Dyadic Processes, Tempestuous Relationships, and System Dynamics

by

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...the term 'many-body problem' takes on new meaning in this context.
Steven H. Strogatz (1988)

Abstract

This paper describes two exercises that are useful in an introductory course in system dynamics. They are centered around two models of a couple engaged in a tempestuous relationship. Although the models are quite simple, the exercises can be used to introduce and practice a surprisingly large number of system dynamics skills.

Introduction

One of the great appeals of the system dynamics paradigm is the emphasis it places on the *intuitive* understanding of the mathematics underlying dynamical systems. This emphasis stems primarily from a modeler's need to identify system structure and relate it to observed behavior. It is important to note however, that it is also responsible for attracting a significant number of people with limited mathematical backgrounds (e.g., no courses in calculus, differential equations, or control theory) to the field. These are people that normally would never consider using, say, differential equations to address a problem, yet find modeling with bathtubs, faucets, and pipes to be an understandable and useful endeavor. While people such as these have always been welcome in the field and, indeed, have been encouraged to join, their presence creates the need for a catalog of simple exercises that can generate insight into some of the well-known relationships between dynamical behavior and mathematical feedback structure. This need becomes even more acute when one considers that most people get exposed to only one semester's worth or, in many cases, a few days worth of formal system dynamics training.

The purpose of this paper is to present two exercises that can be used to teach a large number of system dynamics skills -- including those involving traditional mathematics -- in a short amount of time. They are based on Clarence Peterson's (1988) newspaper account of Steven Strogatz's (1988) article "Love Affairs and Differential Equations," and on Strogatz's original piece itself. Peterson's article describes how Strogatz teaches undergraduates differential equations by, in part, relating them to romantic relationships. Figure 1 presents a copy of this article.[1]

Strogatz's Simple Model

As can be seen in Figure 1, Peterson is both intrigued and perplexed by Strogatz's account of how dyadic relationships can be modeled with differential equations. He reproduces one of Strogatz's models -- a second order, linear, harmonic oscillator [1] that Strogatz relates to a hypothetical relationship between Romeo and Juliet -- and describes its dynamic behavior.

$$dr/dt = -a^*j$$
$$dj/dt = b^*r$$

where: $r(t)$ = Romeo's love/hate for Juliet at time t
$j(t)$ = Juliet's love/hate for Romeo at time t [1]

From the cadence of Peterson's discussion however, it is evident that he has trouble relating the model's structure to its purported behavior. Indeed, at one point he says that

> I telephoned my father, 29 years retired from the math game, to ask if he had any suggestions for making the Strogatz formulations even clearer. Said he:...'you're probably intimidated by the reference to the Shakespearean tragedy. Substitute the Hatfields and the McCoys, and see if that doesn't help.'

It is important to note that Peterson's difficulty -- i.e., the inability to intuitively understand mathematical expressions -- is one that is common to many people.[2] It is also important to note that Strogatz's model can easily be understood via the tools and techniques of system dynamics.

Exercise #1

An interesting exercise in an introductory system dynamics course then, is to hand out Peterson's article and assign students the task of helping him to understand the dyadic relationship between Romeo and Juliet. This exercise forces students to utilize, and perhaps discover for the first time, a surprisingly large number of system dynamics skills.

The first step students must take to help Peterson understand Strogatz's model is to translate [1] into stocks and flows and a system dynamics language. It is crucial that students master this skill if they are to begin developing the ability to intuitively understand the traditional mathematics underlying dynamical systems. Figure 2 shows [1] translated into a STELLA model and Figure 3 lists the corresponding STELLA equations.

Before the model can be simulated and its behavior compared to the "never-ending cycle of love and hate" cited by Peterson, students must answer four important questions: 1) What are the model's parameters? 2) What are the model's initial conditions? 3) What step size should be used for the simulation? and 4) What numerical integration technique should be employed? As Peterson provides no definitive answers to these questions,[3] students must supply their own. This is desirable, of course, because it forces them to think critically about the issues involved.

Parameters and Initial Values

In terms of the model's initial values, students can select any real numbers for Romeo and Juliet's initial love-hate level except zero, which yields a fixed point. It is up to the instructor to determine whether this insight should be explicitly told to students or left for self-discovery in the exercise. Either way, students should come away from the exercise realizing that, if the initial values of both Romeo and Juliet's love-hate are zero (i.e., neutral), they will each stay in that position indefinitely. That is, the parameters "a" and "b," when multiplied by zero, yield no change in either person's affections.

In terms of the values of the model's parameters, Peterson does provide a hint by quoting Strogatz as saying that "The parameters a,b are positive, to be consistent with the story." Thus students are provided with the signs, but not the magnitudes, of the parameters. Peterson's quote may be a bit misleading, however, as the parameter "a" in [1] has a negative sign that causes it to influence the system in a direction opposite to "b," and hence to cause its oscillations. Moreover, if students select zero -- a *nonnegative* rather than a *positive* value -- for the parameters, the model will again yield a fixed point. No matter what values are chosen, however, the instructor should make sure that students are able to discuss, either in written or verbal form, the intuitive meaning of each parameter, and that they can specify the parameters' dimensions and the dimensions of the model's other variables.

Although students do not need simulation to discover that zero is a poor choice for the model's parameters and/or initial values, two important properties of [1] are quite difficult to discover without it. The first involves the relationship between the magnitudes of the model's parameters and the periodicity of its oscillation. The second involves the relationship between its initial values and the amplitude of its oscillation. More specifically, through repeated simulation, students will discover that the absolute values of "a" and "b" are inversely related to the periodicity of the model's oscillation, and that the absolute magnitude of its initial values are positively related to the amplitude of its oscillation. It is particularly important that they discover these properties because they are applicable beyond the exercise. That is, they convey the insight that the character of *any* linear system's oscillation is determined by its parameters and initial values. To ensure that this revelation occurs, the instructor should debrief students after having them systematically vary "a," "b," r(0), and j(0) over a moderate number of simulation runs.

Step Size, Integration Method, and Integration Error

Strogatz's harmonic oscillator produces a large amount of integration error when simulated. As a result, students that select a value for DT that is relatively large (especially if they have also chosen Euler's method of integration and/or parameter values that are relatively large and hence generate a faster rate of change) will see a system that *apparently* generates exploding oscillations. It is all too easy, of course, for them to believe that this is the model's actual behavior. The instructor must make sure therefore, that students cut DT in half after an initial simulation run to see if the system's behavior changes appreciably. Indeed, in the case of Strogatz's model, successively reducing DT alters the appearance of each simulation run dramatically. Moreover, students will find that by reducing the absolute values of "a" and "b" (for a given DT and integration method), they can decrease the model's integration error. The instructor may wish, therefore, to tie this portion of the exercise to Forrester's (1968, p. 6-10) heuristic that DT be less than one half, but greater than one fifth, of a system's shortest first order delay.

The existence of, and trade-offs between, various methods of numerical integration can also be revealed to students in dramatic fashion in this portion of the exercise. This can be accomplished by having students simulate the model first with Euler's method and then with a second and/or fourth order Runge-Kutta method and/or the Adams-Bashforth method, and a constant DT. Students will observe, of course, that switching from, say, Euler's method to a fourth order Runge-Kutta method significantly reduces integration error and increases simulation time.

Dynamic Behavior

Figure 4 presents a time series plot of Romeo and Juliet's emotions. Clearly, with a small enough step size and/or an accurate enough integration method, Strogatz's model generates a "never-ending cycle of love and hate" -- i.e., one with a constant periodicity and amplitude. Given the emphasis on the intuitive understanding of dynamical behavior in the system dynamics paradigm, it is important that students be asked to describe, either in written or verbal form, the dynamics underlying this behavior. Since appealing to Peterson's article is of no help, students will again be forced to think critically about the dynamical story being told.

Essentially students must notice, as Peterson notes and Figure 4 reveals, that Romeo is fickle and that Juliet's affections mimic and lag behind Romeo's. As Romeo's state of love turns from neutral to hate (approximately period 2), Juliet's level of love, although still positive, begins to fall. When Juliet's love hits the neutral point (approximately period 5), Romeo apparently feels that he has been a "heel" long enough and begins to reverse himself. Since it takes some time for Romeo to move back to a state of love, however, Juliet's mimicking affections are driven below the neutral point into a state of hate. In fact, the couple reaches a state of "equal loathing" just after period 6.

When Romeo's affections finally rise past the neutral point (approximately period 8), Juliet apparently begins to feel that Romeo is a "reformed man" and begins to reduce her level of hatred towards him. When she passes the neutral point (approximately period 10.5), however, Romeo's fickleness kicks in and he reverses field. Apparently he feels that it is now ok to ease up because he has increased his love for Juliet long enough to repair the damage to their relationship and convince her to love him again. The cycle, of course, repeats from this point.

Although getting students to provide a description of the oscillations generated by Strogatz's model is an important part of this exercise, the instructor must also make sure that they are able to make the connection between its mathematical structure and dynamic behavior. Essentially this means getting them to recognize that the negative value of Romeo's parameter "a" causes the direction of flow in his rate equation to reverse every time Juliet's affections cross the neutral line. That is, every time Juliet's affections change from positive to negative values, or vice versa. Similarly, students must realize that the positive value of Juliet's parameter "b" causes her to move her rate of flow in the same direction as Romeo's.

Causal Loop Diagram

An additional task that the instructor can assign to students in this exercise is to have them draw a causal loop diagram of Strogatz's model. Such a diagram can add significantly to a discussion of model behavior and is particularly useful for helping students see [1] as a system of equations portraying *feedback* structure.

Figure 5 is a causal loop diagram of Strogatz's model. It is recommended that students be asked to include the flows, as well as the stocks, in the diagram because it enables them to make a smooth transition to Exercise #2 (below). Moreover, it opens the door for an in-class discussion of "problems with causal loop diagrams" -- a topic that arises, among other reasons, from the presence of both rates and levels in the figures (see Richardson 1986).

Inspection of Figure 5 (and, to be safe, Figure 2) reveals that Strogatz's model is a second order, major, negative feedback loop. Since a well-known system dynamics heuristic says that oscillation arises in negative feedback loops with delayed corrective action, the instructor should ask students to find the delay in the system. This is one of the more conceptually difficult tasks in the exercise as Strogatz's model contains no explicit material or informational delays. Nevertheless, a good way to proceed is to note that Juliet's flow is an interrupted version of Romeo's flow. In other words, the integration of Romeo's flow decouples it from Juliet's and causes a delay. Of course, the same process occurs when Juliet's flow is integrated into her stock of love and hate.

Peterson's Puzzle

Returning to Figure 1, it is clear that Peterson was struck (and confused) by Strogatz's claim that Romeo and Juliet "manage to achieve simultaneous love [only] one quarter of the time." Another interesting task therefore, is to have students determine a "clever" way of showing that Strogatz's statement is true. Although there are a number of ways that this can be accomplished, an easy one involves having students plot the levels of Romeo and Juliet's affections against one another on the phase plane. This is shown in Figure 6. Inspection of this figure reveals that only one quarter of the model's orbit passes through the area where both Romeo and Juliet's stocks have positive values. As one might imagine, however, students that have never been introduced to the phase plane will (probably) never think of this solution (despite its ease and clarity). Thus, the instructor must decide how much of a "push" toward the phase plane students should receive prior to starting the exercise.

Problems with the Simple Model

A final task that the instructor can assign to students in this exercise involves having them point out problems with Strogatz's model. Indeed, although it is useful for illustrating many system dynamics concepts, as a dyadic model of a romantic relationship it leaves much to be desired. Although some prompting by the instructor may be necessary, generally students will notice things such as Romeo continuing to increase his level of affection toward love, even after Juliet's affection passes into a state of hate (approximately periods 5 to 7 in Figure 4) -- a reaction that would not necessarily be exhibited by real people. Criticisms such as this can be the source of lively classroom discussion and help students build model conceptualization and critiquing skills. Moreover, critiquing Strogatz's simple model is good preparation for Exercise #2.

Strogatz's General Model

If one pushes past Peterson's article and examines Strogatz's original piece, one finds that he offers a second, more general, model of dyadic relationships [2]. Analogous to Exercise #1 then, students can be asked to analyze this model with the tools and techniques of system dynamics

$$dr/dt = a_{11} \cdot r + a_{12} \cdot j$$
$$dj/dt = a_{21} \cdot r + a_{22} \cdot j$$

where: $r(t)$ = Romeo's love/hate for Juliet at time t
$j(t)$ = Juliet's love/hate for Romeo at time t [2]

Exercise #2

According to Strogatz, much of the fun in analyzing [2] comes from the specification of its parameters. That is, the parameters a_{ik} (i,k = 1,2) can be either positive or negative, and their signs determine the "romantic style" of each participant. Thus (a_{11}, $a_{12} > 0$) would "characterize an 'eager-beaver'" or someone stimulated by both his/her partner's love and his/her own affectionate feelings, and ($a_{21} > 0$, $a_{22} < 0$) would characterize a "cautious lover" or someone excited by his/her partner's love but frightened by his/her own feelings. Strogatz recommends that students be asked to name the two other possible romantic styles (i.e., a_{11}, $a_{12} < 0$; and $a_{21} < 0$, $a_{22} > 0$), and provide "romantic forecasts" for various pairings of styles. Indeed, in terms of the latter task, he poses the question of whether "a cautious lover...[can] find true love with an eager-beaver."

Answering Strogatz's Question

As in the previous exercise, the first step students must take to answer Strogatz's question and analyze his more general model, is to translate [2] into a system dynamics language. Figure 7 shows [2] translated into a STELLA model and Figure 8 lists the corresponding STELLA equations. Inspection of Figure 8 shows that the model has been parameterized to represent the relationship between an eager-beaver (Juliet) and a cautious lover (Romeo).

For the reasons outlined above, it is recommended that the instructor ask students to draw-out the causal loop diagram that corresponds to Figures 7 and 8. Such a diagram is presented in Figure 9. Inspection of this figure reveals that pairing an eager-beaver with a cautious lover yields a feedback structure consisting of a major positive loop, a minor positive loop, and a minor negative loop. In terms of the model's parameters, a_{11} controls the strength of the (Juliet's) minor negative loop, a_{22} controls the strength of the (Romeo's) minor positive loop, and a_{12} and a_{21} jointly control the strength of the major positive loop. Moreover, the sign of each parameter determines, either jointly or individually, the polarities of the loops. Clearly then, a causal loop diagram of [2] can serve as a vehicle for illustrating the difference between minor and major feedback loops and as a backdrop for an analysis of the possible affects of the former on the latter. Intuitively, students must be counseled to realize that these issues are intertwined with the issue of parameter selection and hence, with the specification of romantic styles.

Figure 10 presents a time series plot of the interactions between an eager-beaver and a cautious lover. Inspection of the figure reveals that, given the relative strengths of the loops, the answer to Strogatz's question is that it *is* possible for an eager-beaver and a cautious lover to find true love. Here again, it is recommended that students be asked to describe, either in written or verbal form, why this is so -- i.e., why Romeo's love and Juliet's love *both* grow exponentially. Essentially, students must recognize that although Romeo's minor negative loop (caused by the fear of his own feelings) acts as a drag on the growth of his love, it is not strong enough to override the effects of the major and minor positive loops. Students often find this result curious if all of the model's parameters are of equal magnitude (in absolute value), as in the present case.

Other Romantic Styles

As one might imagine, Strogatz's general model can be used for numerous tasks beyond the analysis of the "cautious lover/eager-beaver case." For example, the romantic styles left undefined by Strogatz can be defined, incorporated into [2], and simulated.

Figure 11 shows the equations, and Figure 12 the corresponding causal loop diagram, for [2] after it has been parameterized to represent an eager-beaver (Romeo) paired with a "Cyrano de Bergerac" (Juliet) -- i.e., a person that is stimulated by his/her own private feelings but repelled by the more public attention given by his/her object of desire ($a_{21} < 0$, $a_{22} > 0$). Clearly this combination yields a feedback structure consisting of a major negative loop and two minor positive loops. The instructor can use this structure to illustrate the well-known system dynamics heuristic that positive feedback loops tend to exacerbate the instability generated by negative feedback loops containing delayed corrective action.

The ever-increasing instability generated by pairing an eager-beaver with a Cyrano de Bergerac can be seen by inspecting the time series plot presented in Figure 13. In this case students should recognize that the positive loops continually give the system's oscillatory tendencies "kicks" or "bursts of energy." The instructor can drive home this point by having them increase the strength of the positive loops (i.e., the values of a_{11} and a_{22}) and re-simulate the model. One caveat, however, is that students will sometimes attribute the explosive behavior of Figure 13 to integration error, rather than to system structure. The instructor should make sure, therefore, that the topic of integration error-generated oscillations versus structurally-generated oscillations gets discussed before or after the completion of the exercise.

Analogous to Figures 11 and 12, Figure 14 presents the equations and Figure 15 the causal loop diagram for [2], after it has been parameterized to represent a cautious lover paired with a "cognitive dissonant" (a_{21}, $a_{22} < 0$). Here, a "cognitive dissonant" is person who is basically fickle and moves his/her emotions in a direction opposite to his/her lover's, but who also has inner feelings that slow down and work against the fickleness. In this case, as shown in Figure 15, the combination yields a feedback structure consisting of a major and two minor negative loops. This structure is useful for illustrating some ideas from control theory.

The main technical insight students can draw from the "cautious lover-cognitive dissonant structure" is that the minor negative loops dampen or "control" the oscillations generated by the major negative loop. This can be seen in time series plot presented in Figure 16. In terms of the romantic relationship, the important insight is that, after starting in a state of mutual love and fluctuating between states of love and hate, Romeo and Juliet end up neutral towards one another -- i.e., in a state of equilibrium or stability. Of note is that this result is transferable to many dyads, whether they consist of interacting people, firms, nations, species, etc.

Problems with the General Model

As in Exercise #1, asking students to critique Strogatz's general model can help them develop additional system dynamics skills. In the case of the cautious lover-Cyrano de Bergerac pairing (Figures 11-13), for example, students should realize that the explosive behavior generated by the model is not sustainable in any real system. Moreover, via prompting by the instructor, students should be able to determine that some limits need to be added to the model to "rein it in" and make it more realistic. Strogatz in fact suggests that the instructor ask students to add nonlinear terms to [2] "to prevent the possibilities of unbounded passion or disdain." Given the emphasis on nonlinearity in the system dynamics paradigm, and the existence of software that makes it easy to test the dynamic effects of new structures, such an assignment would certainly be reasonable. It could also serve as a good introduction to the study of limit cycles.

Additional Twists on the Exercises

Transferability of Structure

In the original newspaper account (Figure 1), Peterson recounts his father's advice to substitute the Hatfields and the McCoys for Romeo and Juliet to help [1] make greater sense. The instructor can use this statement for two purposes in these exercises. The first is as a tool for introducing students to the concept of generic structures -- i.e., identical feedback structures that arise in different systems. In fact, as luck would have it, the interactions between the Hatfields and McCoys is one of the examples used by Richardson (1986, p. 167) in his discussion of problems with causal loop diagrams.[4]

The second use of the senior Peterson's advice is to have students analyze whether or not it makes sense -- i.e., whether substituting the Hatfields and McCoys for Romeo and Juliet in [1] *really* yields a "correct" model of a feud. Indeed, as Richardson (1986) points out, a simple, one loop conceptualization of the interactions between the Hatfields and McCoys is an oversimplification and can lead to problems in defining loop polarity and predicting dynamical behavior. An interesting classroom discussion therefore can arise from asking students to define a "correct" feedback structure for a feud and then interpret it in terms of the Romeo and Juliet story.

As usual, boy + girl = confusion

A Harvard University mathematician has devised a teaching plan that, as he puts it, "relates mathematics to a topic that's already on the minds of many college students: the time-evolution of a love affair between two people."

Harvard's Steven H. Strogatz described the plan in Mathematics magazine under the sexy title "Love Affairs and Differential Equations."

He bases his ill-fated love affair on the story of Romeo and Juliet, except that it's not their families that keep them apart—it's Romeo's fickleness.

The more Juliet loves him, the more Romeo begins to dislike her. But when Juliet loses interest, his feelings for her warm up. She, on the other hand, tends to echo him. Her love grows when he loves her, and turns to hate when he hates her.

According to Strogatz: "A simple model for their ill-fated romance is

$$dr/dt = -aj, \qquad dj/dt = br,$$

where

$r(t)$ = Romeo's love/hate for Juliet at time t

$j(t)$ = Juliet's love/hate for Romeo at time t."

It's important to know that the "positive values r, j signify love; negative values signify hate." It also helps to know that "the parameters a,b are positive, to be consistent with the story."

Or so says Strogatz, who goes on to note that "the sad outcome of their affair, of course, is a never-ending cycle of love and hate; their governing equations are those of a simple harmonic oscillator."

The news is not all bad. According to the equation, the harmonically oscillating Romeo and Juliet "manage to achieve simultaneous love one-quarter of the time."

I telephoned my father, 29 years retired from the math game, to ask if he had any suggestions for making the Strogatz formulations even clearer.

Said he: "I'm mostly into gardening now, son, but my guess is that, lacking a Harvard education, you're probably intimidated by the reference to the Shakespearean tragedy. Substitute the Hatfields and the McCoys, and see if that doesn't help."

Clarence Petersen

Figure 1: Clarence Peterson's Account of Steven Strogatz's Article: Love Affairs and Differential Equations.

Optional Mathematical Rigor

Strogatz's models can be used for additional study tasks that involve the traditional tools and techniques of mathematical dynamics. Due to their relatively advanced nature, they are conceived here as optional additions to Exercises 1 and 2.

A number of points about dynamical systems can be revealed to students by having them solve [1] and [2] analytically. These solutions can then be used to: 1) drive home the distinction between simulated and analytical solutions; 2) show how the behavior of a linear system is merely the sum of the behaviors of its parts (this can be contrasted with the behavior of a nonlinear system); and 3) show precisely the parameters that control a linear system's amplitude and periodicity.

Other topics that might be linked to Exercises 1 and 2 include: 1) the calculation of feedback loop gains in the models; 2) the definition and use of integral and other methods of control in Strogatz's general model; 3) the definition and calculation of the dominant loop polarity in each model (see Richardson 1984); and 4) the calculation of the eigenvalues and eigenvectors for each model and the subsequent relation of them to system behavior.

Conclusion

The field of system dynamics emphasizes the intuitive understanding of the mathematics underlying dynamical systems. This paper has offered two exercises that may be useful in helping students develop their dynamic intuition.

Endnotes

1. An electronically scanned version of this article is available from the author.
2. Including many who *have* had courses in calculus, differential equations, and control theory.
3. Strogatz (1988) does not either.
4. Another good example of the transferability of structure involves a comparison of a simple arms race model (e.g., Forrester 1985) and [2], parameterized to represent two cautious lovers. Both of these structures consist of a major positive loop and two minor negative loops.

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Richardson, G. P. 1984. Loop Dominance, Loop Polarity, and the Concept of Dominant Polarity. Proceedings of the 1984 System Dynamics Conference. Oslo, Norway.

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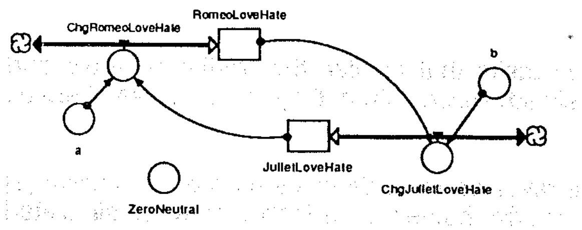

Figure 2: STELLA Representation of Strogatz's Simple Model

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JulietLoveHate(t) = JulietLoveHate(t - dt) + (ChgJulietLoveHate) * dt
INIT JulietLoveHate = 1 {Love-Hate Unit}
ChgJulietLoveHate = b*RomeoLoveHate {Love-Hate Units/Time}
RomeoLoveHate(t) = RomeoLoveHate(t - dt) + (ChgRomeoLoveHate) * dt
INIT RomeoLoveHate = 1 {Love-Hate Unit}
ChgRomeoLoveHate = -a*JulietLoveHate {Love-Hate Units/Time}
a = .5 {%/Time}
b = .5 {%/Time}
ZeroNeutral = 0 {Love-Hate Units}
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Figure 3: STELLA Equations for Strogatz's Simple Model

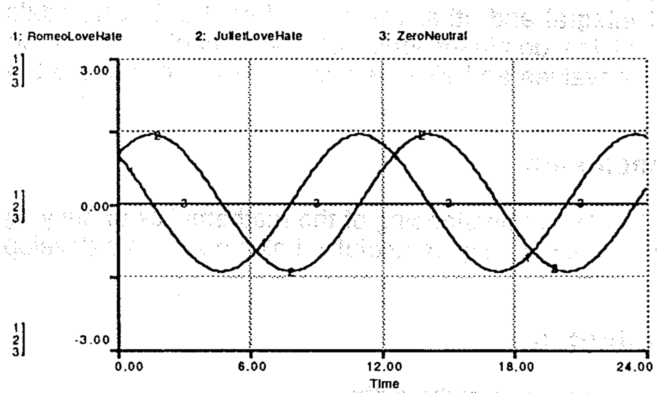

Figure 4: Time Series Plot of Romeo and Juliet's Affections (Strogatz's Simple Model)

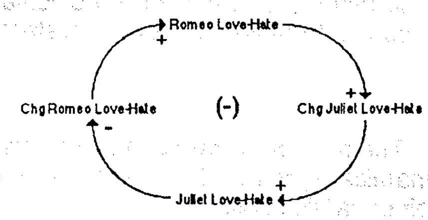

Figure 5: Causal Loop Diagram of Strogatz's Simple Model

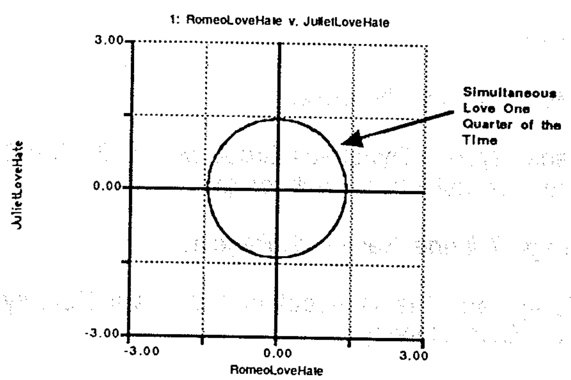

Figure 6: Phase Plot of Romeo and Juliet's Emotions (Strogatz's Simple Model)

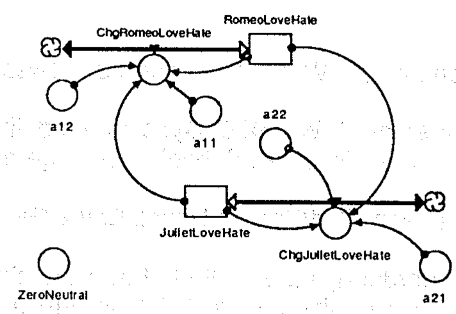

Figure 7: STELLA Representation of Strogatz's General Model

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JulietLoveHate(t) = JulietLoveHate(t - dt) + (ChgJulietLoveHate) * dt
INIT JulietLoveHate = 1 {Love-Hate Unit}
ChgJulietLoveHate = a21*RomeoLoveHate + a22*JulietLoveHate {Love-Hate Units/Time}
RomeoLoveHate(t) = RomeoLoveHate(t - dt) + (ChgRomeoLoveHate) * dt
INIT RomeoLoveHate = 1 {Love-Hate Unit}
ChgRomeoLoveHate = a11*RomeoLoveHate + a12*JulietLoveHate {Love-Hate Units/Time}
a11 = -.1 {%/Time}
a12 = .1 {%/Time}
a21 = .1 {%/Time}
a22 = .1 {%/Time}
ZeroNeutral = 0 {Love-Hate Units}
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Figure 8: STELLA Equations for Strogatz's General Model (Eager-Beaver Paired With Cautious Lover)

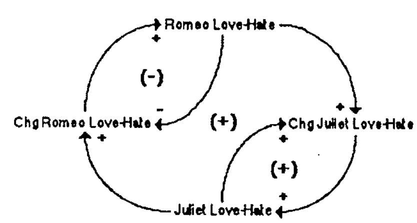

Figure 9: Causal Loop Diagram for Strogatz's General Model (Eager-Beaver Paired With Cautious Lover)

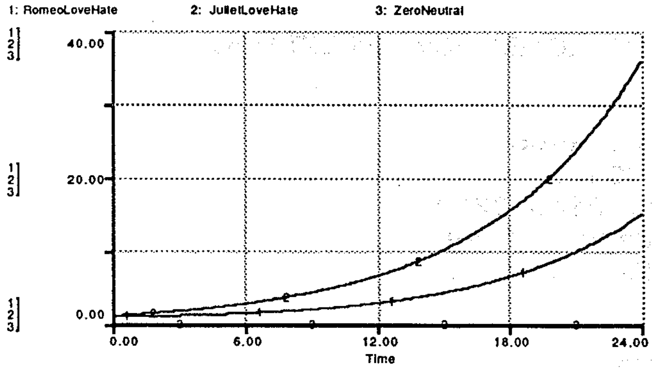

Figure 10: Time Series Plot of Romeo and Juliet's Emotions (Eager-Beaver Paired With Cautious Lover)

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JulietLoveHate(t) = JulietLoveHate(t - dt) + (ChgJulietLoveHate) * dt
INIT JulietLoveHate = 1 {Love-Hate Unit}
ChgJulietLoveHate = a21*RomeoLoveHate + a22*JulietLoveHate {Love-Hate Units/Time}
RomeoLoveHate(t) = RomeoLoveHate(t - dt) + (ChgRomeoLoveHate) * dt
INIT RomeoLoveHate = 1 {Love-Hate Unit}
ChgRomeoLoveHate = a11*RomeoLoveHate + a12*JulietLoveHate {Love-Hate Units/Time}
a11 = .1 {%/Time}
a12 = .5 {%/Time}
a21 = -.5 {%/Time}
a22 = .1 {%/Time}
ZeroNeutral = 0 {Love-Hate Units}
```

Figure 11: STELLA Equations for Strogatz's General Model (Eager-Beaver Paired With Cyrano de Bergerac)

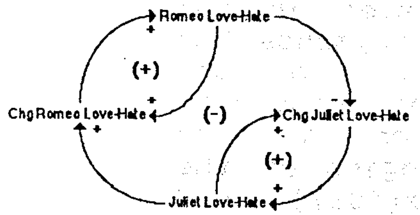

Figure 12: Causal Loop Diagram for Strogatz's General Model (Eager-Beaver Paired With Cyrano de Bergerac)

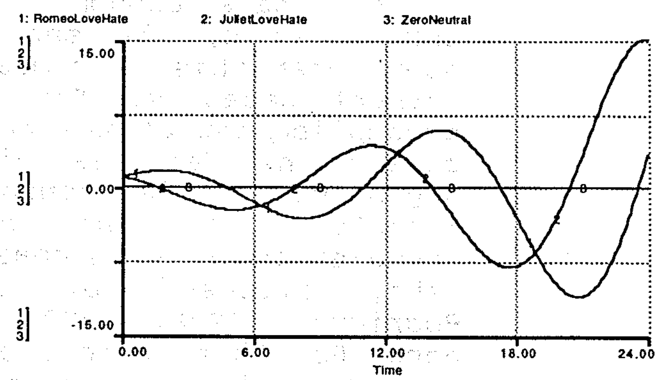

Figure 13: Time Series Plot of Romeo and Juliet's Emotions (Eager-Beaver Paired With Cyrano de Bergerac)

```
JulietLoveHate(t) = JulietLoveHate(t - dt) + (ChgJulietLoveHate) * dt
INIT JulietLoveHate = 1 {Love-Hate Unit}
ChgJulietLoveHate = a21*RomeoLoveHate + a22*JulietLoveHate {Love-Hate Units/Time}
RomeoLoveHate(t) = RomeoLoveHate(t - dt) + (ChgRomeoLoveHate) * dt
INIT RomeoLoveHate = 1 {Love-Hate Unit}
ChgRomeoLoveHate = a11*RomeoLoveHate + a12*JulietLoveHate {Love-Hate Units/Time}
a11 = -.1 {%/Time}
a12 = .5 {%/Time}
a21 = -.5 {%/Time}
a22 = -.1 {%/Time}
ZeroNeutral = 0 {Love-Hate Units}
```

Figure 14: STELLA Equations for Strogatz's General Model (Cautious Lover Paired With Cognitive Dissonant)

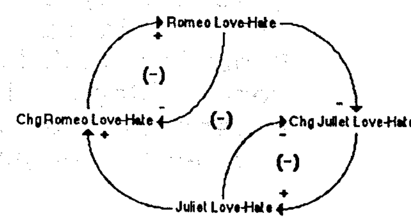

Figure 15: Causal Loop Diagram for Strogatz's General Model (Cautious Lover Paired With Cognitive Dissonant)

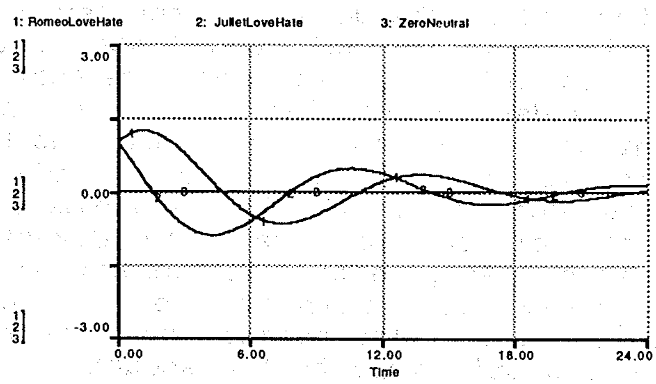

Figure 16: Time Series Plot of Romeo and Juliet's Emotions (Cautious Lover Paired With Cognitive Dissonant)