Designing a manufacturing function as a competitive weapon using the reference approach

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Abstract. The reference approach is a new system dynamics support method. This paper explores the possibility of using this method to design strategies that transform the production function into a competitive weapon. First, the requirements of a manufacturing strategy design tool are identified. Next, the manufacturing strategy of a case study is designed using the reference approach. Based on this application, the possibility of using the reference approach as a tool for the design of a manufacturing strategy is discussed. The analysis concludes that the reference approach is a valuable tool for the computer aided design of a manufacturing strategy.

1. Introduction.

A manufacturing strategy is the pattern of structural and infrastructural changes that will enable a business unit to achieve its desired competitive advantage (Hayes and Wheelwright, 1984; p. 32). While the structural changes are the ones that modify the pattern of the business unit variables, the infrastructural changes are those that change the dynamics of these variables around their pattern (tactical behaviors).

From the classic paper of Skinner (1969), world competitive pressures on business have accentuated the need for improved practice of manufacturing strategy. However, the frameworks for solid research and practice are lacking (Anderson, Cleveland and Schroeder, 1986).

The goal of this paper is to explore the possibility of using the reference approach (Macedo, 1989a) as a tool to help in the design of manufacturing strategies. In section 2, the requirements of a manufacturing strategy design tool are enunciated. In section 3, the reference approach is applied to design the strategy of a well known case study. Finally, section 4 evaluates how well the reference approach fulfills the requirements presented in section 2.
2. Requirements of a manufacturing strategy design tool

The requirements of a manufacturing strategy design tool are listed in table 1. These requirements are derived from observations done in the industry by a group of researchers. In fact, the requirements (a), (c) and (e) of table 1 arise from the works of Skinner. This author suggests a three step approach to design a manufacturing strategy. In the first step, a set of problematic behaviors are identified. In the second step, a set of desired behaviors (manufacturing task) are derived from the corporate strategy. In the third step, the changes necessary to focus the behaviors of the business unit variables on the manufacturing task are designed. In order to identify these changes a rigorous exploration of all possibilities must be done.

The requirements (b) and (f) of table 1 are derived from the works of Hayes and Wheelwright. These researchers indicate that in several leading firms the manufacturing task is not derived from the corporate plan but is instead defined by the manufacturing function. In these firms the competitive strategy depends to a significant degree on the company's manufacturing capability. Hayes and Wheelwright point out that the modification of any manufacturing variable can represent a structural or an infrastructural change. Hence, the typical classification of the manufacturing variables in structural (capacity, location, technology) and infrastructural (workforce system wage, quality control, production planning), on the basis of their nature, has a relative validity.

Finally, the requirements (g) and (h) of table 1 originate from the works of Sink. This author suggests the participation of the employees during the conception of the manufacturing strategy: while the workers propose the changes, the managers approve or refuse them. In addition, Sink introduces a two step approach to conceive the manufacturing strategy. In the first step the patterns of the changes are conceived using an aggregate description of the business unit operations. In the second step, the characteristics of each change are specified after a detailed analysis.

3. Using the reference approach

The reference approach is a simulation by optimization procedure that uses two models, a reference model (block B in annexe 1) and a control model (block D in annexe 1). The reference model is a large scale nonlinear programming model whose normative part is reformulated until the patterns desired by the managers are obtained. The control model is a linear quadratic model with closed feedback solution whose normative part is reformulated until the deviations from the desired patterns satisfy the managers.

The reference approach expresses the manufacturing strategy as a sum of structural and infrastructural changes:
\[ u(t) = u_o - G(t)\delta x(t) - P(t)\delta z(t), \quad 0 \leq t \leq T \quad (1) \]
\[ \delta x(t) = x(t) - x_i \quad (2) \]
\[ \delta z(t) = z(t) - z_i \quad (3) \]

\[ u(t): \quad \text{Manufacturing strategy} \]
\[ u_o: \quad \text{Structural changes} \]
\[ -G(t)\delta x(t) - P(t)\delta z(t): \quad \text{Infrastructural changes} \]
\[ \delta x(t), \delta z(t): \quad \text{Deviations with respect to the desired and exogeneous patterns when the structural changes are introduced in the problematic system} \]
\[ x_i: \quad \text{Desired patterns} \]
\[ z_i: \quad \text{Exogeneous patterns} \]

A close examination of block B in annexe 1 shows that the reference model uses an approximate system dynamics model. Hence, the solution of the reference model, \( u_i,\ 0 \leq t \leq T \), introduced in the original system dynamics model (block A in annexe 1) does not produce the desired patterns but only an approximation. Additional changes that push the observed dynamics towards the desired patterns are necessary. These changes, named infrastructural, are produced by the solution of the control model.

We have briefly introduced the architecture of the reference approach. Because its operationalisation can be found elsewhere (Macedo, 1989a) only one remark is in order: the solution of the control model is (1) above. Hence, it is not necessary to solve the control model as formulated in block D of annexe 1. Instead, the matrix differential equations (A3) and (A5) (annexe 1) must be solved to obtain the auxiliary matrices \( K(t) \) and \( M(t), \quad 0 \leq t \leq T \), which replaced in expressions (A1) and (A2) define the matrices \( G(t), P(t), \quad 0 \leq t \leq T \) necessary to evaluate (1).

Having briefly described the reference approach, the following paragraphs present the results of applying this method to design the manufacturing strategy of a well known case study (Coyle, 1977, chap. 8). These results will be discussed in section 4.

In the mentioned case study, the production start rate (PSR), the factory order rate (FOR) and the inventory of finished products (INV) oscillate when the sales rate (SR) grows 40%. These problematic behaviors are shown in figures 2, 3 and 4 which are obtained by the simulation of the system dynamics model constructed by Coyle with minor modifications. The loops diagram of this model is represented in continuous line in figure 1.

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1 Formula (1) is obtained linearizing the original system dynamics model with respect to the desired patterns \( x_i, u_i, \quad 0 \leq t \leq T \), and using a derivation similar to the one proposed by Johnson (1971).
2 A list of the system dynamics model used is available elsewhere (Macedo, 1989b). However, an aggregate version of this model is the approximate system dynamics model represented in annexe 2.
Figure 1. In continuous line, the original system dynamics model. In dotted lines, the new links created when part of the manufacturing strategy \( \alpha(t) = \alpha_0 + 0.10 \cdot (\text{OBL}(t) - \text{OBL}_1) - 0.20 \cdot (\text{INV}(t) - \text{INV}_1) \) is introduced in the model. Notice the structural changes and the new feedback loops created by the infrastructural changes.
A) The MSDT must focus the behavior of the business unit variables on the manufacturing task (Skinner, 1985, p. 72).

B) The MSDT must generate the manufacturing task when necessary (Wheelwright and Hayes, 1985, p. 103).

C) The MSDT must design a set a manufacturing changes that are consistent between them and with the other functions of the business unit (Skinner, 1985, p. 75).

D) The MSDT evaluates the alternative manufacturing strategies using a multicriteria approach (Kaplan, 1983).

E) The MSDT allows to explore all possible changes in order to identify the ones that constitute the manufacturing strategy (Skinner, 1986).

F) The MSDT allows to identify the structural and the infrastructural changes that constitute the strategy (Wheelwright, 1981).

G) The MSDT allows the participation of the employees of any hierarchical level during the design of the strategy (Sink, 1986).

H) The MSDT uses an aggregate description of the business unit operations in order to design the manufacturing strategy (Sink, 1986).

Table 1. Requirements of a manufacturing strategy design tool (MSDT)

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Fig 2. Production start rate PSR obtained simulating the system dynamics model with the original strategy (- - -) and with the reference approach strategy (---). Also the exogeneous sales rate SR (---) is represented.

Fig 3. Factory order rate FOR obtained simulating the system dynamics model with the original strategy (- - -) and with the reference approach strategy (---). Also the exogeneous sales rate SR (---) is represented.
Fig 4. Inventory of finished products INV obtained simulating the system dynamics model with the original strategy (---) and with the reference approach strategy(----).

Fig 5. Structural changes obtained solving the last reference model: $\alpha_1$ (---), $\beta_1$ (----), $\tau_1$ (-----).

Fig 6. Oder backlog OBL$_t$ (---), Inventory of finished products INV$_t$ (----) and Goods in pipeline PLA$_t$ (-----) obtained solving the last reference model.

Fig 7. Average sales rate ASR$_t$ (---), Average order rate AOR$_t$ (----), Average production level APL$_t$ (-----) and exogeneous sales rate EXOG$_t$ (-----) obtained solving the last reference model.
The final reference model constructed following the reference approach is the one represented in annexe 2, which has as state variables \( x := [\text{OBL}_t, \text{INV}_t, \text{PLA}_t, \text{ASR}_t, \text{AOR}_t, \text{APL}_t] \), as control variables \( u := [\alpha_t, \beta_t, \tau_t] \), and as exogeneous \( z := [\text{EXOG}_t] \). Its solution, illustrated in figures 5 to 7, produces the desired patterns (stabilization of FOR, PSR and INV).

The final values chosen for the penalty matrices of the control model were:

\[
\begin{align*}
\text{Diag } Q(t)_{6x6} &= [98, 278, 132, 5102, 4444, 10000], \ 0 \leq t \leq 30 \\
\text{Diag } R(t)_{3x3} &= [2500, 5102, 160000], \ 0 \leq t \leq 30 \\
\text{Diag } F_{6x6} &= [112, 292, 139, 5102, 5102, 21003]
\end{align*}
\]

Using these penalties, the matrix differential equations (A3) and (A5) were solved and then the values of the gain matrices \( G(t)_{3x6} \) and \( P(t)_{3x1}, \ 0 \leq t \leq 30 \), were obtained. Finally, replacing the mean values\(^3\) of these trajectories in (1), the manufacturing strategy becomes:

\[
\begin{align*}
\alpha(t) &= \alpha_t + 0.10[\text{OBL}_t - \text{OBL}_t] - 0.20[\text{INV}_t - \text{INV}_t] \\
&\quad - 0.15[\text{PLA}_t - \text{PLA}_t] + 0.20[\text{ASR}_t - \text{ASR}_t] \\
&\quad - 0.20[\text{APL}_t - \text{APL}_t] + 0.30[\text{SR}_t - \text{SR}_t] \quad (4) \\
\beta(t) &= \beta_t - 0.20[\text{OBL}_t - \text{OBL}_t] - 0.15[\text{INV}_t - \text{INV}_t] \\
&\quad - 0.15[\text{PLA}_t - \text{PLA}_t] - 0.40[\text{ASR}_t - \text{ASR}_t] \\
&\quad - 0.30[\text{AOR}_t - \text{AOR}_t] - 0.40[\text{APL}_t - \text{APL}_t] \\
&\quad + 0.20[\text{SR}_t - \text{SR}_t] \quad (5) \\
\tau(t) &= \tau_t - 0.05[\text{ASR}_t - \text{ASR}_t], \text{ for } 0 \leq t \leq 30 \quad (6)
\end{align*}
\]

\( \alpha_t, \beta_t, \tau_t : \) Figure 5  
\( \text{OBL}_t, \text{INV}_t, \text{PLA}_t : \) Figure 6  
\( \text{ASR}_t, \text{AOR}_t, \text{APL}_t, \text{EXOG}_t : \) Figure 7

The first three terms of (4), which is only one component of the manufacturing strategy, are represented in dotted lines in figure 1. We notice that the structural changes \( \alpha_t, \ 0 \leq t \leq 30 \), constitute an exogeneous time series that progressively modifies the structure of the original system dynamics. In addition, the infrastructural changes (terms in brackets in formula (4) above) form new feedback loops with the original system dynamics model. Similar observations can be done on formulae (5) and (6).

In the original system dynamics model, we have:

\[
\begin{align*}
\text{PSR}(t) &= \alpha(t) \cdot \text{APL}(t) \quad (7) \\
\text{FOR}(t) &= \beta(t) \cdot \text{ASR}(t) + \tau(t) [\text{WINVD} \cdot \text{ASR}(t) - \text{INV}(t)] \quad (8) \\
\alpha(t) &= 1, \ \beta(t) = 1, \ \tau(t) = 0.25, \ 0 \leq t \leq 30 \quad (9)
\end{align*}
\]

\(^3\) The trajectories of the eighteen elements of \( G(t), 0 \leq t \leq 30 \) and of the three elements of \( P(t), 0 \leq t \leq 30 \), are available from the author. These trajectories were used in all the tests and computations presented in this paper. However, only their mean values are presented in order to alleviate the paper.
When the constant values (9) are replaced by (4), (5) and (6), the instability created on PSR, FOR and INV by the 40% sales rate growth is cancelled, as indicated in figures 2, 3 and 4.

In fact, the structural changes $\alpha$, $\beta$, and $\tau$ are consistent with the goal of stabilization of FOR, PSR and INV as we will shortly demonstrate. Originally, $\alpha(t)$ has a value of 1 and the strategy indicates to dramatically increase it (see figure 5). As a result, the production start rate PSR shifts upwards (see figure 2 and formula 7). This is a logical behavior in response to the 40% sales rate growth. On the other hand, the strategy suggests to immediately increase the original value of $\beta$ (which is 1) and to decrease the original value of $\tau$ (which is 0.25) and, after, to progressively decrease $\beta$ and increase $\tau$, as indicated in figure 5. These simultaneous actions shift upwards the factory order rate FOR (see figure 3 and formula 8). This behavior is also a logical response to the sales rate growth.

4. The reference approach as a manufacturing strategy design tool

In this section we examine if the reference approach satisfies the requirements of table 1. When possible we use the case study of the previous section as a means of demonstration.

a) The manufacturing strategy design tool (MSDT) must focus the business unit variables on the manufacturing task

This requirement is satisfied by the reference approach because the normative part of the reference model and the normative part of the control model must be reformulated until the desired patterns are produced (feedbacks from block C to B and from block F to D in annexe 1). In the case study, this reformulation produced as objective function of the reference model a cumulated function of the quadratic deviations of FOR, PSR and INV with respect to their equilibriums (annexe 2). In addition, inequalities limiting the variation of $\alpha$, $\beta$, $\tau$ were introduced in order to avoid jumps and positivity inequalities on all the state variables in order to eliminate unrealistic behaviors.

b) The MSDT must generate the manufacturing task when necessary

This requirement is satisfied by the reference approach because each solution of the reference model is shown to the managers for acceptance or refusal (block C in annexe 1). Then, the managers can discover that one of these solutions is superior to their initially desired behaviors. In this case, they will adopt this solution as their desired behaviors.

4 When it is impossible to reformulate the normative part of the reference model such that its solution reproduces the desired patterns, there is an inconsistency between these patterns and the structure of the system dynamics model. In this case, the managers must modify their desired behaviors or accept one of the solutions of the reference model.
c) The MSDT must design a set of consistent manufacturing changes

The reference model includes the direct and feedback relations between manufacturing and the other functions of the business because it includes an approximate system dynamics model. In the case study, the direct relation between the warehouse of finished products and the factory is represented by the causal chain FOR-AOR-RBL-ITL-APL-PSR and their feedback relation by the chain PSR-PLA-DFP-INV-FOR (figure 1). In addition, the equations of the approximate system dynamics model are simultaneously (in the variables and time spaces) solved during the optimization of the reference model. Hence, the structural changes obtained optimizing the reference model are consistent between them and with the other functions of the business unit.

d) The MSDT evaluates the alternative manufacturing strategies using a multicriteria approach

Every solution of the reference model and every solution of the control model are multivariate surfaces that are compared against the desired surfaces using many performance indicators (blocks C and F in annexe 1). From this point of view, the reference approach evaluates the alternative strategies using a multicriteria approach. In the case study, each solution of the reference model was checked for the stability of FOR, PSR and INV using a set of performance measurements (overshoot, settling time, accumulated deviation and aggregate reaction - Macedo, 1989b). These performance measurements guided the reformulation of the reference model in order to find a suitable strategy.

e) The MSDT allows to explore all possible changes in order to identify the ones that constitute the manufacturing strategy

The reference approach satisfies this requirement because some parameters of the system dynamics model are chosen as control variables (block A in annexe 1). But, this choice is made after a complete exploration of the structure of the system dynamics model using simulation. Only those parameters that control the problematic behaviors during the simulations are chosen as control variables. In the case study, the parameters α, β and τ were selected as control variables because during the simulations of the original system dynamics model they controlled the dynamics of FOR, PSR and INV.

f) The MSDT allows to identify the structural and infrastructural changes that constitute the strategy

After the application of the reference approach, each control variable \( u(t) \) is expressed in form (1). If the value of \( u_i \) in (1) is identical to the value of the respective control variable in the original system dynamics model and, in addition, 5 System dynamics models are constructed using the following main principle: the dynamics of a variable is the result of loops that push it to growth and loops that push it to decline, a loop being a closed chain of cause-effect relationships (Forrester, 1980).
the concerned elements of $G(t)$ and $P(t)$, $0 \leq t \leq T$, are trajectories different of zero, then the strategy $u(t)$ is constituted by infrastructural changes only. On the other hand, if the value of $u_i$ in (1) differs from the value of the respective control variable in the original system dynamics model and, in addition, the concerned elements of $G(t)$ and $P(t)$, $0 \leq t \leq T$, are zero, then the strategy $u(t)$ is constituted by structural changes only. A mixed strategy $u(t)$, constituted by structural and infrastructural changes, is also possible. This is the case of the manufacturing strategy (4), (5) and (6), whose elements belonging to the matrices $G(t)$ and $P(t)$, $0 \leq t \leq 30$, differ from zero and the values of $\alpha_i$, $\beta_i$, $\tau_i$, $0 \leq i \leq 30$, differ from their values in the original system dynamics model ($\alpha=1$, $\beta=1$, $\tau=0.25$).

As demonstrated, the reference approach does not assume that the modification of a classic structural control variable (capacity, location, technology) represents an structural change. Neither does the reference approach assume that the modification of a classic infrastructural control variable (workforce system wage, production planning, quality control) represents an infrastructural change. Instead, given a control variable (of any nature), the reference approach indicates how to modify it in order to generate the desired behaviors. Because this modification is expressed as a sum of structural and infrastructural changes, it is easy to infer the type of change that the modification of the variable represents.

\textit{g) The MSDT allows the participation of the employees of any hierarchical level}

The reference approach satisfies this requirement when it is applied using two groups: a suggestion multifunctional team and an appraisal management committee.

The suggestion multifunctional team is constituted by the employees (workforce and managers) of any organizational level that are concerned with the problematic behaviors. This group participates in the construction of the system dynamics model and in the identification of the control variables (block A in annexe 1). This participation is consistent with the systemic approach used to build a system dynamics model: any kind of variable responsible for the problematic behavior must be included in the model. The mentioned team must also participate during the reformulation of the reference and control models in order to produce the behaviors desired by the management (feedback from block C to B and from block F to D in annexe 1).

The appraisal management committee is constituted by the managers of the appropriate organizational levels that accept or reject the manufacturing strategy proposed by the suggestion multifunctional team. This committee participates in steps C and F of the reference approach (see annexe 1). In addition, some members of this committee must belong to the suggestion team in order to ensure an effective communication.

We notice that the members of the mentioned two groups can "see" in a monitor the solution trajectories of the system dynamics model, the reference model and the control model. In addition, they can easily understand these dynamics using a loops
diagram (in the case study, the effect of the infrastructural changes can be explained going through the loops diagram of figure 1) and they can aggregate their preferences for the solutions using the nominal group technique. This technique is essentially an structured voting process (Sink, 1986).

h) The MSDT uses an aggregate description of the business unit operations

The heart of the reference approach is a system dynamics model. In fact, an approximation of this model is included in the reference model and its linearization is included in the control model. But, a system dynamics model is constructed using an structural approach: it must reflect the structure of the "laws" governing the problematic behavior (Forrester, 1980). From this point of view, a detailed representation of the reality is not necessary and the model must include aggregate dynamics. For example, in the case study one average product replaces the set of real products. On the other hand, system dynamics models builds on the mental data base (oral, descriptive, acquired experience data base) and where relevant on written and numerical data (Forrester, 1980).

Conclusions

The analysis of the reference approach architecture, as well as the results of a case study, show that this method satisfies most of the fundamental requirements of a manufacturing strategy design tool. Those requirements are derived from the observations carried out in the industry by a group of well-known researchers.

Current tools to design manufacturing strategies are checklists (see for example Skinner, 1985, p. 106) from which the managers select the changes capable of producing the desired behaviors. This selection is largely based on personal experience. However, the human mind has a bounded rationality (Simon, 1976, p. 135) such that alone it can not conceive a set of changes which are necessarily consistent. In addition, the managers misperceive the feedbacks from their decisions (Sterman, 1989) and their desired behaviors change with the experiences (Hogarth, 1981). Finally, it is not evident if a change will be structural or infrastructural (Wheelwright, 1981; Aracil, 1981). The reference approach corrects these shortcomings using a simulation by optimization procedure based on modelling and operations research techniques. At the same time, it explicits the richness of the mental data base, allows for the participation of the employees and transforms the local rational strategies (Morecroft, 1983) into a global and consistent rational set of strategies.

In conclusion, the reference approach proves to be a valuable computer aided design tool. It partially fills the vacuum created by the lack of formal manufacturing strategy design methods. These methods are vital for the survival of business units which are continuously under the pressure of a competitive world.
Annexe 1. The Reference approach architecture.

Problematic behaviors

A
Build a system dynamics model, validate it and choose some of its parameters as control variables. The model becomes:

\[ x(t) = f[x(t), u(t), p, z(t)] \]

\[ z(t) = h[z(t)], \, 0 \leq t \leq T \]

\[ x(0), \, z(0) \text{ known} \]

B
Formulate a reference model:

1) Normative part

\[ \text{Min} \sum_{d=0}^{N-1} I(x_{d+1}, u_d) \cdot D_{d+1} + U(x_N) \]

\[ g(x_{d+1}, u_d, D_{d+1}) \geq 0, \, 0 \leq d \leq N-1 \]

\[ D_{d+1} \leq D_d + 1 \leq D_{d+1}, \, 0 \leq d \leq N-1 \]

2) Descriptive part (approximate system dynamics model)

\[ x_{d+1} - x_d = D_{d+1} \cdot f[x_d, u_d, p, z_d] \]

\[ z_{d+1} - z_d = D_{d+1} \cdot h(z_d), \, 0 \leq d \leq N-1 \]

\[ x_0 = x(0), \, z_0 = z(0) \]

C
Do the patterns \( x_d, \, u_d, \, 0 \leq t \leq T \) satisfy the managers?

Yes

1

D
Formulate a control model:

1) Normative part

\[ \text{Min} \frac{1}{2} \int_0^T [\delta x'(t) \delta z'](t) \delta u(t) \, dt \]

\[ + \frac{1}{2} \delta x'(T) F \delta x(T) \]

2) Descriptive part (\( 0 \leq t \leq T \))

\[ \delta x(t) = A(t) \delta x(t) + B(t) \delta u(t) \]

\[ \delta z(t) = 0 \delta x(t) \]

The solution of the control model gives the strategy:

\[ u(t) = u_t - G(t) \delta x(t) - P(t) \delta z(t), \, 0 \leq t \leq T \]

E
Introduce the strategy \( u(t) \) in the system dynamics model (block A) and simulate it in order to obtain the trajectories \( x(t), \, u(t), \, 0 \leq t \leq T \)

F
Do the deviations \[ [x(t) - x_i] \] and \[ [u(t) - u_i] \], \( 0 \leq t \leq T \) satisfy the managers?

No

G
The strategy is \( u(t), \, 0 \leq t \leq T \)
Annexe 1 (continuation).

Complementary formulae:

\[ G(t) = R^{-1}(t)B'(t)K(t) \]  \hspace{1cm} (A1)
\[ P(t) = R^{-1}(t)B'(t)M(t) \]  \hspace{1cm} (A2)

\[ K(t) = -K(t)A(t) - A'(t)K(t) - Q(t) + K(t)B(t)R^{-1}(t)B'(t)K(t) \]  \hspace{1cm} (A3)
\[ K(T) = F \]  \hspace{1cm} (A4)

\[ M(t) = -K(t)H(t) - M(t)S(t) - A'(t)M(t) + K(t)B(t)R^{-1}(t)B'(t)M(t) \]  \hspace{1cm} (A5)
\[ M(T) = 0 \]  \hspace{1cm} (A6)

\[ A(t)_{n \times n} = \left[ \delta f/\delta x(t) \right] \hspace{1cm} \begin{bmatrix} x_i^t, u_i^t, z_i^t \end{bmatrix} \]
\[ B(t)_{n \times n} = \left[ \delta E/\delta u(t) \right] \hspace{1cm} \begin{bmatrix} x_i^t, u_i^t, z_i^t \end{bmatrix} \]

\[ H(t)_{n \times e} = \left[ \delta f/\delta z(t) \right] \hspace{1cm} \begin{bmatrix} x_i^t, u_i^t, z_i^t \end{bmatrix} \]
\[ S(t)_{e \times e} = \left[ \delta h/\delta z(t) \right] \hspace{1cm} z_i^t \]

\[ \delta x(t) = x(t) - x_i^t \]
\[ \delta u(t) = u(t) - u_i^t \]
\[ \delta z(t) = z(t) - z_i^t \]

Symbols used:

\( x(t) \): Vector (nx1) of state variables dynamics at time \( t \)
\( u(t) \): Vector (mx1) of control variables dynamics at time \( t \)
\( z(t) \): Vector (ex1) of exogeneous dynamics at time \( t \)
\( x_d \): Vector (nx1) of state variables desired patterns at the sampling point \( d \)
\( u_d \): Vector (mx1) of control variables desired patterns at the sampling point \( d \)
\( z_d \): Vector (ex1) of exogeneous patterns at the sampling point \( d \)
\( p \): Vector of fixed parameters
\( f \): Vector (nx1) of nonlinear differential equations
\( h \): Vector (ex1) of nonlinear differential equations
\( g \): Vector of nonlinear inequalities
\( N \): Total number of points used to sample the planning period \( 0 \leq t \leq T \)
\( T \): Upper limit of the planning period \( 0 \leq t \leq T \)
\( D_d+1 \): Interval of time between the sampling points \( d \) and \( d+1 \) in the reference model
\( D_d+1 \): Lower limit of \( D_d+1 \) at the sampling point \( d+1 \)
\( D_d+1 \): Upper limit of \( D_d+1 \) at the sampling point \( d+1 \)
\( I, U \): Nonlinear functions
\( Q(t) \): Penalty matrix (nxn) of relative weights at time \( t \)
\( R(t) \): Penalty matrix (mxm) of relative weights at time \( t \)
\( F \): Penalty matrix (nxn) of relative weights
\( G(t) \): Gain matrix (mxn) of the state variables at time \( t \)
\( P(t) \): Gain matrix (mxe) of the exogeneous at time \( t \)
\( K(t) \): Auxiliary (nxn) matrix at time \( t \)
\( M(t) \): Auxiliary matrix (nxe) at time \( t \)
Annexe 2. The last reference model. Notice that all the equations are valid for 0 ≤ d ≤ N-1 = 30 sampling points except when another number of sampling points is specified.

1) Normative part

\[
\min \sum_{d=0}^{N-1} [WFOR(FOR_d - GFOR)^2 + WPSR(PSR_d - GPSR)^2 + WIN(INV_d - GINV)^2] \cdot D_{d+1} + 27.2[INV_d - GINV]^2
\]
\[\alpha_{d+1} - \alpha_d \leq 0.03, \ 0 \leq d \leq 29; \ \alpha_{d+1} - \alpha_d \geq -0.03, \ 0 \leq d \leq 29\]
\[\beta_{d+1} - \beta_d \leq 0.03, \ 0 \leq d \leq 29; \ \beta_{d+1} - \beta_d \geq -0.03, \ 0 \leq d \leq 29\]
\[\tau_{d+1} - \tau_d \leq 0.03, \ 0 \leq d \leq 29; \ \tau_{d+1} - \tau_d \geq -0.03, \ 0 \leq d \leq 29\]
\[0 \leq \alpha_d \leq 2; \ 0 \leq \beta_d \leq 2; \ 0 \leq \tau_d \leq 2;\]
\[OBL_{d+1} \geq 0; \ INV_{d+1} \geq 0; \ PLAd_{d+1} \geq 0; \ ASRD_{d+1} \geq 0; \ AORD_{d+1} \geq 0; \ RBLd_{d+1} \geq 0;\]

2) Descriptive part (approximate system dynamics model)

\[OBL_{d+1} - OBL_d = D_{d+1} \cdot [\beta_d \cdot ASRD + WINVD \cdot \tau_d \cdot ASRD - \tau_d \cdot INV_d - \alpha_d \cdot APLd]\]
\[INV_{d+1} - INV_d = D_{d+1} \cdot [PLAd / PDEL - EXOGd]\]
\[PLAd_{d+1} - PLAd_d = D_{d+1} \cdot [\alpha_d \cdot APLd - PLAd / PDEL]\]
\[ASRD_{d+1} - ASRD_d = D_{d+1} \cdot [EXOGd / TASR - ASRD / TASR]\]
\[AORD_{d+1} - AORD_d = D_{d+1} \cdot [(\beta_d \cdot ASRD) / TAOR + (WINVD \cdot \tau_d \cdot ASRD) / TAOR - (\tau_d \cdot INV_d) / TAOR - AORD / TAOR]\]
\[APLD_{d+1} - APLd = D_{d+1} \cdot [OBLd / (PAT.TABL) - RBLd / (PAT.TABL) - APLd / PAT]\]
\[EXOGd_{d+1} - EXOGd = D_{d+1} \cdot [-FPT \cdot EXOGd + FPT.GOAL]\]
\[OBL0 = 10; \ INV0 = 6; \ PLAo = 6; \ ASRO = 1; \ ARO = 1; \ APL0 = 1; \ EXOG0 = 1\]
\[RBLd = -0.3 + 11.56AORD - 6.57AORD^2 + 1.33AORD^3; \ 0.5 \leq AORD \leq 1.5\]
\[FORd = \beta_d \cdot ASRD + WINVD \cdot \tau_d \cdot ASRD - \tau_d \cdot INVd; \ PSRD = \alpha_d \cdot APLd\]
\[D_{d+1} = 1\]

Constants: WINVD = 6; PDEL = 6; TASR = 4; TAOR = 4; PAT = 3; TABL = 4; FPT = 0.6; GOAL = 1.4; WFOR = 1000; WPSR = 1000; WIN = 27.2; GFOR = 1.4; GPSR = 1.4; GINV = 6
References


