STATISTICAL METHODS FOR IMPROVING CONFIDENCE IN SYSTEM DYNAMICS MODELS - A CASE STUDY ON BLOOD BANK INVENTORY MANAGEMENT SYSTEMS

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ABSTRACT

The article discusses some statistical techniques applied as confirmatory tools to the System Dynamics modelling and analysis of Blood Bank Inventory Management systems. Instead of using arbitrary means, problem definition and statements are corroborated with statistical methods of correlation and formulation of adjacency matrices. This is extended to the estimation of some of the parameters of the system. Numerical Performance Measures (NPM) used to evaluate the system response to various inputs are discussed. The response of the system is illustrated primarily as time series plots. System trajectories or phase plane plots are presented with statistical inferences in relation to the model. It is concluded that for SD model refinement and analysis statistical techniques can be used judiciously as a confirmatory tool in unison with judgmental evaluation of the system.

INTRODUCTION

System Dynamics models are basically policy design tools used for understanding possible strategies to improve the system behavior. Judgmental methods are usually adopted in building up the model and carrying out the analysis right from problem definition to validation. System Dynamicists believe that what is important both in explaining the dynamics and in designing policies is to have results whose accuracy and validity emerge from qualitative considerations rather than from mathematical exercises. They argue that questions related to system boundaries,correspondence between model structure and real system, model reproduction of the system behavior etc are not really permeable to conventional statistical procedures, especially when dealing with large non-linear dynamic systems found in real life.

J.W.Forrester (1967) argues that statistical techniques can be inconclusive and misleading in System Dynamics modelling. However the authors feel that judgmental methods can equally be inconclusive and misleading if the analyst has not conducted an extensive environmental study of the system with an enormous database to supplement his rather

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philosophical abstractions on causal structures and dynamic consequences. Though failure to pass a statistical test need not be a sufficient condition for rejecting a model it can surely be handy as a confirmatory tool or a reminder to search for flaws in the model. Legasto.A.A. (1980) admits that they can be useful for parameter estimation and statement validation and Graham A.K (1980) gives certain guide lines for statistical procedures especially in model formulation and validation

In the following discussion some methods are presented which can be judiciously used by the modeler for strengthening the validity and representativeness of his model. A case study is conducted on blood bank inventory management system which has been an elusive problem for OR/MS practitioners for the last few decades due to its highly complex nature of interaction of operating variables.

VALIDATION OF A MODEL

Shoukath Ali and Ramaswamy (1991) were the first to analyze the problem from a System Dynamics point of view. Fig (1) shows the time series plots of simulation runs of the model developed by them using DYNAMO. The TABLE input corresponds to the actual data on patients' demand for blood collected from a blood bank in Bombay city. PULSE input is in the form of a sudden moderate hypothetical hike in patients' demand on the 10th day for an additional quantity of 20 units of blood which lasts for 5 days. STEP input corresponds to a
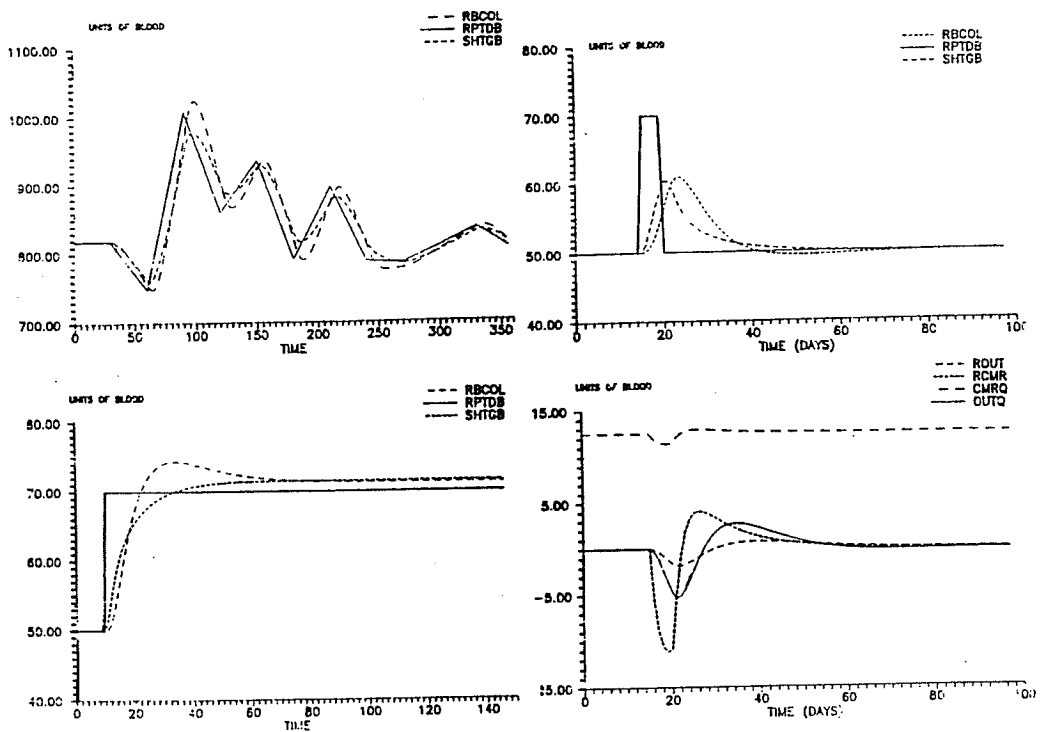
Fig 1. Time series plots of simulation results

aInv	Assigned INVentory	rcmb	Rate of Cross Matching of Blood
cmrq	Cross Match Release Quantity	rrcmq	Rate of Release of Cross Match Qty
dout	Delay in OUTdating	rptdb	Rate of Patients' Demand for Blood
dfa	Delay in Filling at Assigned inventory	rbcol	Rate of Blood COLlection
dfpr	Delay in Filling Physician Requisitions	rLrfb	Rate of Bank Requisition for Blood
dcmb	Delay In Cross Matching of Blood	rtrn	Rate of TRansfusioN
drcmq	Delay in Release of Cross Match Qty	sLtgb	SHorTaGe of Blood
dcmr	Delay in Cross Match Release	trncm	TRaNsfusion − Cross Match ratio
outq	OUTdating Quantity	uuInv	UnAssigned INVentory

Fig 2. Acronyms used In the model

continued hike in demand for 20 units of blood from the 10th day onwards. The acronyms used in the model are shown in Fig.2. Keeping these limitations in mind, the blood bank model has been put to tests for validity and representativeness. The data obtained from the simulation run of the model using the TABLE input is used for this purpose. The input to the model is the actual rate of patients' demand for blood (rptdb) estimated from data collected over a period of 1 year. The output data from the model on rate of blood collection (rbcol) is compared with the actual rate of blood collection in the blood bank. Fig (3) shows the actual rate of blood collection and simulated rate of blood collection. The hypothesis that the mean value of the simulated data x has come from a population (actual data) whose mean is X and whose standard deviation is to be tested. Using test with 5% fiducial limits it is found that the observed difference in means could arise by chance in more than 5% trials. Thus the significance of difference is not established. The Snedecor's F test passes the 10 % fiducial limits thus proving that model truly represents the actual system. The simulated output of outdating quantity is also compared in this manner. Table (I) gives the output results of the statistical analysis carried out. The same procedure may be adopted for comparing other variables such as shortage, cross match release, transfusion rate etc. provided real life data on them are available.
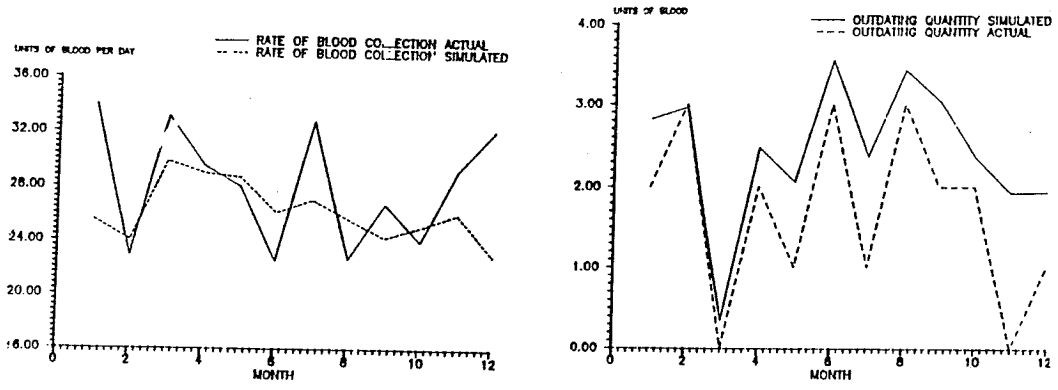

Fig 3. Outdating quantity and rate of blood collection

CORRELATION AND ACCUMULATION PRINCIPLE

Statistical relations can be helpful in suggesting areas to search for causal relations and statistical tests can suggest possible errors. However statistical relations should not serve as substitutes for causal relations and statistical criteria should not be considered sufficient for refutation of a model. Correlation of variables can be taken as an example. Increase in patients' demand for blood may result

ACTUAL RATE OF BLOOD COLLECTION and SIMULATED RATE OF BLOOD COLLECTION

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-----------------------------------------------------------------------
                          Two-Sample Analysis Results
-----------------------------------------------------------------------
                                 Sample 1     Sample 2      Pooled
Sample Statistics: Number of Obs.  12           12           24
                   Average         26.1317      26.1783      27.155
                   Variance        18.8903      4.48378      11.687
                   Std. Deviation  4.34629      2.11749      3.41863
                   Median          26.65        25.825       26.435
-----------------------------------------------------------------------

Conf. Interval For Diff. in Means:    95    Percent
  (Equal Vars.)    Sample 1 - Sample 2   -0.941761 4.84843     22 D.F.
  (Unequal Vars.)  Sample 1 - Sample 2   -1.0069 4.91357   15.9 D.F.
-----------------------------------------------------------------------


Conf. Interval for Ratio of Variances:  0    Percent
                 Sample 1 v Sample 2
-- ----------------- ------------ ------------------------------------


Hypothesis Test for H0: Diff = 1.74    Computed t statistic = 0.152856
                 vs Alt: NE            Sig. Level = 0.879905
                 at Alpha = 0.05       so do not reject H0.
=======================================================================
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COMPARISON OF SIMULATED OUTPUT OF OUTDATING QUANTITY AND ACTUAL OUTDATING

SAMPLE 1 - SIMULATED OUTDATING SAMPLE 2 - ACTUAL DATA

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-----------------------------------------------------------------------
                          Two-Sample Analysis Results
-----------------------------------------------------------------------
                                 Sample 1     Sample 2      Pooled
Sample Statistics: Number of Obs.  12           12           24
                   Average         2.43747      1.66667      2.05207
                   Variance        0.731524     1.15152      0.94152
                   Std. Deviation  0.855292     1.07309      0.970319
                   Median          2.41445      2            2.02865
-----------------------------------------------------------------------

Conf. Interval For Diff. in Means:    95    Percent
  (Equal Vars.)    Sample 1 - Sample 2   -0.0509231 1.59252     22 D.F.
  (Unequal Vars.)  Sample 1 - Sample 2   -0.0533005 1.5949   21.0 D.F.

Conf. Interval for Ratio of Variances:  0    Percent
                 Sample 1 v Sample 2


-----------------------------------------------------------------------


Hypothesis Test for H0: Diff = 0       Computed t statistic = 1.94582
                 vs Alt: NE            Sig. Level = 0.0645626
                 at Alpha = 0.05       so do not reject H0.

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Table.I Statistical results

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in decrease in inventory level. This doesn't mean that decrease in inventory level of blood is the cause for increase in patients' demand for blood. Thus these two variables are correlated in only one direction. Therefore many System Dynamics modelers focus on generic models i.e. they attempt to provide a general theory of the behavior of a class of systems.

Accumulation principle can be used in developing causal relationships in SD models. According to this theory :-

a) All dynamic behavior arises out of explicit flow accumulation in the form of levels or stock variables.

b) Creates a lag between inflows and outflows and between inflows and associated level variables. The phase lag decouples the instantaneous value of a flow from its effect on other variables in a system. Such decoupling accounts for all dynamic behavior in SD models.

Dynamic behavior in Econometrics models arises through distributed lag functions. For example in an Econometrics model let,

$$Y_t = B_o * X_t + B_1 * X_{t-1} + B_2 * X_{t-2} + \ldots + B_r * X_{t-r}$$
$$= \Sigma \ B_i * Y_{t-1}$$

where,

Y_t = collection of blood in units,

X_t = desired change in attitude of people towards blood donation,

B_i = lag weights.

This equation represents the relationship between Y_t, the dependant variable and X_t the independent variable. The parameters B_i are the lag weights which are determined by fitting the variable X and Y. The pattern of the lag weights is called lag pattern. Distributed lag functions represent a general scheme for correlating current values of one variable with past values of another variable. (Bell.J.P and Senge.P.M , 1980)

Even if an explanation is found for establishing a relationship between two variables the Econometrics model is not in a position to explain why there is the presence of delays. For example there is always a delay present between change in the attitude of people towards blood donation and actual collection of blood. Any model representing these variables is forced to give an explanation for this delay. In other words the model is forced to give a causal explanation why a delay occurs between X_t and Y_t and specify the nature of the delay. A causal theory may provide an answer why X_t affects Y_t but fails to explain why there is a delay present. The accumulation principle provides the answer and not the distributed lag functions.

In an SD model the change in attitude of blood donation might lead to a flow of blood into the blood inventory The model might assume a constant average collection time after which collection will be delayed.

This shows that the accumulation principle forces an SD modeler to provide a causal relationship for the dynamic behavior while the distributed lag approach overrides this aspect and considers only the

correlation. Correlation approach can obscure errors in a model while causal explanation provides more points of contact with reality and makes corroboration or refutation more possible. However correlation helps the modeler to establish the subjectively decided relationship between variables. Adjacency matrix method in combination with correlation analysis makes this job easier.

ADJACENCY MATRIX METHOD

Adjacency matrix method i. based on decomposition of the text describing the system into a sequence of inferences. All the nouns and adjective/noun forms in each inference are identified and inserted into a matrix to facilitate selection of variables and polarized relationships.(Camara.S.Antonio, 1991).

The steps involved are:-

1) Break down the description into a series of inferences by looking for inference indicator words such as because,thus,then etc or modal words such as must,can,can not etc. This helps in decomposing a large text into a discrete number of units.

2) Scan for variables in each reference by looking to nouns, adjective/nouns and other combinations involving nouns.

3) Develop an adjacency matrix with the entries previously identified. These matrices are denoted by $A = [a_{ij}]$ and,

$a_{ij} = 1$ if values of x_j depend on x_i and polarity is positive

$a_{ij} = 0$ if x_j is not related to x_i

$a_{ij} = -1$ if x_j depends on x_i and polarity is negative.

This pair wise analysis allows one to infer from the text all the directed relationships and their polarity. Besides this brings out the inconsistencies if any in the verbal description.

4) Translate the adjacency matrix into a causal diagram which is then used for developing the computer model.

This synthetic procedure for identifying variables and polarized relationships enhance the possibility of reaching a more consensual influence diagram and thus a better SD model.

Fig 4 . ADJACENCY MATRIX FOR VARIABLES

	rbcol	rptdb	rrcmq	rtrn	outq	shtgb	uainv
rbcol	–	0	0	1	1	-1	1
rptdb	1	–	-1	1	-1	1	-1
rrcmq	0	0	–	0	1	0	1
rtrn	1	0	-1	–	-1	1	-1
outq	1	0	-1	0	–	-1	-1
shtgb	1	0	0	1	-1	--	0
uainv	-1	0	0	1	1	-1	–

The variables thus identified describing the blood banking system are inserted in a matrix as given below, Fig(4). The influence diagram for the whole system can be easily drawn based on this table. (Shoukath Ali and Ramaswamy.N,1993).

Correlation does not represent the causal relationships between variables but the extent to which two variables are interrelated. However the causal relations attributed subjectively to the variables in the adjacency matrix may be checked with the correlation matrix (Fig 5). For example from the adjacency matrix it is seen that an increase in rate of blood collection (rbcol) causes an increase in unassigned inventory, (uainv). The coefficient of correlation between these two variables is 0.3982. Similarly relations between other variables listed can be checked.

Fig 5.   CORRELATION MATRIX - MODEL OUTPUT

	RBCOL	RPTDB	RRCMQ	RTRN	OUTQ	SHTGB	UAINV
RBCOL	1.0000	0.8994	-0.0746	0.8994	0.7137	0.9778	-0.3928
RPTDB	0.8994	1.0000	-0.4122	1.0000	-0.7783	0.9140	-0.4953
RRCMQ	-0.0746	-0.4122	1.0000	-0.4122	0.6241	-0.0202	0.8069
RTRN	0.8994	1.0000	-0.4122	1.0000	-0.7783	0.9140	-0.4953
OUTQ	0.7137	-0.7783	0.6241	-0.7783	1.0000	-0.6181	0.8511
SHTGB	0.9778	0.9140	-0.0202	0.9140	-0.6181	1.0000	-0.2442
UAINV	-0.3928	-0.4953	0.8069	-0.4953	0.8511	-0.2442	1.0000

PARAMETER ESTIMATION

When parameters of a model are too aggregated to be set reliably from available unaggregate data statistical techniques are used. i.e restructure the model so that its parameters corresponds to observable changing characteristics of the system.(Graham.A.K,1980). A few plausible methods are presented here for arriving at the parameters of the system and evaluating them.

Delay in outdating (dout) is one of the most important parameters in the model. This represents the time elapsed between the collection of blood from the donor's body and the discarding of blood after it decays. As per the standards specified by the American Association of Blood Banks (AABB) the age of whole blood is 35 days under standard preservation conditions. These standards are internationally accepted and followed in India too. Thus a value of 35 is assigned to dout. When blood components are considered the value of dout will assume

values corresponding to their age.

Another parameter of importance is delay in cross match release of blood (drcmq) which is the period elapsed between cross matching of blood for a particular patient for transfusion and returning of this quantity without transfusion to the unassigned inventory or free inventory. When this delay is more chances of outdating is more. drcmq is dependent on trncmsince,

$$trncm = \frac{\text{Cross match release quantity}}{\text{Cross match quantity}}$$

Thus it becomes necessary to determine both drcmqand trncmThe data on average cross match release quantities and cross match quantities were taken and trncmwas computed. Delphi method suggested by Graham.A.K (1980) was used to determine the value of drcmq as 2 days. The effect of varying drcmq on outdating of blood was also studied by the authors.(Shoukath Ali and Ramaswamy,1993).

Other important parameters of the system are dfpr, the delay in filling physician requisitions and dfa, delay in filling requisitions from assigned inventory. As no blood bank keeps records of these information, expert opinion was sought on the values of these parameters and average values fixed as 0.125 days. Thus collection of real system data and estimating the mean yields model parameters reliably. Statistical tests like t test may also be conducted to check if the values have really come from the real system.

SYSTEM TRAJECTORIES

System Trajectories can be drawn for studying the response of the system. They can be of two types namely, time series plots and phase plane plots.Plots of level variables against each other with time as parameter are called system trajectories or phase plane plots. (Aburdene, 1982). The Fig (6) shows phase plane plots obtained by plotting rate of blood collection, $x_1(t)$ against level variables. $x_2(t)$ i.e. unassigned inventory level (uainy), and outdating quantity (outq) . All the above variables are time response outputs from the model developed for blood bank inventory system. This process is repeated for all possible initial conditions of $x_1(0)$ and $x_2(0)$.

A second way is by iso-cline method which is based on slope of the trajectories in the phase plane. The ratio (dx_2/dx_1) represents the slope of the trajectories in x_1 ,x_2 plane. Aided by equation for (dx_2/dx_1) the regions of (x_1-x_2) planes that have the same values for the trajectory slope are determined . With constant values attached to (dx_2/dx_1) the slope becomes an algebraic equation of the form ,

$s = f(x_1, x_2)$ or the equation gives loci for states of equal slope.

Let $dx_1/dt = x_2$ and $dx_2/dt = 0.25 * (x_1-x_2)$

Equilibrium conditions are $x_1 = x_2 = 0$. Thus trajectories should approach the origin of the $(x_1 - x_2)$ plane. This can be shown by forming the following equations:-

$$\frac{dx_2/dt}{dx/dt} = \frac{dx_2}{dx} = \frac{x_2}{-0.25\,x\,-x}$$

Letting,

$s = dx_2 / dx_1$ and solving for x_2 results in

$$x_2 = \frac{-0.25x_1}{s + 1}$$

where,

s = slope of the trajectory.

This type of sketches can be used to quantitatively compare the results either with analytic solutions or with solutions obtained by simulation.
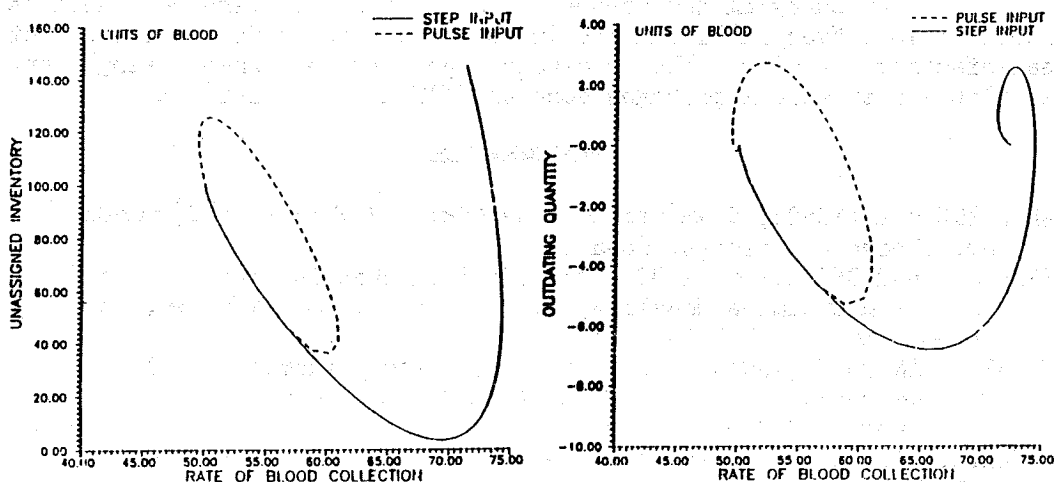

Fig.6 Phase plane plots

NUMERIC PERFORMANCE MEASURES

Computation of certain numerical values are useful to know how well the system has performed during the simulation run. Single performance measures (PM) or multiple performance measures (MPM) may be calculated from the model output and examined. Coyle.R.G (1980) discusses various such measures like cumulative values, point values etc. The values may be obtained either by introducing macros into the

model or by external computation.

The cumulative value of transfused quantity is thus obtained as 9502.9 from the model run using a macro in the following form:

```
1   crtrn.k   crtrn.j   dt * (rtrn.jk)
n   crtrn =   rptdb
a   maxui.k   max (uainv.k, omaxui.k)
1   omaxui.k   omaxui.j   dt * (maxui.j   omaxui.jk dt)
n   omaxui = 0
```

where,   crtrn= cumulative rate of transfusion
          rtrn = rate of transfusion
          rptdb= rate of patients' demand for blood and
          uainv= unassigned inventory.

From the actual data collected the value of total quantity of blood transfused is found to be 9502 units. Thus the method provides a validation scheme as well.

RESULTS AND CONCLUSION

The methods described are mostly statistical in nature, though, some are outside the conventional definition. The use of these techniques need not necessarily bring out positive results always. But an attempt at changing the parameter values and averaging constants is worth trying. Even small changes in the model structure may bring out satisfactory results. The analysis can be performed using any available statistical packages such as STATGRAPHICS, SPSS etc.

REFERENCES

ABURDENE.M.F (1988), *Coputer Simulation of Dynamic Systems* , Wm.C.Brown Publishers, Iowa.

BELL.J.A and SENGE.P.M (1980), Methods for enhancing refutability in System Dynamics Modeling, *TIMS Studies in Management Sciences*, Vol.14,61-73.

CAMARA.S.ANTONIO (1991), Formulation of System Dynamics models, *Proceedings of the 1991 Int.S.D. Conferenece. Conference* 82-87.

COYLE.R.G (1978), *Management System Dynamics*, John Wiley.

FORRESTER.J.W (1967) *Industrial Dynamics*, M.I.T.Press, Cambridge, Massachusetts.

GRAHAM.A.K (1980) Parameter Estimation in System Dynamics modeling, *TIMS Studies in Management Sciences*, Vol.14, 125-142.

LEGASTO.A.A and MACIARIELLO.J.A (1980), System Dynamics : A Critical Reciew, *TIMS Studies in Management Sciences*, Vol.14, 23-44.

SHOUKATH ALI.K and RAMASWAMY.N (1991), Setting Inventory levels for a centrally located blood bank of a metropolis – A Simulation Approach, *Proceedings of 1991 Int S D Conference*

SHOUKATH ALI.K and RAMASWAMY.N (1993), Simulation of Blood bank Inventory Management Systems : System Dynamics Approach, *Unpublished paper*.