A SYSTEM DYNAMICS APROACH TO KALECKI'S MODEL

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ABSTRACT

This paper examines the behavior of the gross investment considerig the linear version and two no-linear versions from Kalecki's model. This model assumes that there is an average gestation lag of investment and it is formuled by means of a mixed differential-diference equations.

In each version, moreover we analyze, the influence that a parameter has on the dynamics.

The response of the model is different, there are cyclos but, also, there are monotonic behaviors.

INTRODUCTION

Frequently, the differential equations and difference equations are chosen to form economic models. However, these equations are not adequates for the treatment of some economic phenomena with lags or no-continuous, for example.

This paper is found from a macroeconomic model stated by Kalecki and formuled as mixed differential-difference equation. This fact, is a consequence of an assumption about the existence of a gestation period of investment.

The relations between the variables of the model permits to determine a feedback process and then, the model can be analyzed using system dynamics. So, the flow diagram shall contain two levels: gross investment and capital stock. But, moreover, we shall include a first-order delay to consider the gestation period of investment.

The purpose of this paper is analyze the dynamics of this model when we consider that the response of the gross investment-capital stock ratio is linear (as Kalecki)[1], exponential or sigmoidal. In each case, also we shall examine, that occurs when a specific parameter in modified.

Simulations were carried out on a Hewlett-Pakard computer usig Mathematica.

KALECKI'S MODEL

The model concerns a closed economic system without trend. The total real income of capitalist is determined by the relation

$$B = C + A$$

where C is capitalists' consumption, while A coincides with gross capital accumulation. Capitalists' consumption can be related to capitalists' income by means of the linear function: $C = C_1 + \lambda B$, where C_1 is a positive constant and λ is a positive constant too, but smaller than 1. The total real income of capitalist may then be written as:

$$B = \frac{C_1 + A}{1 - \lambda} \; .$$

Regarding investment, Kalecki assumes that there is an average gestation lag of investment equal to a positive constant θ. The gestation period of investment is the time interval between the decision to invest and the delivery of the finished capital goods. More precisely, in each investment three stages can be distinguished:

- investment orders, they are called I;
- production of capital goods, that is gross capital accumulation, call it A;
- deliveries of finished capital goods, call them L.

[1]Our version is take from Gandolfo (1980)

Given the assumption made above, we can **write:** $L(t) = I(t-\theta)$.

To find the relation between A and I, Gandolfo[2] assumes that A can be written in the form:

$$A = \frac{1}{\theta}\int_{t-\theta}^{t} I(z)\,dz.$$

The meaning of equation above is that the output of capital goods at time t is equal to an average of the orders placed in the interval $(t-\theta, t)$.

Differentiating this last equation, we obtain:

$$\dot{A} = \frac{1}{\theta}\left[I(t) - I(t-\theta)\right]. \tag{1}$$

If we call K the capital stock, its first derivate with respect to time is its net increment, so that

$$\dot{K} = L(t) - U, \tag{2}$$

where U indicates physical depreciation[3].

To close the model we need an investment function. According to Kalecki, there are two main determinants of investment: the gross profit rate B/K and the money rate of interest wich he calls p. However, such variables, in Kalecki's opinion, do not influence the absolute level of investment but rather its level relative to the capital stock, that is the ratio I/K; in fact, when B and K increase in the same proportion, so that the ratio B/K remains unchanged, I probably rise. Thus we have the equation:

$$\frac{I}{K} = f\left(\frac{B}{K}, p\right).$$

In the absence of external actions and except for situations of "financial panic", the money rate of interest usually varies according to the general business conditions, which are represented by B/K. Thus the money rate p can be assumed to be an increasing function of B/K, and consequently f is a function of B/K only. Since B is proportional to $C_1 + A$, we can write:

$$\frac{I}{K} = \Phi\left(\frac{C_1 + A}{K}\right),$$

where Φ is an increasing function and it must be such as to admit zero or negative values. Kalecki assumes that Φ can be taken as linear.

[2] Kalecki seems to consider this fact as a consequence of the existence of a gestation lag, while Gandolfo thinks that it more correct to regard it as a separate assumption.

[3] Kalecki assumes that in the periode under considetation U is a constant.

Figure 1 show the causal diagram of the model.

The following section examines three behaviour assumptions of the function Φ, linear, exponential and sigmoid.

THE INVESTMENT ORDERS-CAPITAL STOCK RATIO

Given the definition of Φ, if we take a linear approximation (fig. 2), we have:

$$\frac{I}{K} = m\frac{C_1 + A}{K} - n\,, \qquad m > 0, \quad n > 0,$$

thus, $I = m(C_1 + A) - nK$. Differentiating both member with respect to time and substituting $\dot{A}$ y $\dot{K}$ from (1) and (2), we have:

$$\dot{I} = \frac{m}{\theta}\big(I(t) - L(t)\big) - n\big(L(t) - U\big),$$

with $L(t) = I(t - \theta)$. Figure 2 shows, the flow diagram of the model, that it includes two levels: capital stock and gross investment. The capital stock satisfies (1). The diagram, also, contains a first order information delay which finds the value of the investment at $(t - \theta)$.

The investment satisfies a new differential equation, if the function Φ is assumed to be of exponential type (fig. 2):

$$\frac{I}{K} = me^{\frac{B}{K}} - m - n\,, \qquad m > 0, \quad n > 0,$$

now, $I = K\big(me^{\frac{B}{K}} - m - n\big)$. Differentiating this expression, we have:

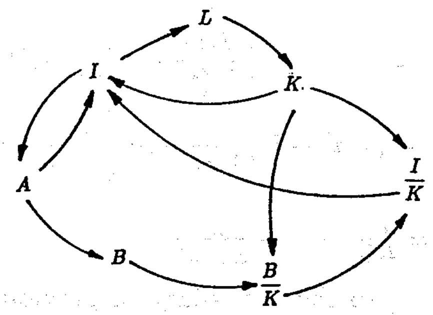
$$\dot{I} = \frac{\big(L(t) - U\big)I(t)}{K(t)} +$$
$$+\left(\frac{I(t)}{K(t)} + m + n\right)\left[\big(I(t) - L(t)\big) - \big(L(t) - U\big)\ln\left(\frac{\frac{I(t)}{K(t)} + m + n}{m}\right)\right],$$

once again $L(t) = I(t - \theta)$.

Finally, if the function Φ satisfies an equation of sigmoidal growth (fig. 2):

$$\frac{I}{K} = \frac{M}{\alpha + e^{\frac{n}{m} - \frac{B}{K}}} - n - \frac{M}{\alpha + e^{\frac{n}{m}}}\,,$$

where α and M are chosen, so that, when the gross investment-stock capital ratio takes the value $\frac{n}{m}$, the function Φ must be null (as linear case) and, moreover,



Causal Diagrame.

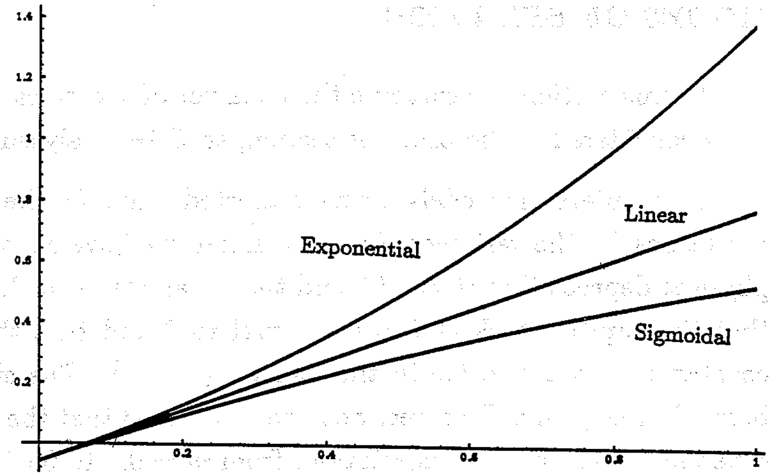
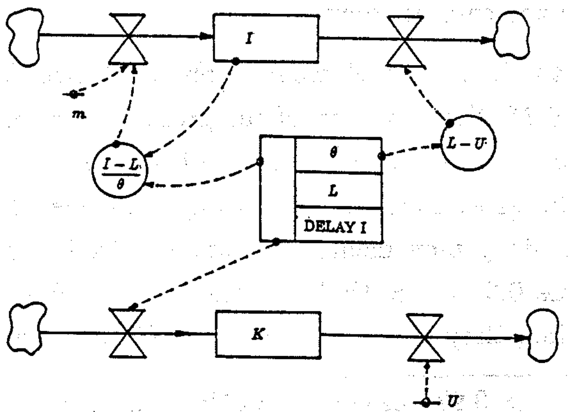


Figure 1.



Flow Diagrame.

Now, the investment verifies a differential equation non-linear, too.

$$\dot{I} = \big(L(t) - U\big)\frac{I(t)}{K(t)} + \big(I(t) - L(t)\big)J^2(t)\left(\frac{M}{J(t)} - \alpha\right)\frac{1}{M} -$$
$$- \big(L(t) - U\big)\left(\frac{n}{m} - \ln\left(\frac{M}{J(t)} - \alpha\right)\right),$$

donde $J(t) = \frac{I(t)}{K(t)} + n + \frac{M}{\alpha + e^{\frac{n}{m}}}$.

According to the flow diagram previous, we can obtain, the flow diagram of the exponential and sigmoidal cases. Only, it is necessary to redefine the auxiliary variables, that, now, they take the no-linearity of the model.

MODES OF BEHAVIOR

In this section, we examine the behavior of the gross investment. The three cases considered in the previous section, shall be analysed.

To simulate the models, we have selected values for the parameters and initial conditions for the variables. In this manner, we have choosen $\theta = 1$ months, the physical depreciation $U = 0.01$ and the parameter $n = 0.06$. We have assumed that the capital stock at $t = 0$ is equal to 2 and that the gross investment is constan and equal to 0.5 in the interval $(-1, 0\,]$. The simulation periode has been of three years. However, we have considered that the m parameter can take different values in the computation: from $m = 0.1$ to 0.97[4] with a step 0.01.

Along every simulation prodess, the capital stock remains positive. We analyze the behavior of the gross investment, but a treatment, simular, we could do with the capital stock.

In the linear model, we have observed three kinds of different behaviors. So, if $m < 0.75$, the evolution of the gross investment is monotic and decreasing, it tends to a number next to zero. But, if m increases, then this number decreases. Now, the gross investment is always positive. If, $m = 0.75$, the model shows an oscillatory movement, very smooth, that it tends to zero. When m reachs the value 0.9 is sure that the model has cycles. For $m = 0.95$ the oscillations are fading, they are steady around 0.97 and the are growing at $m = 0.98$. The

[4]If $1 > m > 0.97$, the expression of ln, in the differential equations of the exponential and signoidal cases, can not be determined.

average period from peak to peak is about 19 months. In the gross investment-capital stock phase plane the differential equations for m next to 0.972 represent a curve closed; the curves are spiral towards inside or outside if $m = 0.95$ and $m = 0.98$, respectively.

In the exponential model, we observe that two cases are possible at $t = 35$ months. The gross investment can tend to a number next to zero, but positive or to infinity. This last case occurs when $m \geq 0.9$. If m decreases, the movement is cyclical, the oscillations are fading, the gross investment takes positive, zero and negative values. If m follows decreasing, the behavior of the model is (as linear case) monotic and decreasing; once again, the gross investment is always positive.

Choosing $M = 100$[5], in the sigmoid case, we obtain that, at first of the evolution, the gross investment decreases until it reachs a minimun (no positive). This behavior does not depend, of the value chosen for m. However, the moment where the minimun is reached and the value of this minimun is independenty of m. Later, when the time increases, the gross investment can tend to 0.01. There is oscillatory movement if m takes values greather than 0.8. Now, the oscillation periode is, again next to 19 months.

The 4, 5, and 6 figures, show the behavior of gross investment for $m = 0.95$, 0.80, 0.45, when, we consider the linear, exponential and sigmoidal case.

Finally, according to equation that satisfies the capital stock, it is possible assume that the physical depreciation is not constant. If, we assume this fact, then, we have a new equation:

$$\dot{K} = L(t) - U\ K(t).$$

Thus, substituting this equation in the differential equations of the gross investment, we have a new situation that it can be computed. However, these new assumptions, substantially, do not change the behavior of the models.

[5]Seems that the value of the M parameter is not significant in the evolution of the model.

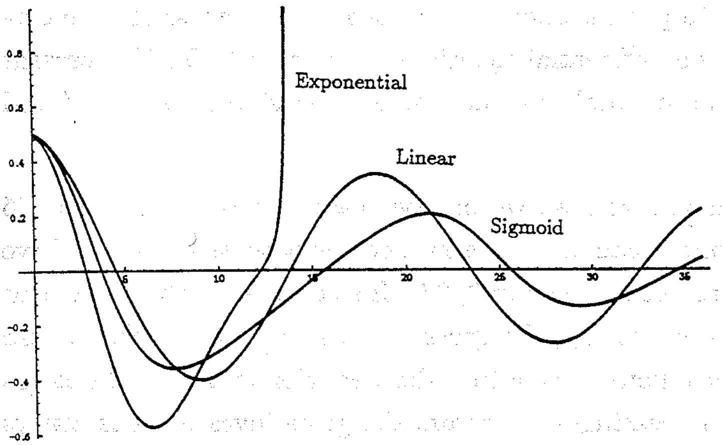


Figure 4.

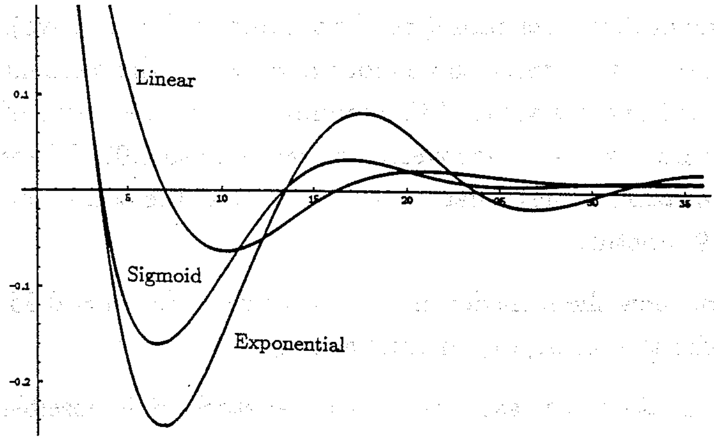
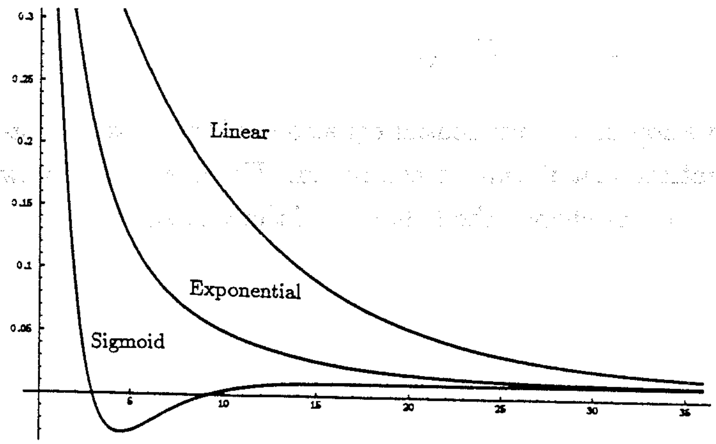


Figure 5.



CONCLUSIONS

By following a system dynamic approach, we have simulated the behavior of the gross investment when it satisfies the fundamentaly assumptions gives by Kalecki.

We have observed different behaviors. So, the gross investment can exhibit growth (exponential case), decay (linear and exponential case) and, also, oscillatory movements. This last behavior, appear in all the cases. Moreover, the oscillation periode is, always, around 19 months. Remark, that only in the linear case appears, in the gross investment-capital stock phase plane, closed curves.

The m paremeter seems to be one responsible of these behaviors. But the type of function, that we use by determine the behavior of the gross investment-stock capital ratio, has also an important influence.

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