MACROSYSTEM APPROACH FOR MODELLING OF REGIONAL DYNAMICS

Yu.S.POPKOV

The Institute for System Studies
of the USSR Academy of Sciences
9,Prospect 60 let Oktyabria
Moscow,117312,USSR.
telex 411237 POISK SU
tel. 135-42-22
fax. 007/095/938- 2209

Abstract

Formulation of the general scheme for developing dynamic models of macrosystems with selfreproduction and resources exchange,using the local equilibria principle,is proposed.Regional system,where reproduction and exchange processes have significantly different relaxation times,is considered.The modelof regional system,where dynamic exchange processes are described as the evolution of local stationary states,is presented.Calculation,classification and bifurcation of equilibrium points of regional dynamics are studied.Results of computational experiments are given.

1.Introduction

During last decades we have observed growth of interest in re-gional dynamics problems,stimulated by the accelerated rates temporal -spatial regional development.The regional system is a system of human settlements (centers),linked by network,through which exchanges of goods,manpower,information,intellectual resources are realised intensively.The specific properties of regional system are the existence of close links among settlements and their resource interdependence.

Interraction between regional settlements (centers) is realized in different forms.We consider one of them,when reproduction of some resources at centers is accompanied by their redistribution among centers.

The problem of regional dynamic modelling is very popular. It may be seen from large number of papers dealing with various aspects of this problem (see,for example ,Prigogine,Herman,Allen 1977;Allen,Sanglier 1979;Wilson 1981;Roy,Brotchie 1984;Weidlich, Haag 1988;Popkov 1989).

The approach to regional systems modelling pursued below is based on the natural phenomenology of interaction between selfreproduction at centers and resources exchanges among them. We use the general procedure for describing dynamics of macro-systems with selfreproduction and local-stationary distribution,proposed by Popkov(1988,1989).

2. General scheme

Consider the macrosystem consisting of *n* interlinked blocks, in which system elements are located. The macrosystem elements are of two types: specific for each block and nonspecific, i.e. common for all blocks. The structure of links between blocks may be different for both types of elements. Examples of these structures are shown at fig.1. Two interdependent processes are realised in the macrosystem: selfreproduction at blocks and distribution among blocks. The specific elements of each block and of blocks linked to it, as well as the nonspecific elements participate in the selfreproduction process. A state of this process is characterized by some parameters, considered as the state variables. These variables may be the numbers of specific and nonspecific elements at block, their concentration or distribution over some scale. Below the deterministic selfreproduction processes are considered.

The distribution process is related to the exchange between blocks of universal (the same for all blocks) product, reproduced in these blocks. Such a product may include nonspecific elements, set of goods, some universal good, which is used for pricing the other goods.. Below stochastic distribution processes are considered.

The state of each macrosystem block at time t is characterized by generalized state variables for specific elements $z^i(t) = \{z_{i1}(t),\dots,z_{im}(t)\}$, for nonspecific elements $x^i(t) = \{x_{i1}(t),\dots,x_{im}(t)\}$ and the average flows of universal good $y^i(t) = \{y_{i1}(t),\dots,y_{in}(t)\}$.

<u>Assumption 1.</u> The relaxation time of distribution process is much less then the relaxation time of the selfreproduction process.

This assumption makes possible to apply the local equilibria principle (De Groot, Mazur, 1964) and to characterize the distribution process by its local stationary state $Y^*(t) = \{y^*_{ij}\overline{(t), i, j=1,n}\}$, where $y^*_{ij}(t)$ is a local-stationary flows of the universal good.

The selfreproduction process is the result of interaction among specific and nonspecific elements (including consumption of the universal good). The flow $G(z,x,Y^*)$ and the flow $F(z,x,Y^*)$ constitute the aggregate characteristic of the selfreproduction process.

Then the dynamics of the selfreproduction process may be described with the following system of differential equations:

$$\dot{z} = G(z,x,Y^*); \quad \dot{x} = F(z,x,Y^*) \tag{1}$$

In applied studies the system

$$\dot{z} = G(z,x) + AY^*; \quad \dot{x} = F(z,x) + BY^*, \tag{2}$$

is widely used, where A,B - constant matrixes.The matrix of flows Y^*, characterizing the local-stationary state of stochastical distribution of the universal good among blocks, is included into right sides of the equations (1,2).

Consider the phenomenology of stochastic process of the local-stationary state formation. Let $w_{ij}(t) = \Delta t\ y_{ij}(t)$ be the volume of the universal good, transfered from block i to block j during time interval Δt, $u_i(t)$ - the summary volume of the universal good, transfered from i during Δt. Since the universal good is produced at macrosystem blocks, it appears natural to assume that $u_i(t)$ depends on the states of block i and other blocks, which exchange the specific elements with block i, i.e. $u_i(t) = u_i(z^i(t), z^{s(i)}(t), x^i(t))$. Distribution of the universal good among macrosystem blocks during time interval Δt, characterized by the matrix $W(t) = \{w_{ij}(t)\}$, is accompanied by consumption of r resources. Let their stores be $Q_k(t) = \Delta t\ q_k(t)$, $k=\overline{1,r}$, where $q_k(t)$ is the maximal volume of resource k, which may be consumed in unit of time (maximal flow of resource k).

Resources consumption and their stores are linked as

$$\phi_k(W(t)) \le Q_k(t), \quad k\in\overline{1,r}; \tag{3}$$

where $\phi_k(W)$ is a function of consumption of resource k. Below we consider the linear consumption functions:

$$\sum_{i,j=1}^{n} c_{ijk} y_{ij} \le q_k(t), \quad k \in\overline{1,r}; \tag{4}$$

where c_{ijk} is a resource k consumption per unit of time.

We assume that the universal good is produced in portions and $w_{ij}(t)$ is the number of these portions, transfered from i to j during Δt. Let p_i be the probability of producing one portion of the universal good at block i; $b_{i/j}$ - probability of transfering one portion of universal good from block i to block j. From the definition of macrosystem state variables it follows that the set of states for the distribution process is the set of pairs (i,j) - 'producer - consumer of good'. Then $a_{ij}=p_i b_{j/i}$ the probability of a portion getting to the state (i,j). The states (i,j) characterize by capacities G_{ij}.

Assumption 2. The parameters of distribution processes are

$a_{ij}=a_{ij}(z,x)$; $c_{ijk}=c_{ijk}(z,x)$; $q_k=q_k(z,x)$; $G_{ij}=G_{ij}(z,x)$.

Thus, numerous random distributions of the universal product portions over states (i,j) determine flows $W^*(t) = \{w^*_{ij}(t), i,j=\overline{1,n}\}$, which characterize the distribution process macrostate. Due to the Assumption 1 the distribution $W^*(t)$ is established during short time interval Δt while the blocks state variables $z(t),x(t)$ remain the same.

The local stationary macrostate of the distribution process may be obtained as the state with maximal entropy under resources constrains (see Wilson 1967;Popkov 1980). In this case the time interval Δt is assumed sufficiently less than the selfreproduction process relaxation time, but long enough for the macrostate to be formed from multiple realizations of the microstates (in physical sence, Δt is infinitezimal).

Since the local stationary state is given by the point of entropy maximum, and not by its value, the entropy function may be determined by flows Y:

$$H(Y,z,x) = - \sum_{i,j=1}^{n} y_{ij} \ ln \frac{y_{ij}}{\tilde{a}_{ij}(z,x)} , \qquad (5)$$

where

$$\tilde{a}_{ij}(z,x)=a_{ij}(z,x)G_{ij}(z,x). \qquad (6)$$

Then the model of the distribution process local stationary states becomes

$$H(Y,z\ x) \Rightarrow max, \qquad (7)$$

$$Y \in \mathcal{D}(z,x). \qquad (8)$$

The set of feasible states is

$$\mathcal{D}(z,x) = \{Y: \sum_{i,j=1}^{n} c_{ijk}(z,x)y_{ij} \le q_k(z,x),\ k=\overline{1,r}\} \quad (9)$$

Thus the dynamic model of the macrosystem with selfreproduction and local-stationary redistribution (DMSR) becomes

$$\dot{z} = G(z,x,Y^*(z,x)); \quad \dot{x} = F(z,x,Y^*(z,x)), \quad (10)$$

$$Y^*(z,x) = \operatorname{argmax}\ (H(Y,z,x)\ \ Y\in\mathcal{D}(z,x)). \quad (11)$$

The structural scheme of DMSR is shown at fig.2.

3. Dynamic model of regional system

Consider the regional system consisting of n interlinked centres with capacities $E_1,\dots,E_n$. We characterize the system state by population $x_i(t)$ at center i and migration flow y_{ij} from center i to center j. The capacities $E_1,\dots,E_n$ and population volumes satisfy the following unequalities:

$$0 \le x_i(t) \le E_i,\ \ i\in\overline{1,n}. \quad (12)$$

Assume that the biological reproduction of centers population takes place significantly slower, than the mechanical migration among centers, This is a hypothesis, and for some countries it is confirmed by the empirical studies.

The migration from center i to center j is stimulated by the attractiveness of center j for the residents of center i. The attractiveness is determined by many factors and various methods are proposed to estimate it (for example, Weidlich, Haag, 1988). We use the stochastic model of migration and estimate attractiveness via probability a_{ij} of resident choosing the pair of centres: i - "old" residence, j - "new" residence. The probability a_{ij} depends on population allocation, i.e. $a_{ij}=a_{ij}(x)$.

This dependence may take different forms but certain properties are typical. So, if the population of "new" residence equals zero or maximal capacity of this center, then the attractiveness of center j equals zero:

$$a_{ij}(x_1,\dots,x_{j-1},0,x_{j+1},\dots,x_n)=0, \quad (13)$$

$$a_{ij}(x_1,\dots,x_{j-1},E_j,x_{j+1},\dots,x_n)=0,\ \ i,j\in\overline{1,n}. \quad (14)$$

Assume, that only fixed share h_i of population in center $i$participates in migration. This share of population will be called mobile.

Introduce the notation:

$$P_i(t) = \sum_{j=1}^{n} y_{ji}(t)\ ,\quad Q_i(t) = \sum_{j=1}^{n} y_{ij}(t). \tag{15}$$

Considering the environment as the additional center numbered 0 we obtain

$$\sum_{i=0}^{n} P_i(t) \ge \sum_{i=1}^{n} Q_i(t).$$

The process of changes population centres may be described by the following system of differential equations:

$$\dot{x}_i = T_i(x_i) + \sum_{j=0}^{n} y^*_{ji} - \sum_{j=1}^{n} y^*_{ij},\quad i\in\overline{1,n};$$

where $T_i(x_i)$ is the reproduction function; $T_i(x_i)\ge 0$; $T_i(0)=0$.

According to the general scheme of DMSR formulation, the matrix $Y^*=[y^*_{ij}]$ is determined as

$$Y^*(x) = argmax\ \{\ H(Y,x)\quad Y\in D(x)\ \}, \tag{16}$$

where

$$H(Y,x) = - \sum_{i=1,\, j=0}^{n} y_{ij}\ ln\ \frac{y_{ij}}{a_{ij}(x)e}\ , \tag{17}$$

$$\mathcal{D}(x)=\{y_{ij}:\ \sum_{j=0}^{n} y_{ij}=h_i x_i;\ \sum_{i=1,\, j=0}^{n} c_{ijk}y_{ij}=q_k(x);\ i\in\overline{1,n};\ k\in\overline{1,r}\} \tag{18}$$

Considering the structure of the feasible set (18), notice that the first group of equalities is implied by the assumption that all mobile population of center i participates in the migration. The second group of equalities accounts for resources consumption. For example, these expenditures may be stipulated by travel costs, new housing construction, creation of additional working places. Here it is assumed that resources consumption is proportional to the flows y_{ij} and the coefficients of proportionality c_{ijk} do not depend on distribution of the population over centres. Such a dependence may take place only for the total stores q_k of resources.

Due to the special structure of the feasible set $\mathcal{D}(x)$ (18)

$$\dot{x}_i = \tilde{T}_i(x_i) + \sum_{j=1}^{n} y^*_{ji}\ ,\quad i\in\overline{1,n};$$

where

$$\tilde{T}_i(x_i) = T_i(x)-h_i x_i.$$

We assume that $T_i(x_i) < h_i x_i$.

Consider the problem (15-18). Using Lagrange method we obtain the following system of differential equations:

$$\dot{x}_i = \tilde{T}_i(x_i) + \sum_{j=1}^{n} a_{ji}(x) h_j x_j \phi_{ji}(x), \qquad (19)$$

$$\phi_{ij}(x) = \frac{exp\,(-\sum_{k=1}^{r} \lambda_k c_{ijk})}{\sum_{s=0} a_{is}(x)\, exp(-\sum_{k} \lambda_k c_{isk})} \geq 0\ , \quad i \in \overline{1,n}, \quad j \in \overline{0,n}, \qquad (20)$$

$$\sum_{i=1,\ j=0}^{n} a_{ij}\ (x)\ c_{ijk} \frac{h_i x_i\ exp\ (-\sum_{k=1}^{r} \lambda_k\ c_{ijk}\ )}{\sum_{s=0}^{n} (x_{is})\ exp\ (-\sum_{k=1}^{r} \lambda_k\ c_{isk}\ )} - q_k(x) = 0,$$

$$k \in \overline{1,r} \qquad (21)$$

Hence, the equations (19-21) describe the dynamical model of population reproduction and migration in the regional system.

4. Model of regional system with linear reproduction

Consider the regional system, where reproduction function $T_i(x_i) = \alpha x_i$, mobile population share $h_i = h$ and resources constraints in the model of migration are omitted. Then the equations (19) come to:

$$\dot{x}_i = \sum_{j=1}^{n} \frac{a_{ji}(x)}{\sum_{k=0} a_{jk}(x)} h x_j - (h-\alpha) x_i, \quad i \in \overline{1,n}. \qquad (19')$$

Consider the case when emigration exceeds reproduction $(\alpha < h)$ and assume that the attractiveness of center j depends only on its own population:.

$$a_{ij}(x) = v_j(x_j) = x_j(E_j - x_j), \quad j \in \overline{1,n}. \qquad v_0 = E_0^2.$$

Transform the system (19'), substituting the variables $x_i E_0$ for x_i, t/h for t. We obtain

$$\dot{x}_i(t) = \frac{x_i(e_i - x_i) \sum_{k=1}^{n} x_k}{1 + \sum_{k=1}^{n} x_k(e_k - x_k)} - \frac{h-\alpha}{\alpha} x_i, \quad i = \overline{1,n}; \qquad (22)$$

where $e_i = E_i / E_0$.

The system (22) belongs to the class of differential equations $\dot{x}_i = x_i \Phi_i(x)$, for which the nonnegative ortant $\Re^n_+ = \{x \in \Re^n |\ x_i \geq 0,\ i = \overline{1,n}\}$ constitutes the invariant set.

We study the singular points of this system and their evolution depending on change of centers capacity. It is shown that system (22) have $2n_I$ positive solutions

$$x_i^{\pm} = e_i - \frac{e_a}{2} \pm \frac{1}{2}\sqrt{e_a{}^2 - [e^{(n)}]^2},\quad i\in\overline{1,n};$$

where:

$$e_a = (\sum_{i=1}^{n_I} e_i)/n_I,$$

$$e^{(n)} = [2(h-\alpha)n_I\ \alpha]^{1/2},$$

n_I is the number of positive coordinateof singular points. The positive singular points $x_i^{\pm}$ exist if the following unequalities are valid:

$$e_a \geq e^{(n)},$$

$$e_i > \frac{e_a}{2} \pm \frac{1}{2}[\,e_a^2 - (e^{(n)})^2\,]^{1/2},\ i\in\overline{1,n_I}. \qquad (23)$$

In (Popkov,Shwetsov,1990) the analysis of situations connected with existing of positive singular points fordifferent centers capacity is realized.

The inequalities (23) divide the $(e_1,\ldots,e_n)$ area of parameters values into several subareas with different numbers of equilibria. The smooth variation of parameters within one of these areas provides the smooth shift of equilibrium position. If the $(e_1,\ldots,e_n)$ parameter point crosses an area boundary, the equilibrium bifurcations occur, such as birth,disappearance or merging of equilibrium points. The transformations of trajectories structure, implied by the equilibrium bifurcations, provide the qualitative changes in system behaviour. Thus the global properties of system are changing through the equilibrium bifurcations.

In (Popkov,Shwetsov,1990) full description of the bifurcation of equilibrium positions in this system is obtained. On fig.3 is shown the results of the investigation of equilibrium positions of the regional system consisting of two centers. The phase pictures illustrate the evolution of equilibrium points for various centers capacity. When the capacities are small, system degrades due to population emigration to environment (fig.3a,b).For some value of capacity determined by conditions (23) the nontrivial equilibrium points arise, at the beginning

- one point (fig.3c) and then - two points (fig.3d). The point "A" on this picture is stable, and the point "B" is not stable. Notethat the phase plane is divided into two parts. One of them is area of the attraction of the trivial point and the other - of point "A". When the center capacities increase then size of "A" attractiveness-area increases too (fig.3e,f).

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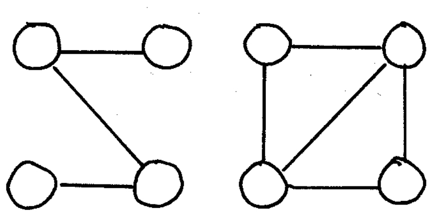

Fig 1

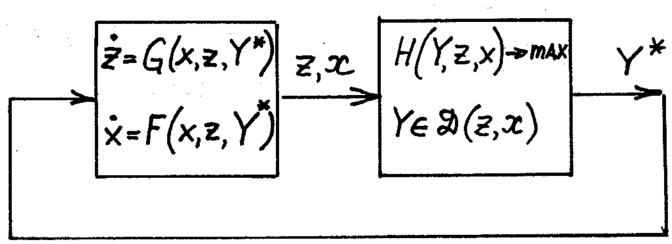

Fig 2

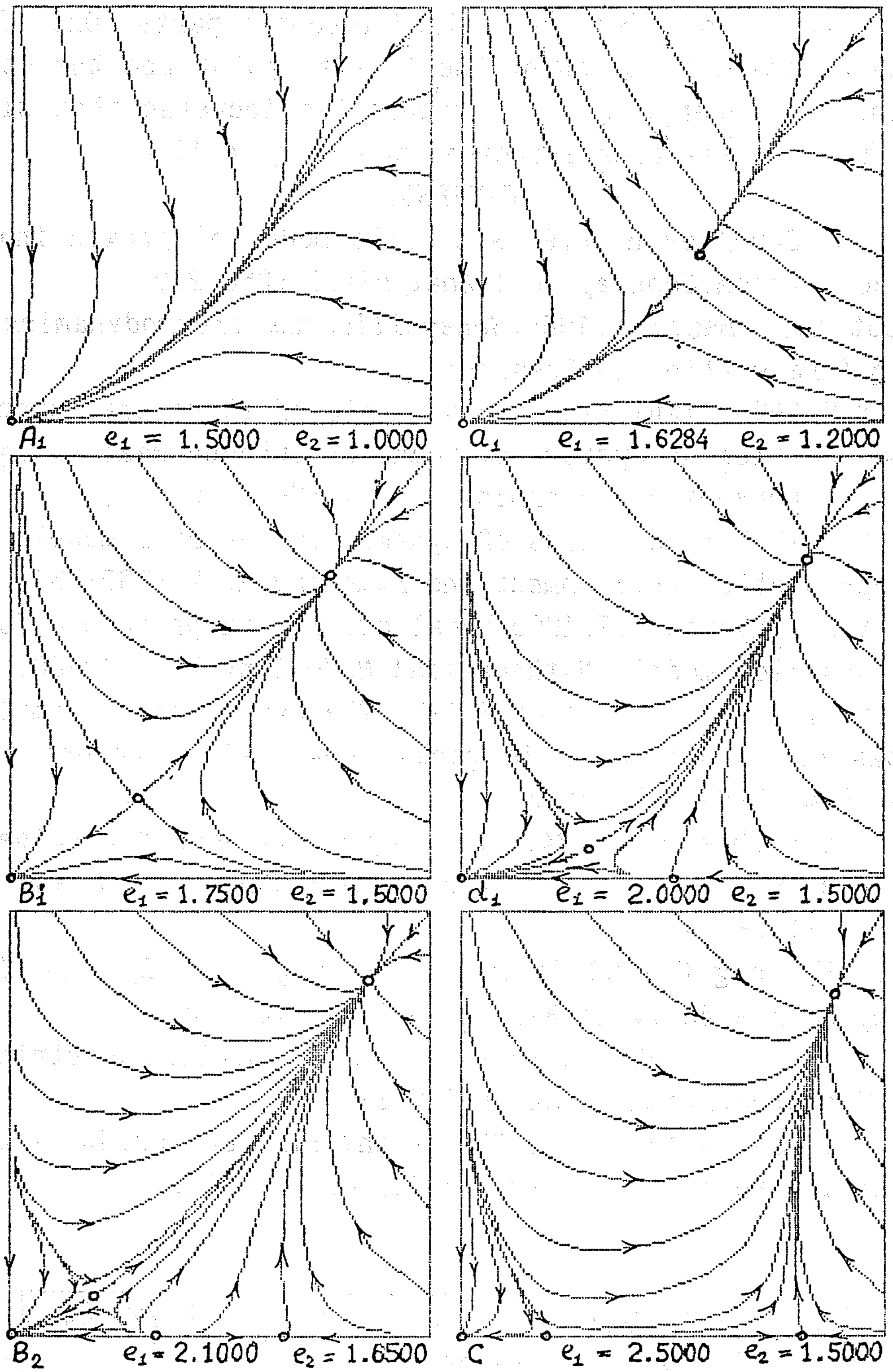
A1 e1 = 1.5000 e2 = 1.0000
a1 e1 = 1.6284 e2 = 1.2000
B1 e1 = 1.7500 e2 = 1.5000
d1 e1 = 2.0000 e2 = 1.5000
B2 e1 = 2.1000 e2 = 1.6500
C e1 = 2.5000 e2 = 1.5000

Fig 3.