CROSS ANALYSIS OF DYNAMIC MODELLING OF AN ECONOMICAL ENERGY SYSTEM AND A DISSIPATIVE PHYSICAL SYSTEM

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ABSTRACT

Actual Dissipative Systems are not workable without energy supply and energy dissipation. Among these systems are economical systems as well physical systems such as fluidized bed. A System Dynamics Analysis of a simple macro-economical system related to the energy sector of a country shows causal loops including energy supplies. This analysis can be applied to a fluidized bed. This system is a classical engineering dynamic system. Due to its complexity it has been highly investigated on the basis of time averaged regimes but a few studies exist about the true dynamical aspect. This is mainly due to the fact that the basic equations are complex. Several length and time scales exist simultaneously. The theoretical equations should comprise the classical hydrodynamic equations for fluid containing solid particles in suspension. The complete set of theoretical equations is for the time being not available. From analogies with the economical systems it is possible to identify causal loops which are not usually considered in the classical modelling of a fluidized bed, i.e interaction with energy supply. Introducing this phenomena and using a very simple equation it is possible to show that space structuration can occur. Then by using a simplified non-linear hydrodynamic equation a chaotic complex behavior, in agreement with experiments, can be simulated. The analysis of the model shows that one of the origin of the difficulties to use the models lies in the existence of several time and space scales. This kind of analysis is helpful to understand fluctuant behavior of many other systems including economical systems.

THE PROBLEM

Many mathematical tools used for economical sciences or human sciences come from exact sciences such as thermodynamics.

Little has been done, up to now, in order to apply tools specifically developed for economical systems, i.e. *"index"*, *"indicators"*, *"causal loop"* to classical physical systems.

This paper shows how a *"System Dynamics"* analysis applied to a macroeconomical system can help to improve the modelling of pure engineering system such as a fluidized bed.

ECONOMICAL MODEL

The simplified economical model is based on macroanalysis of the energy sector of a country.

The energy supplies include: importation, and domestic resources.

Energy is consumed for:

- domestic transportation,
- international exchanges,
- organization of the energy sector,
- internal development,
- well being and goods production.

Uncertainty on energy supply needs to partly control energy importation. This can be done through:

- investment in oil producer countries,
- control of oil or gas transportation.

These two conditions are directly dependent on the development of the country as well as its growth rate.

Internal energy consumption is directly linked to the level of development. This energy consumption can be considered by this fact as an *"indicator"*or *"index"* of the economical activity of the country for the purpose of the present analysis.

Two countries defined as above will have exchanges. These exchanges can be purely economically or energy related, i.e. energy or equipment related to energy.

Exchange capabilities of one country are directly dependant on its internals development. A poor country cannot *"invest"* in many other countries while a rich country can do that. These exchanges can be related to the *"energy indicator"* as defined above.

Regarding the international transportation a developed country is able to afford to get airline companies, this sector consumes energy but at the same time gives a larger geographical influence, i.e. space structuration.

Finally a causal type diagram can be summarized by the figure 1.

The energy consumption is in fact dissipated into heat: engines, heating, so that the energy consumption is fully comparable to the entropy source of classical physical system.

Without energy these systems are not workable: energy is compulsory but of course not sufficient.

Regarding the figure 1, world energy resources are limited so that there is a competition in order to control external energy supplies. Two oil importer countries will have limited possibilities of investment and joint ventures in an oil exporter country. A too high energy demand from one country could be taken out of the possibility for the other.

FLUIDIZED BED

A fluidized bed is composed of solid particles (sand, catalyst, etc.) suspended into an up flow of fluid (gas or liquid). The drag forces counterbalance the gravity force (Figure 3).

The controlling parameter is the fluid flow. At low flow rate the particle, bed is fixed, then it becomes fluidized i.e. particles are suspended in the flow. Bubbles and channels appear at higher flow rate. Finally the particles are transported at very high flow rate.

Fluidization is characterized by the pressure drop. This is the main characteristic of fluidization. The other characteristic is the void fraction, this increases up to one when transportation occurs.

The two parameters fluctuate around average values. The void fraction can be related to the various regimes: fixed bed, bubbling, channelling.

At least four length scales are to be simultaneously considered:

- the molecular scale for the fluid flow,
- the grains scale,
- the bubbles and channels scales,
- the equipment scale.

Writting classical fluid mechanics equations is difficult, even with simplified basic equation. Another difficulty lies on the definition of the border conditions, i.e. fluid distribution, bed height, particles shapes, etc..

For the time being modelling cannot be disconnected from a pure phenomenological analysis.

Most of the experimental results and equations use time and space averaged values but very few can be found for time and space fluctuations.

CAUSAL DIAGRAM

The fluidized bed is considered as series of cells. Each of these cells are characterized by dissipated energy. The regimes: fixed bed, fluidized, bubbling etc ... depends on this energy. One cell interacts with two other cells according to viscosity like interactions. It has been already shown how dissipated energy per unit of time and volume can be consider as an *"activity index"*. Bes (1982),

Figure 2 shows the causal diagramme. This diagramme comprises influences on gas feed, which is normally not considered.

Comparison with the macroeconomical system is as follows:

- In both cases energy dissipation is compulsory, without energy the two systems are not workable.

- Energy dissipated into heat, through particles shocks and viscosity, should be compared to wasted energy.

- The two systems develop an internal organization which is directly linked to energy consumption.

- Energy dissipation is connected to the development of length scales. Domestic and international transportation for the macroeconomical system, bubbles and channels for the fluidized bed.

- A self organization occurs in both systems. The use of a fraction of energy for self organizations permits to decrease energy intensity compared to a non-organized system.

Fluidized bed compared to a fixed bed dissipated less energy for the same fluid flow rate. This behavior can be related to a Curie-Prigogine Thermodynamic principle.

Following the analogy with the macroeconomical system, the fluidized bed should not be considered independently from the fluid supply. Energy supply is not hexogeneous.

BASIC MODELLING

The basic equation for fluidizations are a steady state equation, i.e. force balance.

a) Equipment scale

Pressure force - gravity force = 0 (1)

b) Particle scale

Drag force - gravity force = 0 (2)

which define the minimum fluidization velocity.

Most of the phenomenological equations are derived from steady state balance.

However as fluctuations obviously occur this equation should be the consequence of a non-steady state equation.

The momentum balance is a way to find one of these equations.

The simplest equation is:

$$dv/dt = kV_g - Kv - mg \qquad (3)$$

For minimum of fluidization a true steady state exists: i.e. the particles are at rest so that $v = 0$ and $dv/dt = 0$.

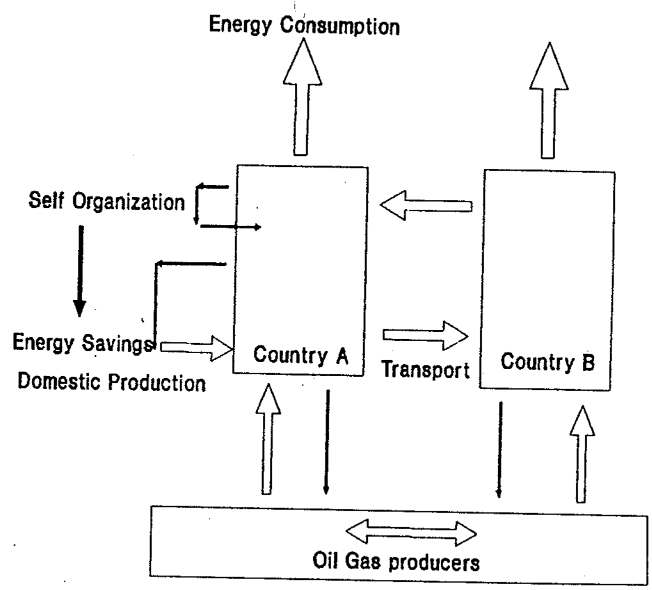
Finally the equation becomes

$$dv/dt = kV - Kv \qquad (4)$$

where V is the excess gas velocity over the minimum of fluidization.

This equation is however unable to represent any fluctuations or space structuration.

The fluidized bed is considered as series of cells with feedback on fluid feed. When the fluid flow increases in a given cell, it decreases in the other consecutive cells. Two consecutive cells could be with the following conditions.



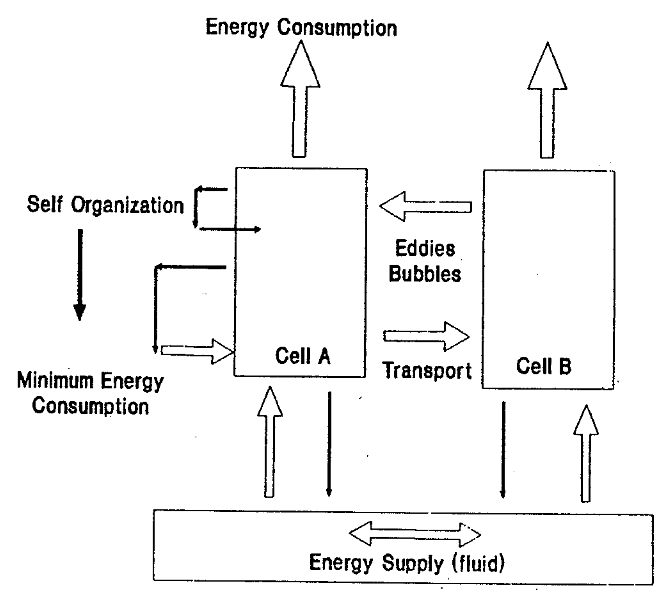


Figure 1 : Economical system

Figure 2 : Fluidized bed

$$V_n + V_{n+1} = V_o \quad (5)$$

$$dv_n/dt = k\,V_n - Kv_n \quad (6)$$

$$dv_{n+1}/dt = k(V_o - V_n) - K\,v_{n+1} \quad (7)$$

These equations are compatible with the two following steady states.

State 1: $V_n = V_o$, $V_{n+1} = 0$

State 2: $v_{n+1} = V_o$, $v_{n+1} = 0$ (8)

These conditions could represent series of alternate channeling zones ($V_n = V_o$) and fixed bed zones ($V_n = 0$). This is the case, for fluidization of sawdust.

This kind of representation is of course very simplified but however better than the initial equation as at least one space organization can be represented.

v_n or v_{n+1} represents the velocity of the particles or clusters of particles. Along time the averaged value should be zero with fluctuations around zero.

The dissipative effect can be seen as resulting from acceleration phases, by the gas, followed by dissipation of kinetic energy by shocks. V_o is linked to the energy supplied to the fluid, the following general equation has been proposed.

$$d\,a_n/dt = k'a_o - k\,a_n \quad (9)$$

where a_n, a_o are the square roots of the dissipated energy per unit of time and volume of fluidized phase. In agreement with a generalized non-steady state reaction - diffusion equation. a_n has been proposed to be a *"dissipative indicator"* or a *"dissipative index"*.

The two following equations have the same steady state.

$$da_n/dt = (k'a_o - k\,a_n) \quad (10)$$

$$da_n/dt = f(a_n)(k'a_o - k\,a_n) \quad (11)$$

but the second one is non-linear and as such could have much more complex behavior especially when this one is used under its time discretized form.

This equation can be put under a logistic form.

$$x(t+1) = g(x)(1-x) \quad (12)$$

The simplest case $g(x) = 4\,ux$ is the most commonly used to illustrate a chaotic numerical process. Hao Bai Lin (1990)

Working only with steady state or time and length averaged values makes the complex behavior hidden from equation point of view in spite of an obviously fluctuant system.

These general considerations open ways to consider discretized equations, or cellular automata, Gabsi (1990), as actual models instead of considering only differential equations to be true models.

MODELLING

A more complex model can be considered using analogy with hydrodynamic Navier_Stokes equation using viscosity. The equation is of the following type:

$$\frac{\partial v}{\partial t} = \left(k\Delta P + \mu \frac{\partial^2 v}{\partial z^2} \right) \tag{13}$$

The adaptation to a fluidized bed will be, CHEHBOUNI (1985)

$$\frac{\partial v}{\partial t} = g(v)\left[k(V-v) + \mu \frac{\partial^2 v}{\partial z^2} \right] \tag{14}$$

In order to consider that the particles are not moving out of the bed, on time averaged basis, only time fluctuating solutions should be considered.

Finally the equation using the dissipative indicator is:

$$\frac{\partial a}{\partial t} = g(a)\left[k(A-a) + \mu \frac{\partial^2 a}{\partial z^2} \right] \tag{15}$$

A simple discretized equation is:

$$[a(t+1)] = K\,a[A_n - a] + Da(a_{n+1} + a_{n-1} - 2a) \tag{16}$$

The simple cases, $D = 0$

or $a_{n+1} = a_{n-1} = A$ and $K = 0$ will give a logistic like function.

Coming back to the basic equations in steady state conditions. k and μ depends on the void fraction, this latter depending on the fluid flow rate which depends itself on the energy supplied to the pump or compressor.

MODEL ANALYSIS

A basic equation can be written under the following way.

$$x(t+1) = x[K\,A_n(1-x) + D(x_n + x_{n+1} - 2x)] \tag{17}$$

Iterative numerical processes permit to calculate "x" profiles versus the rank of cells.

Due to the non linearity complex behavior occurs with the following typical cases:

- At low value of A, there is no fluctuations when A increases fluctuations appear.
- The system is highly dependant on initial conditions, difference of 10^{-8}, destabilized completely the system which can pass from a time converging process to a chaotic process.

"A_n" represent the energy supplied to the system, and has been taken so that

$$\Sigma A_n = NA_o$$

Figure (4) shows example values of "*x*" versus the length position at different time. This emphasizes the structurations and the fact that for some external constraints, "*A*", two consecutive cells are rigidly linked. When "*A*" increases the general trend of one cell, is kept, but with fluctuations, compared to the neighbor cells.

CONCLUSION

An economical system and a physical system could have similar causal diagramme. From this analogy, it is possible to improve, at least qualitatively the model of the physical system in order to better fit with actual behavior.

The use of *"index"* of *"indicator"* permits to get an overview about the trend of the systems. The use of the simplified physical model permits to emphasizes, space and time chaotic structuration which is in good qualitative agreement with actual behavior.

As a consequence the fluidized bed model could lead to some help to understand complex economical behavior which are not necessarily originated in complex equations.

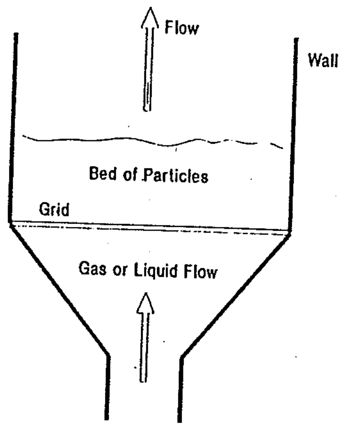


Figure 3 : Fluidized bed principle.

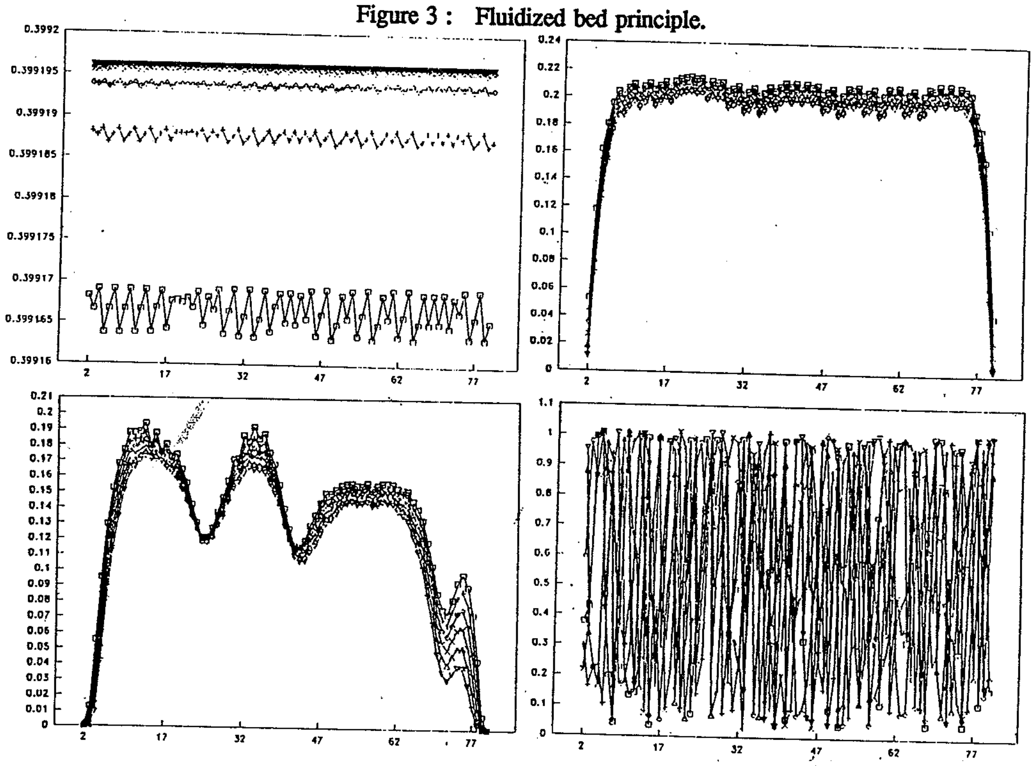


Figure 4 : **Four examples of dynamic simulation**

Houzontal axis : cells rank
Vertical axis : dissipative index

NOTATIONS

a	:	dissipative index, time t
a_n	:	dissipative index, volume n, time t
a_o	:	dissipative index, averaged value
$a_{(t+1)}$	:	dissipative index, time t+1
A	:	dissipative index, fluid
D	:	coefficient (dimension defined by the equation)
$f(a_n)$	:	mathematical function of a_n
g	:	gravity constant, (m/s/s)
$g(a)$	:	function of a
$g(x)$	:	function of x
k, k'	:	coefficient (dimension defined by the equation)
K	:	Coefficient (dimension defined by the equation)
m	:	mass (m)
n	:	rank of a cell, or a volume
N	:	total number of cells
p	:	Pressure (P_a)
t	:	time (s)
v	:	velocity, particle (m/s)
V	:	velocity, gas or liquid (m/s)
x	:	variable
z	:	length position (m)
μ	:	viscosity (m^2/s)
$_g$	:	gas (or fluid)
$_n$	:	rank of a cell
$_o$	:	constant or averaged value

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