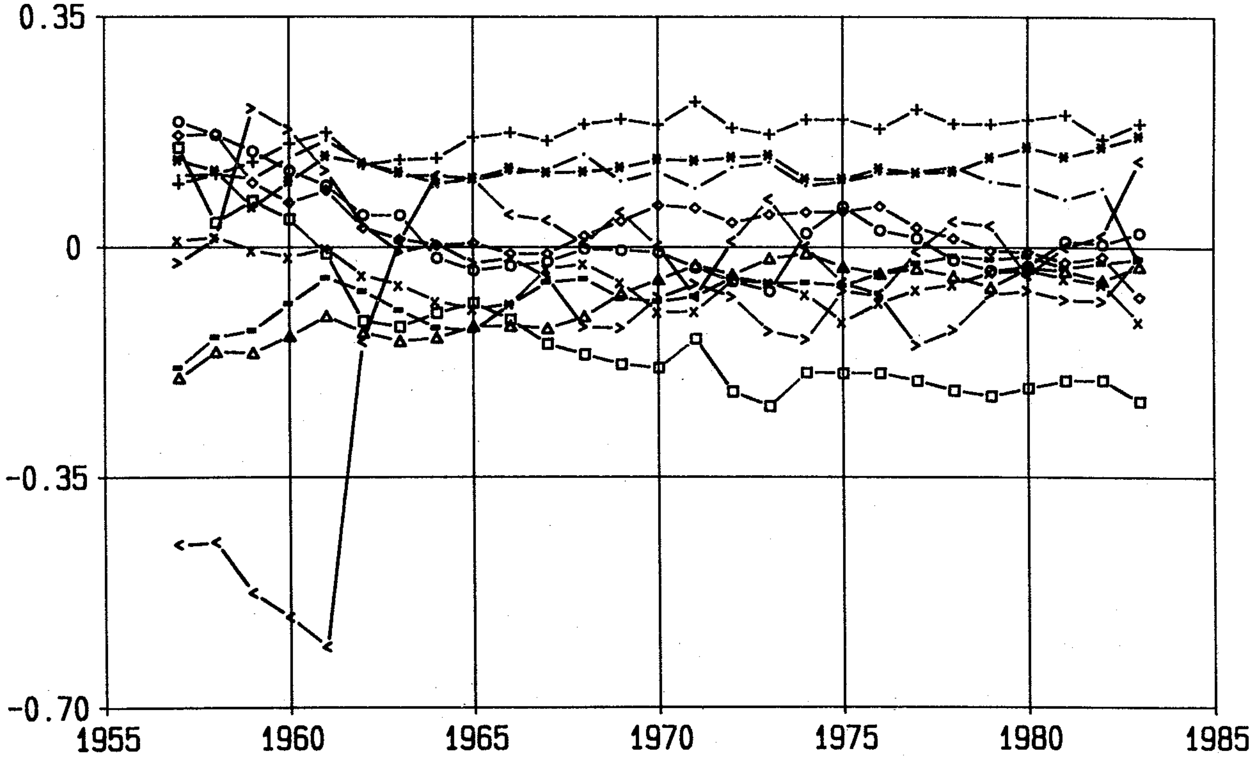


Fig. 11

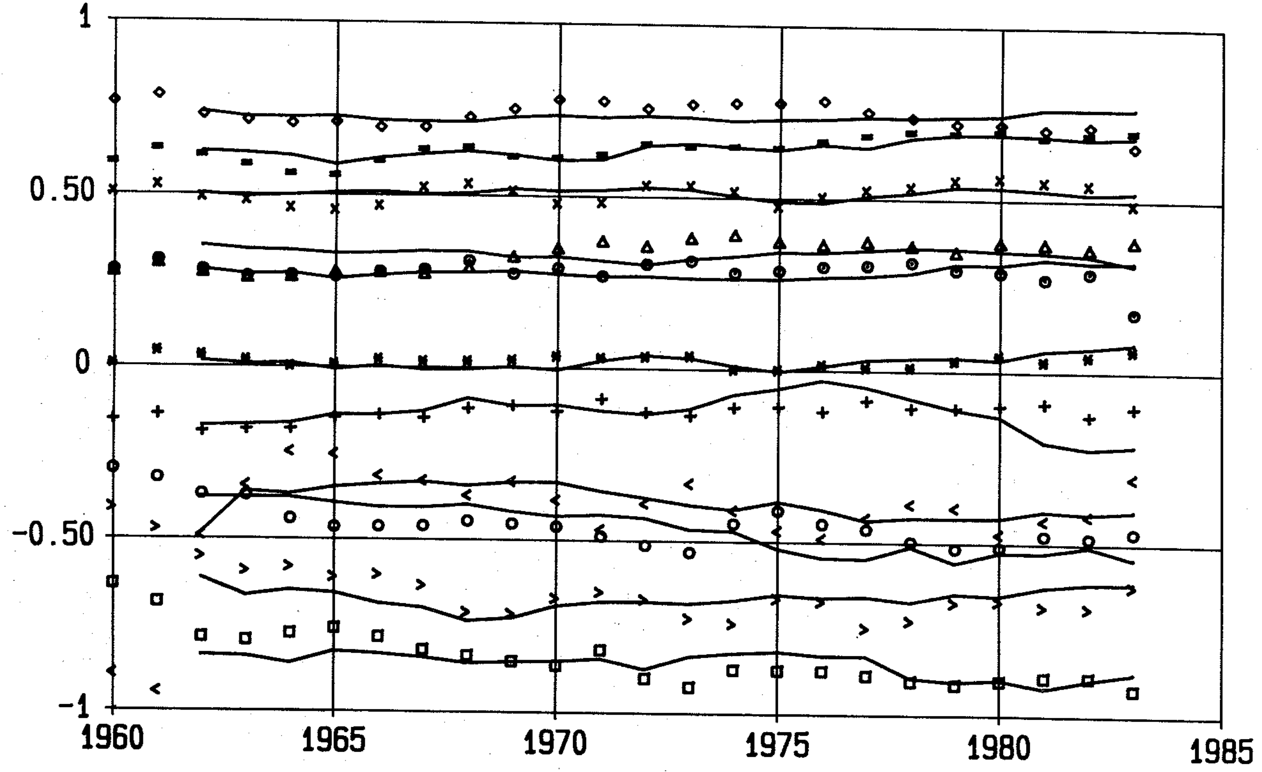


Fig. 12