Approach on Educational Economics by Using the Non-equilibrium System Theory

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Abstract

Under the framework of non-equiluibrium theory, a model is adopted to analyse the educational economic system of China. By dividing the education levels into four parts, the high education, the basic education, the secondary education and the vocational -technical education, we study the relation between educational systems and economics. The development of chinese education and its economical benefits have been calculated and discussed.

The theory of non-equilibrium system is given by I.Prigogine[1-3], and its application to economical systems is discussed by P.Allen[4-6]. This paper is the first time on using this framework to treat an educational economic system.

We will study the educational economic system as an open system. This system involves several subsystems with complex interaction inside, and also the system has its effects from the environment.

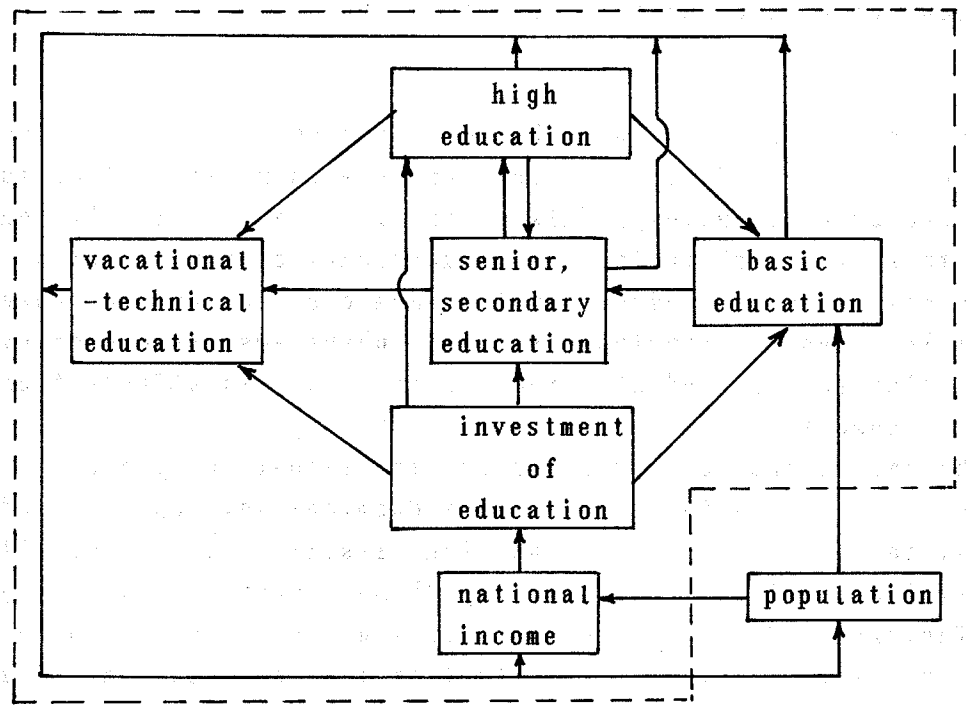
The educational economic system is rather complex, it includes the policy thinking , the economical development, and the different parts of education itself. For a detail quanitative treatment is difficult.[7] But when we simplify some factors, and discuss the interesting parts as a modle, we can get some meaningful results from the calculation and discussion.

We will simplify the system and give the model in Sec.1. The mathematical method is given in Sec.2. The calculating results are given in Sec.3. Some discussion on the problem will be given in Sec.4

I .The model of the educational economic system

For the discussion on the relation between education and economy, we regard the educational economic system as an open system. There are interation between the subsystems, also the system is linked closely with the environment. We will simplify the problem and only concentrate on the relation between investement and development of the educational economic system.

Investment of education is one part of national income, originally coming from gross national product. This investment will be divided into several parts for different levels of education, the basic education, the secondary education, the vocational education and at last the high education. In our problem for calculation we will take basic education as primary education added junior secondary education. The result of the educational investment will change the character of science, technique and culture etc. for the educational people. At last, the national income is contributed by the people with different educational levels. We can model these relations as following graph.



The graph gives a describtion of the simplified model of the educational economic system. The development of the education is restricted by the investment of education, a part of the national income. The education with its scale and different levels provides training labors and experts for the

productive area. Then the national income will be increased. The education and the economy exhibit the improving and the restrictive relation for each other.

These are also the relation between the different levels of education. The high education provides the teachers for other levels and also for itself. The lower educational level provides the students for the higher educational level. The population of the society also influences the education as an external parameter via enrollment of the basic education.

The graph thus roughly gives the model of interaction between the education and economy. Mainly we concentrete on the benefits of the investment and the development of the education. The subsystems of the educational economic system are interacted each other with depending, improving, and restricting relation. The system will be evolving under these conditions. We will give the mathematical discription for the model in the next section.

II. The mathematical method for the description of economic educational system.

For the discussion of the economic educational system given by the graph in Sec.1, we introduce a mathematical model based from the non-equilibrium system theory. [1,4] Thus we write the variables, parameters and equations for the system as following.

1.Variables

X(1,J), X(2,J), X(3,J), X(4,J) denote the numbers of the students in year J, these students separately belong to high education, vocational-teachnical education, senion secondary education and basic education;

X(5,J) writes as the investment of education in year J;

X(6,J), X(7,J) as the national incomes in year J+1 and year J-1 separately;

P(J) as the population of year J;

Q(J) as the numbers of labors for year J.

P(J), Q(J) both are exognous parameters of the economic educational system.

2. Additional variables

T(1,J), T(2,J), T(3,J), T(4,J) denote the numbers of the teachers in year J, these teachers separately belong to high education, vocational-technical education, senior secondary education and basic education;

Q(1,J), Q(2,J), Q(3,J), Q(4,J) denote the numbers of graduate students of the social labors in year J, these graduate students also come from high education, vocational -technical education, senior secondary education and basic education separately, and Q(5,J) denote the number of the labors in the society who haven't get any kind of educational degree;

G(J) writes as the quantity of the educational inverstment provided by the goverment in year J.

3. Parameters.

A(1,1), A(2,1), A(3,1), A(4,1) denote the average numbers of students per each teacher for different educational -technical education, senior secondary education and basic education;

A(5,1), A(5,2), A(5,3), A(5,4) denote the fees per each student separately for the four educational levels;

α_1 , α_2 , α_3 , α_4 denote the ratios of educational inversment separately for the four educational levels with the sum $\sum_{i=1}^{4}\alpha_i=1$;

We have the relations between the parameters written above

$A(1,2)=\alpha_1/A(5,1)$, $A(2,2)=\alpha_2/A(5,2)$,

$A(3,2)=\alpha_3/A(5,3)$, $A(4,2)=\alpha_4/A(5,4)$;

m(1,J), m(2,J), m(3,J), m(4,J) are the rations which denote the graduates divided by envolments in year J srparately for the four educational levels;

m(o,J) is the ratio which denotes the admitted children of the primary schools divided by the population;

n(1,J), n(2,J), n(3,J), n(4,J) denote the ratios between students admitted and envolments for four educational levels separately;

We also have the notations

A(1,3)=m(1,J)/n(1,J), A(2,3)=m(3,J)/n(2,J),

A(3,3)=m(4,J)/n(3,J), A(4,3)=m(0,J)/n(4,J);

η_1 , η_2 , η_3 , η_4 are the ratios of graduates for the educational levels, these graduates are able to study in the advanced educational level.

A(6,1), A(6,2), A(6,3), A(6,4)are the values of production from the labors educated by high education, vocational -technical education, senior secondary education and basic education separately, A(6,5) is the value of production from the labors who haven't educated from above educational levels;

A(i,0) with i=1,2...6 is the time scale for the evolution of X(i,J);

$A(i,6)$ with $i=1,2...6$ is the external parameter which exhibits the effects to the system from the enviorment.

4. Some additional variables which can be described by the variables and the parameters.

Let $X(1,0)$, $X(2,0)$, $X(3,0)$, $X(4,0)$ as the initial numbers of graduates of the four educational levels, S as the average death rate of the population, then we have

$Q(1,J)=\sum_{i=0}^{J} m(1,i)X(1,i)\ (1-s)^{J-i}$, $m(1,0)=1$;

$Q(2,J)=\sum_{i=0}^{J} m(2,i)X(2,i)\ (1-s)^{J-i}$, $m(2,0)=1$;

$Q(3,J)=\sum_{i=0}^{J}\{[m(3,i)X(3,i)-n(1-i)X(1,i)-n(2,i)X(2,i)](1-s)^{J-i}\}$, $m(3,0)=1, n(1,0)=0, n(2,0)=0$;

$Q(4,J)=\sum_{i=0}^{J}\{[m(4,i)X(4,i)-n(3-i)X(3,i)-n(3,i)X(3,i)](1-s)^{J-i}\}$, $m(4,0)=1\ n(3,0)=0$;

$Q(5,J)=Q(J)-\sum_{i=1}^{4} Q(i,J)$;

$G(J)=E[X(7,J)-F\ P(J)\ \ln\ (1+X(7,J)/F\ P(J))]$.

Here E is the maximum ratio for the educational inverstment in the natioal income, it depends on the situation of the economy and social development; F is the parameter which denotes the necessary living spends for the people in average.

5. Equations

Now we describer the educational economic system as a Logistic type system and have the equations as following

$$dX(i,t)/dt=A(i,t)X(i,t)[1-X(i,t)/N(i,t)]+A(i,6),\ i=1,2...6 \qquad (1-6)$$

$$dX(7,t)/dt=1/2\,[X(6,t)-X(7,t)] \qquad (7)$$

Here we have

$$N^{-1}(1,t)=1/3\{[A(1,1)T(1,t)]^{-1}+[A(1,2)X(5,t)]^{-1}+[A(1,3)\eta_1 X(3,t)]^{-1}\};$$

$$N^{-1}(2,t)=1/3\{[A(2,1)T(2,t)]^{-1}+[A(2,2)X(5,t)]^{-1}+[A(2,3)\eta_1 X(3,t)]^{-1}\};$$

$$N^{-1}(3,t)=1/3\{[A(3,1)T(3,t)]^{-1}+[A(3,2)X(5,t)]^{-1}+[A(3,3)\eta_1 X(4,t)]^{-1}\};$$

$$N^{-1}(4,t)=1/3\{[A(4,1)T(4,t)]^{-1}+[A(4,2)X(5,t)]^{-1}+[A(4,3)\eta_1 X(5,t)]^{-1}\};$$

$$N^{-1}(5,t)=1/2\{[\sum_{i=1}^{4} A(5,i)X(i,t)]^{-1}+[G(t)]^{-1}\};$$

$$N^{-1}(6,t)=\sum_{i=1}^{5} A(6,i)\ Q(i,t).$$

6. Some explainations for the equations

By useing the non-equilibsium theory we get the equations (1)-(7) to describe the educational economic system written above. Equation (1) shows the development of the high education. The students of high educationare restricted by three condition, the munbers of teachers, the funds and the resources of

students. Here $A(1,1)T(1,t)$ denotes the numbers of students restricted by the teachers, i.e the numbers of teachers provide the possibility of enrolments for high education. $A(1,2)X(5,t)$ denotes the restriction of the students from the funds, and $A(1,3)\eta X(3,t)$ denotes the restriction of the high education from the resources of students. In equation (1), $N(1,t)$ combines these three factors and restricts the development of $X(1,t)$. Among these three factors, the shortage factor has much strong restriction for the $X(1,t)$. Thus we have the explaination of equation(1), we can explain equation (2)-(4) as the same.

In equation (5), $N(5,t)$ is constructed by two terms. The first term $\sum_{i=1}^{4} A(5,i)X(i,t)$ describes the demand of educational investment for the different levels of education at time t. The second term $G(t)$ describes the possible supply of educational investment decided by the national income, the policy of education, and some othes. The equilibrium of the demand and supply gives the restriction of educational investment.

In equation(6), $N(6,t)=\sum_{i=1}^{5} A(6,i)Q(i,t)$ describes the potential of economic development provided by the labors of different eduational levels.

Equation (7) describes the effects of time delay. The educational investment in year J is a result from the national income of year J-1, and the national income in year J+1 will be influenced by the construction of the educational labors in year J.

Equations (1)-(7) are cooperated each other. These equations are the mathematical description for the educational economic system given by the graph in Sec 1. We will discuss the results from the equations in succeed sections.

Ⅲ. The calculation on Chinese educational economic system.

Now we use the model of educational economic system to discuss the development of education in China. In this paper, we concentrate the period from 1951-1957 and 1977-1983, as an illustration for the model given above.

The calculation of the model is going as following. We give the initial condition for the equations. The parameters in the equations are taken as constants in certain period. Then we use the equations to calculate the data of the variables. By adjusting the parameters we make the correspondence between the real data and calculating data. Thus we can illustrate the

model and get the policy meanings from the value of parameters which are adjusted during the calculation.

The calculation gives good results. All the variables get the calculating data with the error less than 10% comparing to the real data. Here we give the examples for the students of high education ($X_1(t)$ as calculating data, $X_{10}(t)$ as real data) and educational investment ($X_5(t)$ as calculating data, $X_{50}(t)$ as real data).

year	$X_1(T)$	$X_{10}(T)$	$[(X-X_0)/X]$ %
1951	1.45	1.53	-5.3
1952	1.78	1.91	-7.6
1953	2.24	2.21	5.4
1954	2.73	2.53	7.3
1955	3.17	2.88	9.1
1956	3.79	4.03	-6.3
1957	4.44	4.41	0.6
1977	6.43	6.25	2.8
1978	8.60	8.58	0.4
1979	10.31	10.20	1.0
1980	11.05	11.44	-3.5
1981	11.73	12.79	-9.1
1982	11.24	11.54	-2.7
1983	12.35	12.07	2.2

year	$X_5(T)$	$X_{50}(T)$	$[(X-X_0)/X]$ %
1951	0.75	0.74	1.3
1952	0.99	0.90	8.7
1953	1.17	1.28	-9.7
1954	1.34	1.38	-3.3
1955	1.49	1.41	5.5
1956	1.69	1.65	2.4
1957	1.88	1.95	-3.8
1977	5.14	5.30	-3.2
1978	6.38	6.56	-2.9
1979	8.27	7.70	6.9
1980	9.56	9.42	1.5
1981	10.52	10.25	2.5
1982	11.30	11.57	-2.4
1983	12.20	12.79	-4.8

The other variables also have good results for the correspondence between calculation data and real data. We omit to write here and turn to the parameters. We write some parameters from the calculation.

The numbers of students per each teacher for different educational levels A(i,1) write as

	A(1,1)	A(2,1)	A(3,1)	A(4,1)
1951-1957	8.0	15.3	22.0	34.0
1977-1983	4.5	10.0	18.0	27.0

The average fees per each student in Unit Yan for different educational levels A(5,i) write as

	A(5,1)	A(5,2)	A(5,3)	A(5,4)
1951-1957	1200	400	150	15.5
1977-1983	2400	1200	200	40.0

The ratios of educational investment for the four educational levels α_i(%) write as

	α_1	α_2	α_3	α_4
1951-1957	20.4	14.0	6.0	54.3
1977-1983	19.4	10.8	16.0	60.0

For the analysis of an educational economic system, naturally we are interesting in the problem of calculating results of educational investment and the economic benefits of education. The educational investment of China has been calculated and compared with the real data. We give the results for the ratio of educational invesment to the national income,. We write educational investment in year T as G(T), and the national income in year T-1 as X(T), than the ratio Y(T)=G(T)/X(T). The comparation of some calculating results Y(T) to the real data Yo(T) is given as following.

year	Y(T)	Yo(T)	(Y-Yo)/Y%
1971	1.94	1.77	9
1972	2.01	1.93	2
1973	2.01	2.13	-6
1974	2.10	2.15	-2
1975	2.09	2.21	-6
1976	2.16	2.17	0
1977	2.63	2.43	8
1978	2.75	2.73	1
1979	2.97	2.93	1
1980	3.18	3.23	-3
1981	3.30	3.46	-5
1982	3.40	3.38	1
1983	3.54	3.56	-1

The calculation also can give some forecast for the educational investments on the different levels of the national income.

X(T)	G(T)	Y(T)%
500	17.28	3.46
800	34.46	4.31
1000	47.05	4.70
1500	80.85	5.31
2000	116.67	5.83
2500	153.71	6.15

Now we write some results for the economical benefits for the education. This model can directly calculate the value of production in average for the labors of different educational levels.The calculating results is the following.(Unit,Yan)

year	average	high education	vocational-technical education	senior secondary education	basic education	others
1977	671	1784	1394	1115	836	502
1978	754	1965	1535	1228	921	553
1979	825	2113	1650	1320	990	594
1980	880	2229	1742	1393	1045	627
1981	914	2294	1792	1434	1075	645
1982	953	2379	1859	1487	1115	669
1983	1028	2550	1992	1593	1195	717

Ⅳ.discussion.

The model given above suggests a mathematical analysis on the educational economic system based on the non-equilibrium theory. The calculating results fulfil the real data, thus the model is available. From the model and the parameters, we can understand the operating mechanism for the educational economic system. Although the analysis is given under some simplified conditions, we still can get the useful informations on the relations between the economy and education, especially the investment, the benefit,and the development of the education.

Here we give some discussions on the benefits of the education.From the table of educational benefits given in Sec. 3, we can see that the labors contribute different productive values according to their .different educational levels. The higher educated labors produce higher values than the lower educated labors in average. We can rate the difference from the

table. If we let the productive value of uneducated labors(or below the primary education level) as a basic index named 1.Than we have the productive values index as 3.4, 2.6, 2.1,1.6 for the educated labors of high education,vocational-technical education, senior secondary education and basic education separately. these are the productive value index in average and can be gotten from the table as a result of calculation in eductional benefits.

The another comment is the global benefits of education. How can we estimate the eduvational benefit as a whole for the national income ?This is a rather basic problem for the exterior benefits of the education. Now in this model of educational economic system, we can calculate the productive values of different educated labors, thus we can get the global benefits for the education from the above calculation.In the situation of China, the benefit ratio of education takes 30% in year 1983, also we can get the benefits ratio for other years. This is the first time we calculate the educational benefits in Chinese situation. The other interesting problems also can be discussed under the framwork of non-equilibrium theory.

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Reference

1. Nicolis,G. and Prigogine,I. 1977. Selforganization in Non-equilibrium Systems.Wiley, New York.

2. Nicolis,G. and Prigogine,I. 1986. Exploring Complexity Freeman, New York.

3. Prigogine,I.and Petrosky,T.Y. 1987. Physica 147 A 33

4. Allen,P.M. and Sanglicer,M. 1981. J.Social biol.Struct. 4,263.

5. Allen,P.M. Engelen,G. and Sanglier,M. 1985. Environment and planning, Series B12(1), 1.

6. Engelen,G. Allen,P.M. and Sanglier,M. 1983. A new spatial and Dynamic model of an urban system, Report.

7. Windham,D.M. 1979. Economic Dimensions of Education. National Academy of Education.