The Structure-Behavior Diagram:

Understanding the relationship between structure and behavior in complex dynamic systems.

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ABSTRACT

One of the major challenges in system dynamics, and a prerequisite for successful policy design, is to establish an understanding of the relationship between the structure and the behavior of complex, dynamic systems. In this paper, we propose the utilization of a combination of a phase- and a time-plot, called a structure-behavior diagram, to obtain such an understanding. To demonstrate its usefulness, we apply the diagram in order to explain the transient behavior of a simple ecological system and illustrate the dynamics of its modes of behavior under changing harvesting intensity. Our discussion focuses on equilibria and shifts in the dominant polarity and indicates a qualitative difference between the two with respect to resistance to bifurcations.

THE PROBLEM: THE RELATIONSHIP BETWEEN STRUCTURE AND BEHAVIOR

System dynamics can be considered a methodology that bridges the gap between an understanding of the structure and an understanding of the behavior of dynamic systems. As such, it is being extensively utilized in education, research, and management. The method helps us identify the modes of behavior of a system, based upon an investigation of the underlying structure, and facilitates the identification of system structure, based upon its behavior characteristics. Understanding how structure relates to behavior, is a fundamental prerequisite for policy design, i.e. the redesign of structure to modify purposely the systems behavior. Most people do not possess the mathematical qualifications required to carry out a formal analysis of other than trivial dynamic systems - let alone interpret the result and its practical implications. Traditionally, there has been an extensive utilization of graphical forms of representation, as well as of simulation, in system dynamics. This has enabled more people to describe and analyze a wide range of complex models, based upon an intuitive systems understanding.

A fundamental barrier against learning systems thinking is the abyss existing between a structure description that captures the static aspect of the system and a behavior description that represents its dynamic aspect. The transition between the two constitutes a non-trivial cognitive leap. Typically, therefore, most basic system dynamics courses incorporate an element of graphical integration, to enhance our understanding of the most generic process that links structure to behavior in dynamic systems - the integration process.

A series of additional systems properties constitute hurdles we must overcome in order understand complex systems and their behavior. Among the most essential ones are feedback, delays, and non-linearities. Over the last couple of years, we have become increasingly aware of the fact that relatively simple, deterministic, continuous systems may exhibit surprisingly complex modes of behavior (May, 1976) (System Dynamics Review, 1988). Therefore, it has become an educational challenge to establish an understanding among mathematicians as well as non-mathematicians of the relationship between structure and behavior in such systems.

In doing so, we should make use of the diagrams that have actually proved effective in mathematics. Moreover, it is important that software, commonly applied by system dynamicists, offer a variety of user-friendly ways to generate these kinds of diagrams. Finally, we must develop a technique for making use of these diagrams for educational purposes. The most common diagram applied in mathematical studies of differential equation systems, is the phase diagram that portrays the relationship between state variables (Arnold, 1973). A phase diagram captures the structure of the system. Consequently, it matches well a formal, mathematical systems description. From a phase diagram, the dynamic characteristics of a system, such as its critical points, can be identified. Usually, however, mathematical textbooks seldom synthesize such structural descriptions with a description of the dynamic solution, so as to make the behavior consequences of the underlying structure transparent. At most, separate descriptions of the two systems aspects may be offered as a description of certain qualities of dynamic systems (Edwards et. al., 1989, chapter 7) - not as a way to illustrate the curriculum of differential equations in general.

This paper presents a class of structure-behavior diagrams, designed to contribute to the kind of qualitative mathematical understanding we have been advocating. The diagrams, exhibited in their generic version in figure 1, provide a combined description of the structure and the behavior of systems. The educational potential of this form of presentation is illustrated using the diagram to portray the first order, non-linear ecological model, shown in figure 2 (obtainable from the author), - one that represents a population subject to harvesting. In spite of its structural simplicity, the model exhibits surprisingly complex behavior modes, characterized by shifts in the dominant feedback loop polarity and bifurcations that result in a non-trivial behavior. For a formal treatment of these concepts, see (Richardson, 1984, 1986).

The structure, however, is general enough to encompass a wide variety of systems, focussing on the possible depletion of a renewable resource, such as that of renewable energy resources, the resource of natural atmospheric purifiers and the genetic material of the rain forest. Predator-prey systems are also of this kind (Forrester, 1969) (Schwartz, 1990). They are of particular importance because of the interdependencies among various species: The erosion of one resource may effect a series of species in a food-chain. The economic effects may be devastating, moreover, if one or more of these species constitute an industrial resource-base, exemplified by fisheries of various kinds, as illustrated by The Fish Bank Game (Meadows, 1990). If there is no coordination in the utilization of interdependent resources, industries may erode each other's resource base. In spite of this fact, very few multi-species models have been developed for such industries. Partly this is true because of the inherent complexities of these systems. The graphical technique, described in this paper, is intended to facilitate an understanding of these kinds of interdependencies.

It also helps us understand the draining of renewable resources, recognized in personal and organizational development. The dynamics of personal energy and the prevention and control of worker burn-out (Homer, 1985) is illustrated by the MIT tooling game (Sterman, 1984). The need for a critical mass and the threat represented by head-hunters, and break-outs leading to herd migration may be addressed using the same kind of models.

Skilled mathematicians may find this paper trivial. Most of the technical, mathematical terminology has been left out because our very purpose is to demonstrate how mathematical intuition can be built without making use of formalisms. Moreover, this paper has been written to encourage others to use system dynamics and simulation as an exploratory tool in learning and teaching.

To summarize, this paper is distinguished from traditional mathematical descriptions of dynamic systems in three ways: First of all, it combines intimately structure and behavior diagrams. Secondly, the structure diagrams utilized are not ordinary phase-diagrams exhibiting the relationship between the level (state) variables of the system, but one in which the net rate (differential) of each level is plotted against its level. Finally, we treat shifts in polarity dominance as critical points along with equilibria. The mathematical background for these decisions can be found in (Richardson, 1984, 1986).

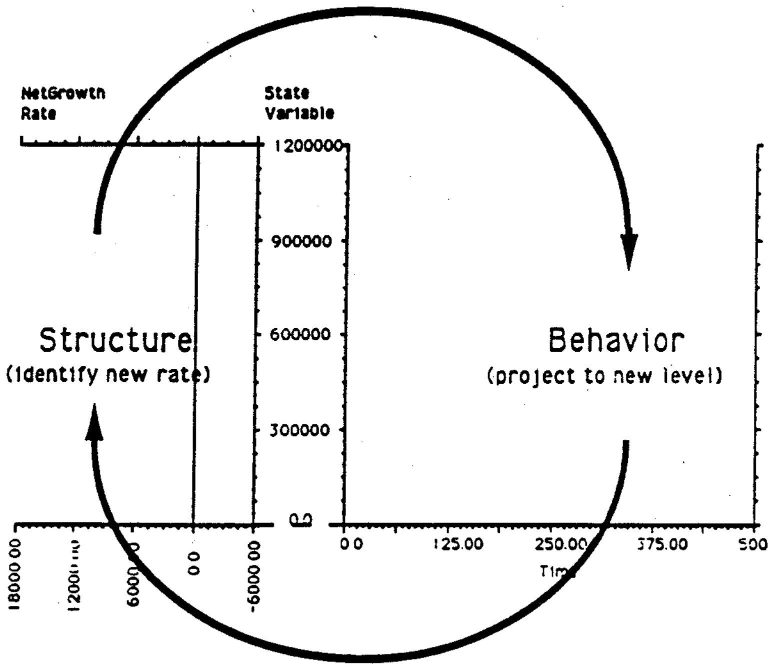

Figure 1: The structure-behavior diagram

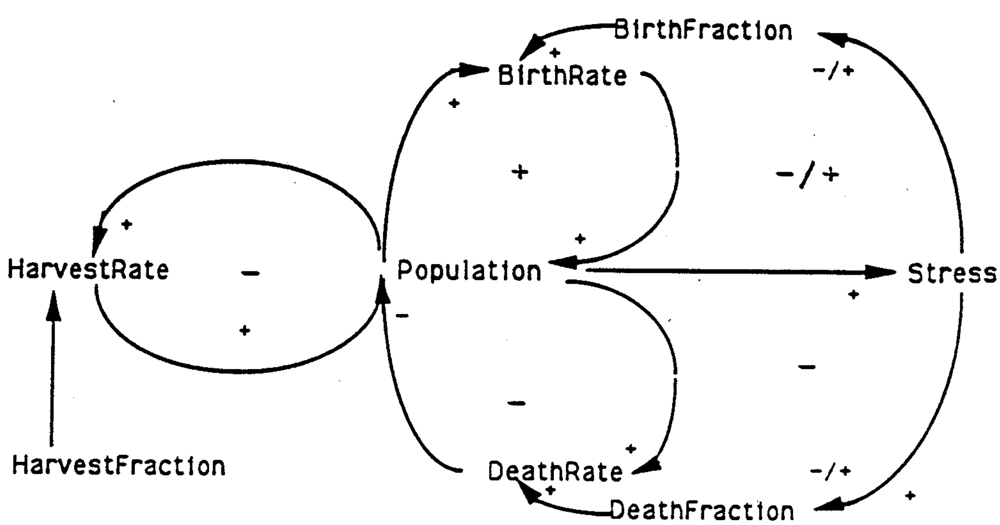

Figure 2: An ecological model

A POPULATION SUBJECT TO HARVESTING

Our model portrays a population subject to harvesting. The variable BirthRate is determined by Population through a positive feedback loop who´s strength is determined by the BirthFraction. The variable DeathRate is determined by Population through a negative feedback loop who´s strength is determined by the DeathFraction. As the population increases towards the finite carrying capacity of its environment, stress builds up and causes the birth- and death-fractions to be non-linearly modified. Correspondingly, the relative strength of each loop changes, potentially causing shifts in the dominant loop-polarity. In figure 3, we portray the structural relationship between the (size of the) population on the one hand and the rates of change of that population under various harvesting intensities on the other hand. Figure 3a exhibits the non-linear natural birth and death rates for various population levels, resulting from the influence of stress, along with the net natural growth rate. Note the initial S-shaped birth rate.

In each of the figures 3b - e, we have applied a constant harvesting intensity, i.e. we harvest proportionally to the size of the population, whereby the net growth rate is being lowered from its natural trajectory. In figure 3b the harvesting intensity is moderate (0.0095), causing the net growth rate to be positive across a relatively wide range of population values. In figure 3c, harvesting is so intense (0.0136) that the net growth rate is zero for a single population value, else negative. In figure 3 d, harvesting intensity has been slightly increased (to 0.014) so as to lift harvesting rate above the natural growth rate for all values, resulting in excess harvesting across all population values. In figure 3e, the intensity has been increased substantially (to 0.019) so as to cause an extensive excess harvesting.

As demonstrated in figure 1, and focussing on the resulting net growth rates, including harvesting, we are now ready to combine the phase diagrams of figure 3 with the corresponding time plots. The results, portrayed in figure 4, provide a means by which we can enhance our understanding of the intimate relationship between structure and behavior in this system. Note that, in general, the net growth rate, described by the phase diagram, determines the slope of the population trajectory in the time plot. At any time, on the other hand, the net growth rate will be determined by the size of the population as described by the phase diagram.

THE STRUCTURE-BEHAVIOR DIAGRAM

The phase diagram enables us to partition the population axis using the maxima, minima, and zeros of the net growth rate. In our example the zeros have been named A, B, and C. We carry this partitioning over into the time plot exhibiting the behavior of the model. The zeros constitute equilibria (where the population remains constant), represented by dotted lines. The size of the population is divided by these equilibria into three different ranges, *I*, *II*, and *III*. Additional dotted lines represent the maxima and minima of the net growth rate constituting shifts in loop polarity dominance. The size of the population is further subdivided by these shifts into sub-ranges a and b. Our model exhibits its richest behavior under moderate harvesting intensity, figure 4 b. A brief discussion of this behavior will constitute our point of departure for the story told in the next section of this paper:

When the size of the population lies above the carrying capacity of its environment, its mortality will rise above the fertility, and the population will decrease. Due to the resulting stress reduction, it will do so at a diminishing rate. Thus, in spite of the increase in birth/death rate ratio, the negative polarity dominates all along. As the population approaches the carrying capacity, net growth approaches zero, and the population approaches an equilibrium, A, below which it does not fall. In equilibrium, there is a balance between birth rate and death rate. In the structure-behavior diagram, this development takes place in the upper *Range I*, characterized by a negative polarity dominance and a lower, stable equilibrium.

If the size of the population is represented by B, clearly no growth is possible and the population remains in equilibrium, B. This is an unstable equilibrium, because a small increase in the population will make it continue to grow. If the population is slightly larger than B, the net growth rate will be increasing. Thus the positive polarity dominates. As the net growth approaches its maximum, the increase in population growth is attenuated, indicating that the positive polarity looses momentum. In

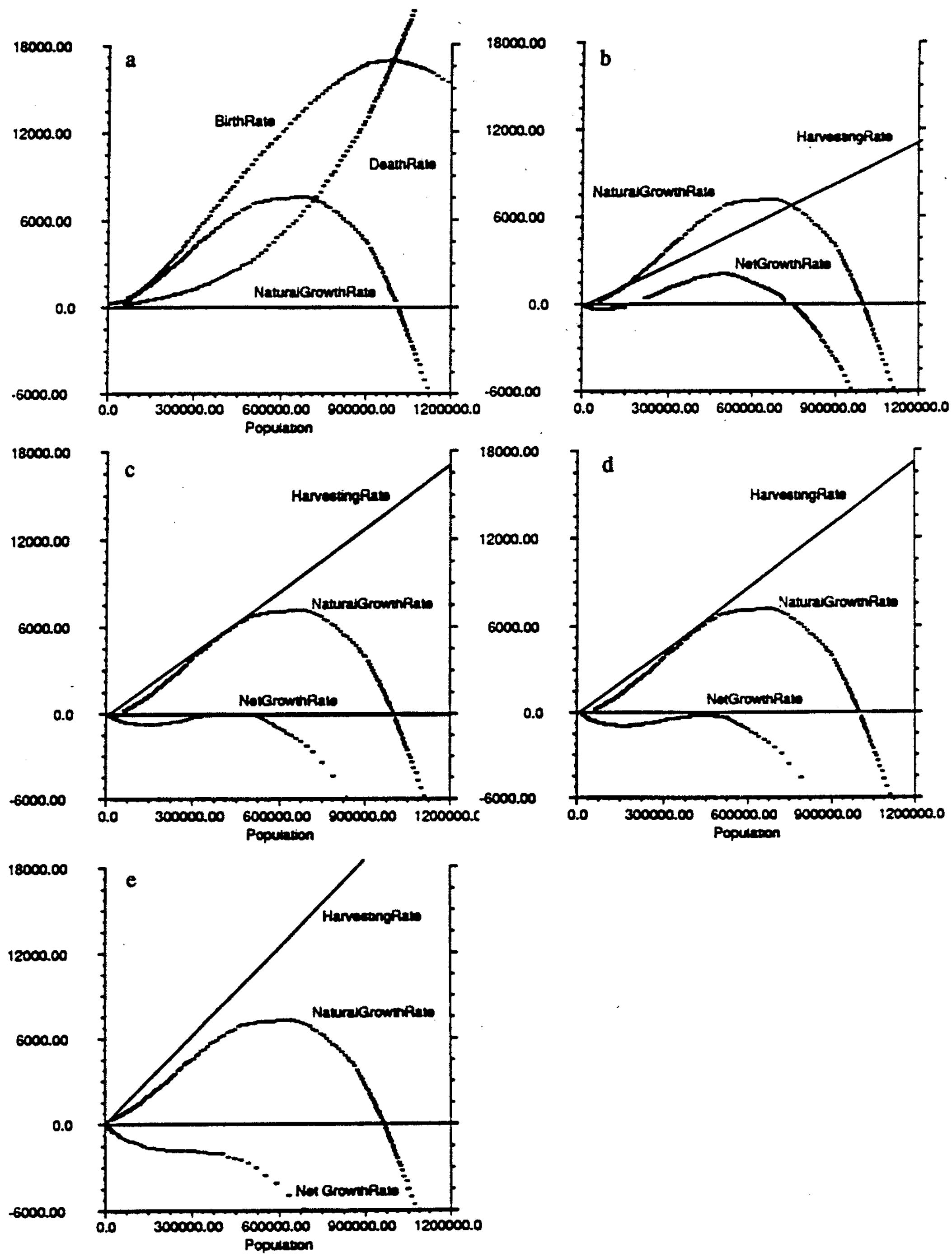

Figure 3: Natural Birth and death rates, Natural growth rate, Net growth rate, and Harvesting rate

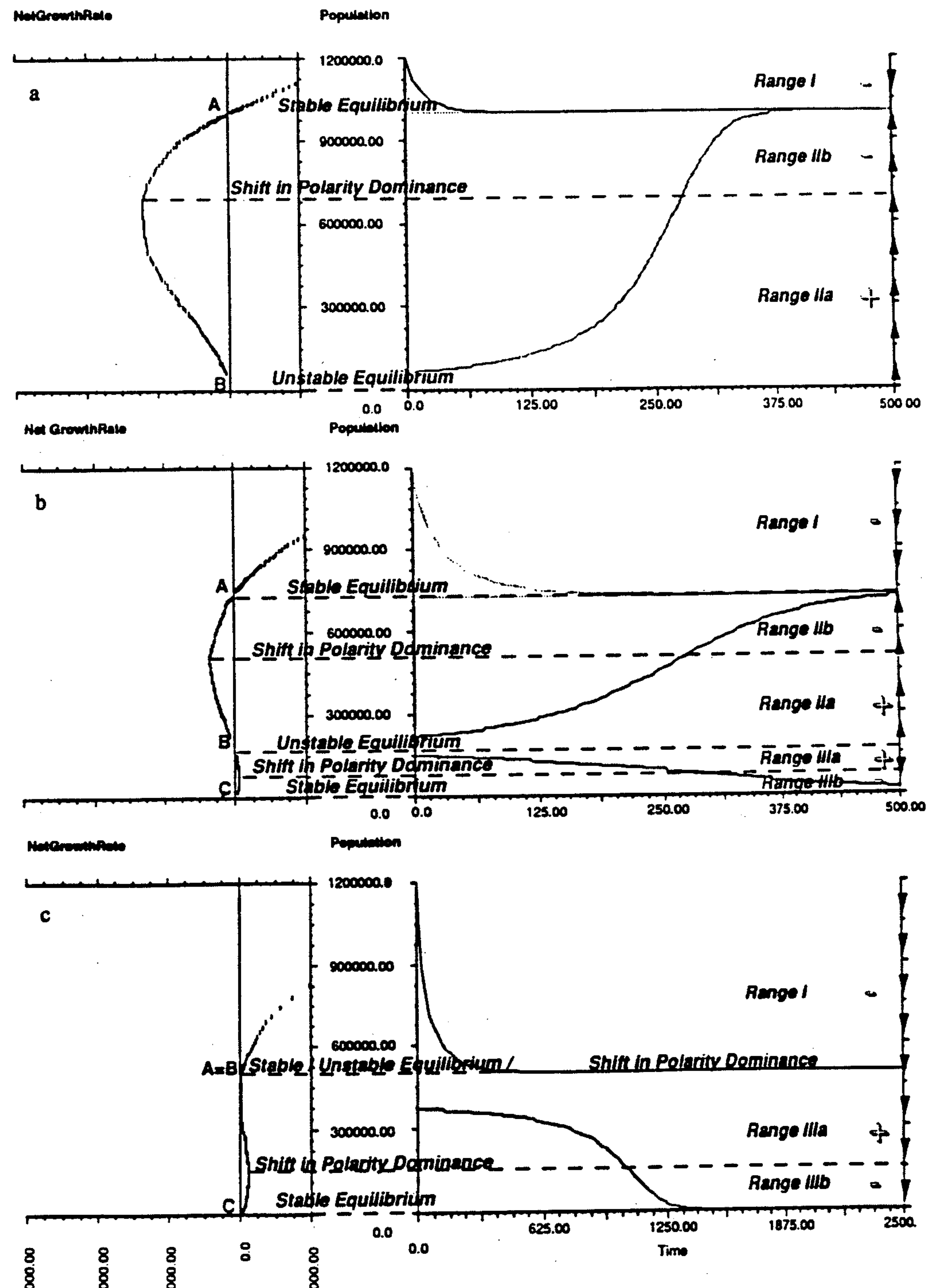

Figure 4: The ecological system portrayed in a structure-behavior diagram

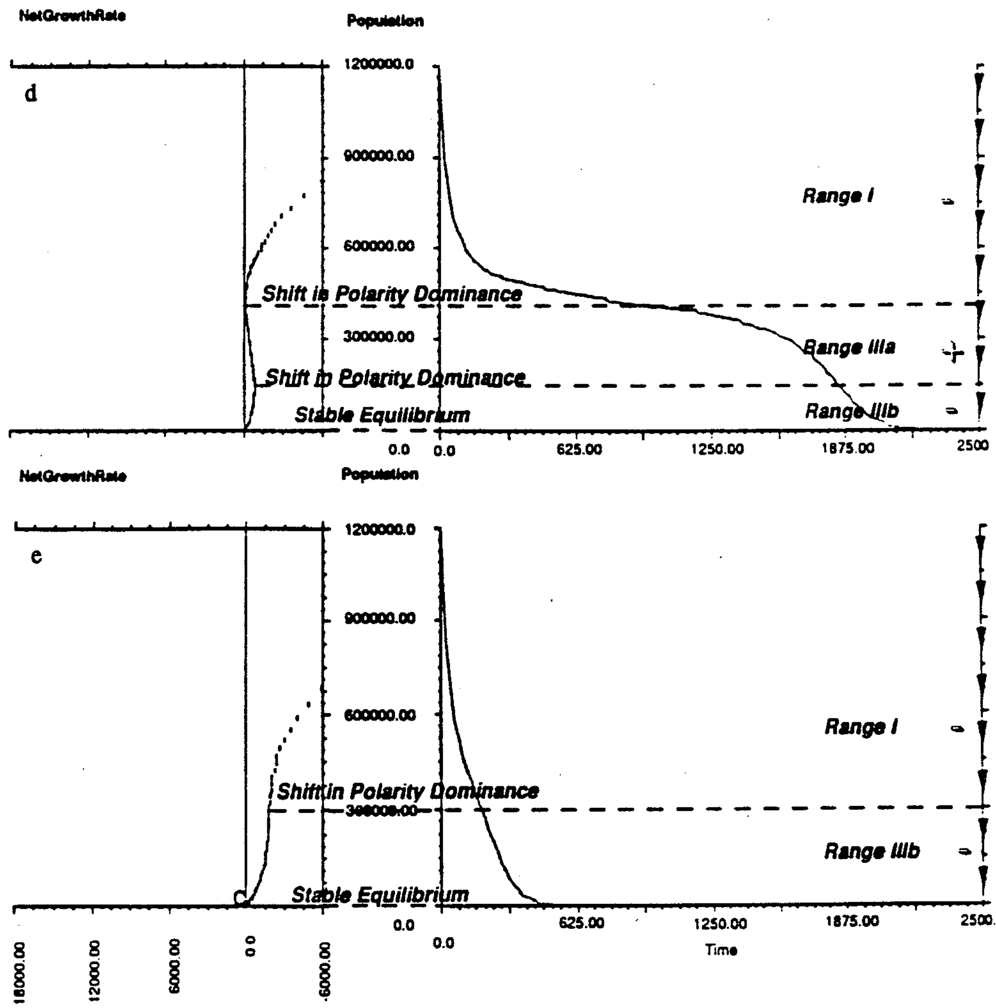

Figure 4 (continued): The ecological system portrayed in a structure-behavior diagram the structure-behavior diagram, this development takes place in the lower *Range IIa*, characterized by a lower, unstable equilibrium, a positive polarity dominance, and an upper shift in polarity dominance.

A point of inflection is reached as the polarity dominance shifts from a positive de-stabilization to a negative stabilization. From then on, the growth rate decreases towards zero. As the growth rate approaches zero, the decrease in growth rate is amplified, indicating that the negative polarity gains momentum. Consequently, the population approaches its stable equilibrium, A, from below. In the structure-behavior diagram, this development takes place in the upper *Range IIb*, characterized by a lower shift in polarity dominance, a negative polarity dominance, and an upper, stable equilibrium.

If the size of the population falls below B, then the net growth rate is negative. As the population decreases, so does the growth rate. The fall in growth rate is attenuated, however, leading the rate to a minimum. Thereafter, the growth rate increases towards zero. As it approaches zero, this increase is being amplified. If the population is extinct, clearly no growth is possible and the population remains in equilibrium C. C is stable because if the population is increased by only a small amount, it will die out. Sustainable growth requires the population to be lifted above its critical size, represented by B.

If the population is relatively small and falling, net growth rate will be negative and decreasing. Thus the positive polarity dominates. As the net growth approaches its minimum, the increase in the erosion of the population is attenuated, indicating that the positive polarity looses momentum. In the structure-behavior diagram, this development takes place in the upper *Range IIIa*, characterized by, an upper unstable equilibrium, a positive polarity dominance, and a lower shift in polarity dominance.

A point of inflection is reached as the polarity dominance shifts from a positive de-stabilization to a negative stabilization. From then on, the growth rate increases towards zero. As the growth rate approaches zero, the increase in growth rate is amplified, indicating that the negative polarity gains momentum. Consequently, the population approaches the state of equilibrium, C, from above. In the structure-behavior diagram, this development takes place in the lower *Range IIIb*, characterized by, an upper shift in polarity dominance, a negative polarity dominance, and a lower, stable equilibrium.

A STORY TOLD USING THE STRUCTURE-BEHAVIOR DIAGRAM

Suppose, first, that harvesting intensity decreases towards zero, say as a consequence of diminishing demand. In that case, the system will drift towards the situation described in figure 4a: The unstable equilibrium, B, has drifted towards and merged with C, causing an expansion of *Range II*, corresponding to the contraction of *Range I* and the degenerated *Range III*. Zero population has turned into an unstable equilibrium. The non-existing harvesting, represented in figure 4a, constitutes no ecological hazard.

Suppose, on the other hand, that harvesting intensity increases, say as a consequence of profitable yield from moderate harvesting. Gradually, then, the system will drift towards the situation described in figure 4c: The unstable equilibrium, B, has drifted towards and merged with A, causing an expansion of *Range I* and *III* corresponding to the contraction and degeneration of *Range II* into the single point A=B which retains the A-property, constituting a stable equilibrium from above, and the B-property, constituting an unstable equilibrium from below. More importantly, note that A=B also constitutes a shift in polarity dominance, the one corresponding to maximum net growth rate, situated between the two equilibria. The relatively intense harvesting, represented in figure 4c, is potentially hazardous. If the population is above A=B, then it will rapidly approach that equilibrium. A minor disturbance will cause the population to fall through into a positive loop that accelerates the drift towards extinction.

If the harvesting intensity is slightly increased, the situation portrayed in figure 4d will result: The equilibrium A=B has vanished, a bifurcation has occurred, and the population has no support for stabilization, let alone net growth. More importantly, note that the shift in polarity dominance, previously situated between A and B, remains. That is, it survived the bifurcation. It remains to see whether it will be subject to a bifurcation at a later stage in this process. The second shift in polarity dominance, characterized by minimum net growth rate, remains a divider between the sub-ranges *IIIa* and *IIIb*, Considering the development 4a - d, note that if we got used to the prey population stabilizing every time we increased the harvesting intensity, then we would hardly become alarmed by the initial segments of the state-trajectories of figure 4c and 4d, apparently approaching an equilibrium floor, supposedly supported by a growth region (*Range II*). This is partly due to the negative polarity dominance characterizing *Range I* and the smooth transition into a the positive polarity dominance of *Range III*.

If harvesting intensity is increased substantially beyond the situation described in figure 4d, we approach the one described in figure 4e. In that case the net growth rate is not only negative all over, but is monotonically decreasing, flattening out in a single point. In the course of this process, the two shifts in polarity dominance left in figure 4d have converged, squeezing between them the degenerated *Range IIIa*. Here we are at the brink of another bifurcation: Another slight increase in the harvesting intensity will cause the shifts to vanish leaving the systems behavior to be characterized only by the negative polarity dominance of *Range I* and *IIIb* which consequently merge.

SUMMARY AND CONCLUSIONS

The modes of behavior of the ecological system, described in this paper, is dynamically modified as harvesting intensity increases. To the extent that these modifications include bifurcations, they are quite dramatic. In figure 5a we obtain a complete description of these dynamics by plotting the bifurcations as a function of the harvesting intensity. In figure 5b, we have added indicators + and - for the dominant polarities in various regions, along with arrows indicating the direction in which the population moves. In many real cases it is impossible to determine with absolute certainty the size of the population and the trajectories of the bifurcations. In figure 5c we have indicated how the latter uncertainties can be illustrated. Figure 5 can be considered an important tool for policy design and for the development of information systems designed to support such policies. Given an estimate of the population, we are able to find a harvesting intensity that we are confident will allow us to avoid the extinction of this population. Note that, as an important by-product of this discussion, we may tentatively conclude that shifts in dominant polarity tend to be more persistent than equilibria, i.e. they are more resistant to changes in parameter values (e.g. harvesting intensity) that drive the model towards bifurcations. Equilibria may bifurcate even though the corresponding shifts in polarity dominance remain. We have illustrated how, in some cases, the structure-behavior diagram can help us obtain a non-mathematical, nevertheless proper, understanding of the relationship between the structure and behavior of non-linear feedback systems. We have indicated how to illustrate the concepts of equilibria, shifts in polarity dominance, their bifurcations, and their impact on the transient behavior of dynamic systems. We have done so using a very simple example. In order to apply the diagram effectively to more complex models, we need to refine the diagramming technique and implement the technique in a modeling and simulation software environment.

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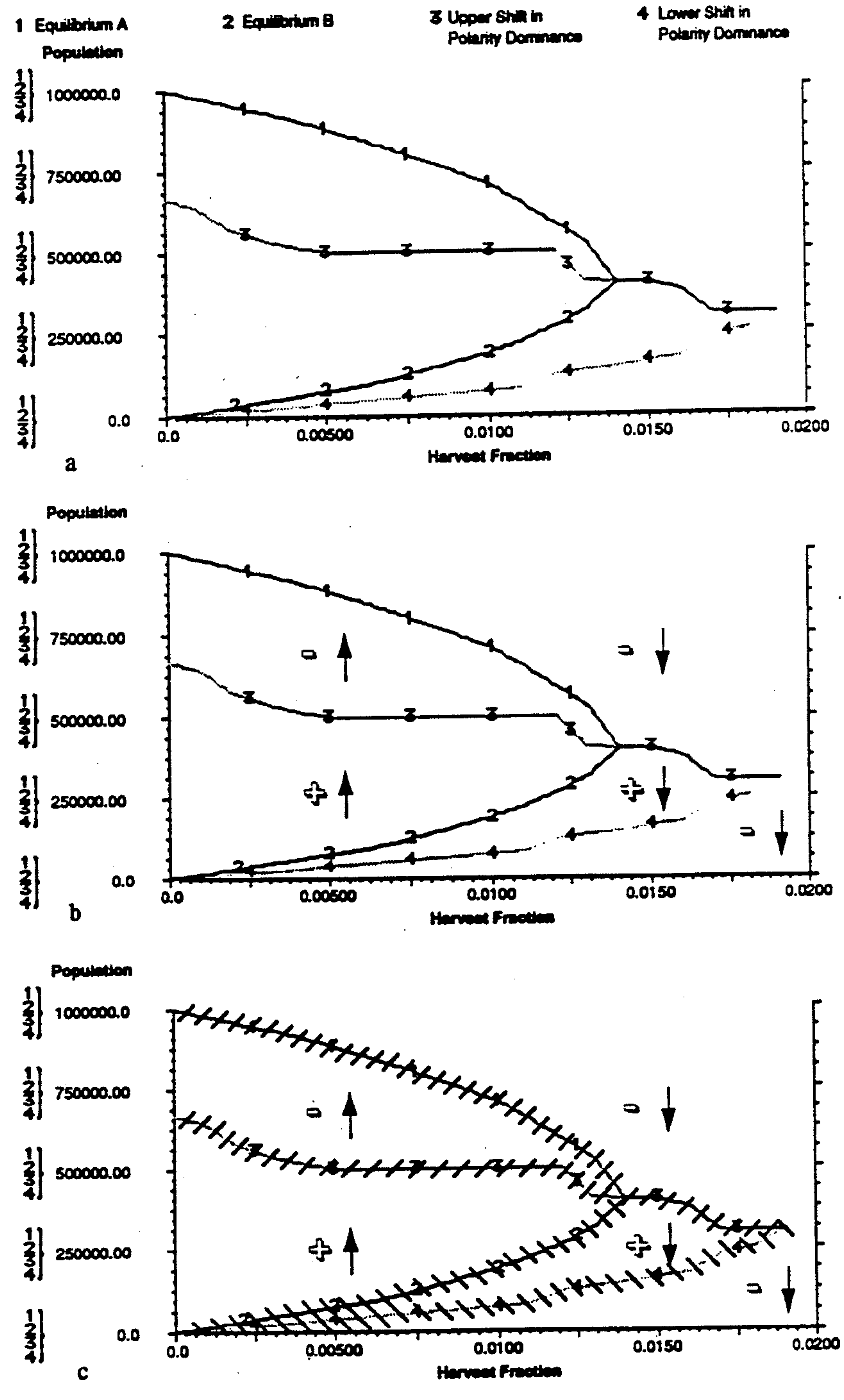

Figure 5: Summaries of bifurcations and regions bounded by bifurcation