System Dynamics and Learning Curves

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Abstract

Learning curves are met in a wide variety of industrial situations. They have become particularly important in modern business strategy because product life cycles are constantly reducing as companies seek to gain competitive advantage via a rapid response to customer demands. The paper describes a family of System Dynamics models that have been found particularly appropriate in modelling and forecasting the performance of business organisations; including the performance of manufacturing systems and the penetration of new products into the market place.

The System Dynamics learning curve models have a servomechanism analogue that yields valuable insight into the parameter estimation problem. The models are required under two quite different circumstances. The first is based on historical information where the model is to be added to a company or consultancy data base. The second is for on-line forecasting and control of a business activity. An enhanced stability least squared error predictor is described which covers both requirements. The paper concludes with industrial applications of the System Dynamics models.

System Dynamics and Learning Curves

INTRODUCTION

The concept of learning is generally accepted in a wide ranging set of disciplines and professions. Learning, in the dictionary sense, means the possession of knowledge via study. This conjures up the thought of individual human learning. The individual's output in such circumstances is difficult to quantify, although conceptualised as being time dependent. As an individual learns more with time, his level of knowledge grows. The time dependent output may be modelled via a learning curve following a classic 'S'-shape. It is this hypothetical model that is often cited by most authors on learning curve theory especially psychologists. The curve may model an individuals level of knowledge throughout his life time, or may just represent his performance for a single task. The learning curve may be made up of a combination of single tasks, either taken sequentially or simultaneously.

Such a human learning concept may be transferred over to "group" learning, where individuals, learning particular tasks, integrate them in order to obtain a group output. Therefore the learning of the group is dependent on the interaction of the individual learning components and the complexity of the overall task performed. The integrating mechanism may be via formalised work procedures or via a group leader.

For an individual or group then the concept of system learning arises intuitively. When talking of "soft" systems, such as people, organisations or any group of individuals, this concept is easy to understand. But when talking of "hard" systems this becomes a bit more difficult. After all it is not easy to imagine an inanimate object such as a machine tool learning. Clearly there is no such thing as a perfect hard system that requires no human interaction, so that some softness, and thereby a component of human learning, will undoubtedly exist. Nevertheless, even an hypothetically ideal machine tool would go through an improvement phase prior to steady state operation. The existence of failures would typically occur in a "bath tub" fashion. Such a reliability curve suggests that, when inverted, the performance of the system would follow an 'S'-shaped pattern. So hard/soft systems (such as a Flexible Manufacturing System) may be accommodated by the learning, or improvement, concept.

LEARNING CURVE MODELS

The ability to mathematically model the learning curve has found favour in a wide ranging set of (industrial) applications. These include human operator training and learning performance [Bevis, 1970, Baran, 1986], the diffusion of innovations and product life cycles [Mahajan & Peterson, 1985, Tchijov & Norov, 1989, Yelle, 1980, Edquist & Jacobsson, 1988, Butler, 1988], the introduction of advanced manufacturing technologies (AMT) [Towill et al, 1989], the reliability aspects of a nuclear power station [Sabri & Husseiny, 1979] and the thermal efficiency of power stations [Sharp & Price, 1990, Naim & Towill, 1990]. This in itself tends to underline the versatility of learning curves.

Modelling the learning curve allows insight into the system under study. It provides a simplistic representation of the system's time varying performance so that causal relationships may be determined, comparisons made with similar systems and forms the basis of decision making policies. In conjunction with modelling, many applications require a forecasting capability. Given a limited amount of performance data downstream from saturation, a model may predict, both in the long-term and short-term, likely system performance in the future.

But perhaps due to the very different applications there has resulted from different researchers no recognisable "standard" curve fitting practice. In fact, the type of curves fitted, the methods of fitting and forecasting are just as wide ranging as their applications. Even the names given to the learning curve differ, such as 'S'-shaped curves, saturation curves, diffusion curves and experience curves, although all have similar mathematical properties.

Various models have been advocated to describe the growth of dynamic systems, some of the most common as indicated in Table I which includes both trend line and smoothing models. The merits, or otherwise, of such models in describing growth, and subsequently in forecasting (both short- and long-term), have had considerable exposure. Although a certain model may be adopted as it provides the "best fit" for a particular application of many of these models have one or more of the following snags associated with them;

- inability to predict a final saturation level despite the fact that is what happens in the real world
- holding of meaningless parameter values which have no interpretation in the real world
- require data near the saturation level prior to obtaining realistic predictions
- gives only short range forecasts despite requiring much data

Model	Equation
Straight line	$Y = a + bt$
Parabolic	$Y = a + bt + ct^2$
Power	$Y = at^b$
Exponential	$Y = ae^{bt}$
Logistic	$Y = a/\left(1 + be^{-bt}\right)$
Gompertz	$Y = a + be^{ct} + dt^2$
von Bertalanffy	$Y = a\left(1 - be^{-ct}\right)^3$
ARMA	$Y = ARMA(p,q)$

Table I: Some commonly used learning curve models

Besides this, such models are used in an ad hoc manner with no recognised methodology by which a particular model may be chosen. Various models may be attempted until the "best fit" model is found.

This paper highlights the hypothesis put forward by Towill [1982] that a family of generic System Dynamics models, as indicated in Figure 1, may be used to represent, and control, the dynamics of "soft"/"soft-hard" systems, just as they are used in the engineering of "hard" systems. In particular, with care, the disadvantages previously mentioned of the models in Table I may be avoided.

THE SYSTEM DYNAMICS LEARNING CURVE MODELS

The analogue between the learning curves of industrial systems and the responses of "soft" systems has been noted in the literature. For example;

> *"It appears that the stages of economic growth.... are analogous to the growth of knowledge in a given field.... We will further elaborate on the subject by showing that there exists analogies between the stages of OR/MS knowledge growth and the kinetics of chemical reactions."* [Reismann & Xu, 1992]

While such analogies are beneficial, industrial systems learning curves are probably better compared with "hard" engineering systems. As Towill has pointed out, in particular with reference to production start-up, many engineers become plant on-line managers as well as attaining position where an appreciation of system dynamics is required. This is especially true for those graduates *"who have mastered control engineering ....* [with an] *ability to model existing*

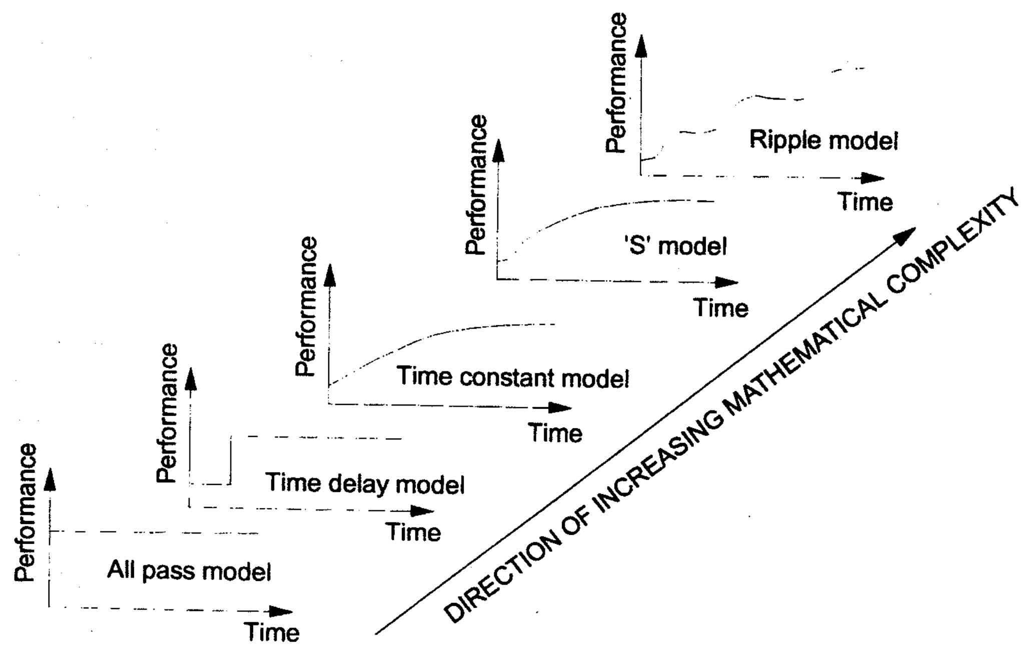

Figure 1: Family of transfer function based learning curve

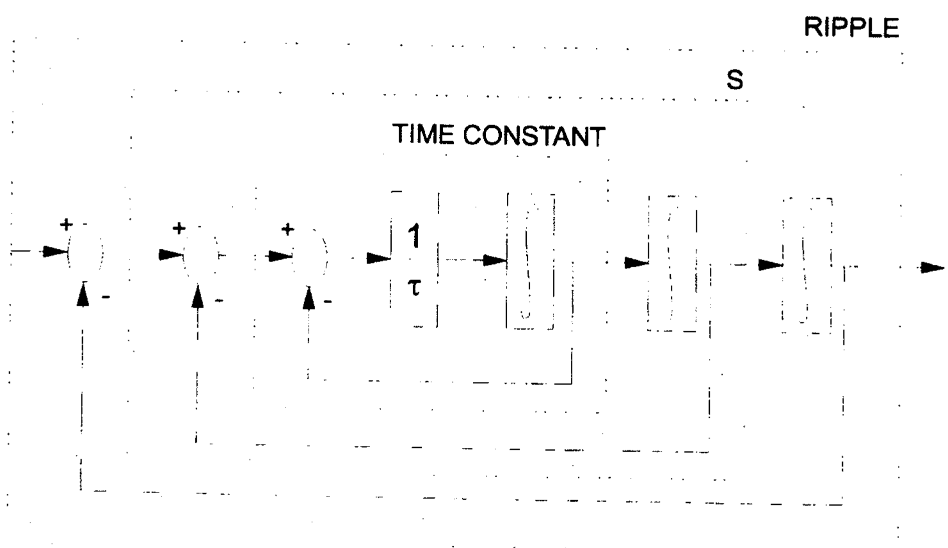

Figure 2: Hierarchical block diagram representation of the family of system dynamics learning curve models

systems from scanty experimental data together with an appreciation of problems associated with real time control." [Towill, 1991]

The suite of System Dynamics learning curve models shown in Figure 1 are derived from the unit step responses of simple transfer function models. The block diagram model representation of those transients in the family that adequately replicate learning behaviour, namely the Time Constant, 'S' and Ripple curves, are given in Figure 2. The hierarchical nature of the family lends itself well to a structured methodology by which an analyst is able to select the most appropriate transient by which to model a particular application. Thus, if the Time Constant model proves to be inadequate for a particular application then the analyst has a logical line of progression along the family of System Dynamics models based on increased model complexity.

The equations defining the transients of the latter three models are given by;

Time Constant Model

$$Y_m(t) = Y_c + Y_f\left(1 - e^{-t/\tau}\right) \qquad \ldots 1$$

Generalised 'S'-curve

$$Y_m(t) = Y_c + Y_f\left(1 + \frac{\tau_1}{\tau_2 - \tau_1}.e^{-t/\tau_1} - \frac{\tau_2}{\tau_2 - \tau_1}.e^{-t/\tau_2}\right) \qquad \ldots 2$$

Limiting 'S'-curve ($\tau_1 = \tau_2$)

$$Y_m(t) = Y_c + Y_f\left[1 - \left(1 + t/\tau\right)e^{-t/\tau}\right] \qquad \ldots 3$$

Ripple model

$$Y_m(t) = Y_c + Y_f\left(1 - e^{-t/\tau}\right) + A.\sin(\omega.t + \phi)e^{-t/\tau_d} \qquad \ldots 4$$

$Y_m(t)$	= model output at time, t
Y_c	= model output at time, t = 0
$Y_c + Y_f$	= model output at time, t = ∞
τ	= time constant, a measure of the rate of learning
τ_1,τ_2	= time constants, the ratio of which describes the 'S'-curve characteristics
A	= amplitude of cyclical component
ω	= frequency of cyclical component
ϕ	= phase shift
τ_d	= damping coefficient

SYSTEM DYNAMICS LEARNING CURVE APPLICATIONS

Since the Time Constant model was first used in 1970 [Bevis, 1970] it has been utilised in many industrial applications. These include; various manual and assembly tasks [Hitchings & Towill, 1975], the modelling of nuclear power station operator reliability [Sabri & Husseiny, 1979], determining the average thermal efficiency of nuclear power stations [Sharp & Price, 1990, Naim & Towill, 1990], capacity assessment of process plants [Brennan & Stephens, 1985], and the start-up of FMS [Towill et al, 1989].

The widespread application of the Time Constant model has been possible through the development of a number of sophisticated curve fitting techniques. These include a Taylor series expansion method for least squared error (LSE) estimation of exponential functions, LSE search methods, Kalman filters and maximum likelihood. Of these the LSE search method may be the simplest to apply although it is rather cumbersome and time consuming in terms of convergence properties especially if the solution space is too wide or localised optima are reached. Kalman filters have been found to be effective but require a considerable number of data points and an appreciation of the statistics of the data prior to curve fitting.

The Taylor series LSE expansion method is that favoured by the authors as it is simple to apply and is relatively easy for managers to understand as it is based on simple regression and as with all methods the aim is to minimise the error between the model and the raw data. This method has subsequently been used, and adapted, in the development of curve fit algorithms for the 'S'-shaped and Ripple learning curve models. Appendix I gives the curve fit sequence for the 'S'-curve that utilises parallel adaptation in order to improve the robustness of the algorithms [Naim, 1993].

The aim of modelling the learning curve is to glean information from the model itself *and* from the residuals scattered around the model. Costs may then be estimated and forecasts projected. In order for the analysis to be of any use in instigating action about the process it must be a component in an overall management information and control system (MICS). Towill [1985] proposed a simple structure for a computer-based MICS for on-line manufacturing system start-up. This has been developed to take into account the learning curve associated with the diffusion of new products and is shown in Figure 3. Such a model therefore takes into account the systems view of a business in which disparate company functions integrate to develop mutually fulfilling strategies.

The model not only accounts for distinct and separate manufacturing or marketing changes but also the knock-on effect of one on the other as well as change occurring in parallel. An important point to note is that the target learning curves have to be realistic. Imposed learning *"has an adverse affect when the imposed learning pace is slightly faster than the unpaced [natural] learning rate [due to the individual attempting to keep up with the imposed rate and thereby leading to stress and poor performance], and has no effect when the imposed pace seem to be too difficult"*, the individual reverting back to his natural pace. [Globerson & Seidmann, 1988]. This can also be extended to group and organisational learning.

The structure and interactions between the functions are obviously more complex in practice but the system boundary under consideration is as shown in the causal loop diagram of Figure 4. This shows the open-loop cascade effect of efficient project management; leading to a rapid, smooth start-up learning curve and therefore a steep cost function curve that ultimately results in an effective product diffusion learning curve [Yelle, 1980].

PRACTICAL EXAMPLES

The implementation of condition monitoring (C-M) techniques to a continuous casting (CONCAST) steelworks

The start-up tonnage output for a particular CONCAST steel mill [James, 1988] over a 60 month period is shown in Figure 5. The time scales relate to the period for which data was available and not necessarily from the actual start date of production. Sequential models are required in this particular case with the boundary between phases distinguished by the implementation of C-M tools. The Time Constant model is suitable for the first phase of production, up to Month 29, forecasting a final saturation level of 39.23 thousand tonnes per month (95% of which will be reached by Month 75).

The implementation by the company of C-M techniques is preceded by a downturn in production, as may be expected owing to disruptions during the installation of C-M technology. An improvement in output follows which may be modelled via the Limiting 'S'-curve which predicts a final performance output of 47.11 thousand tonnes per month. The final level of attainment is

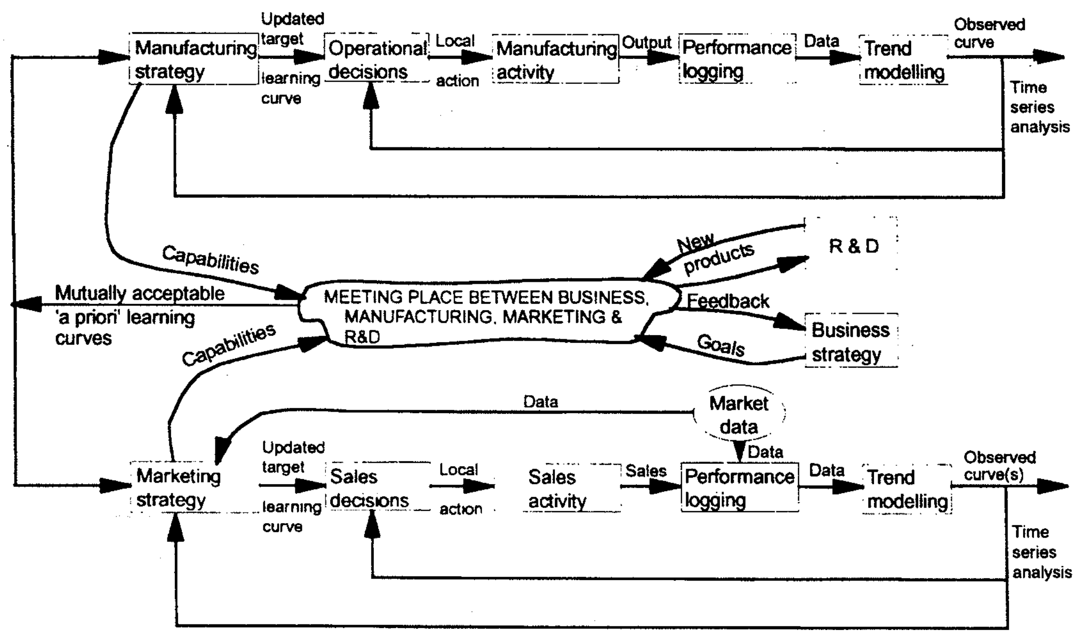

Figure 3: An integrated model of industrial start-up and new product diffusion

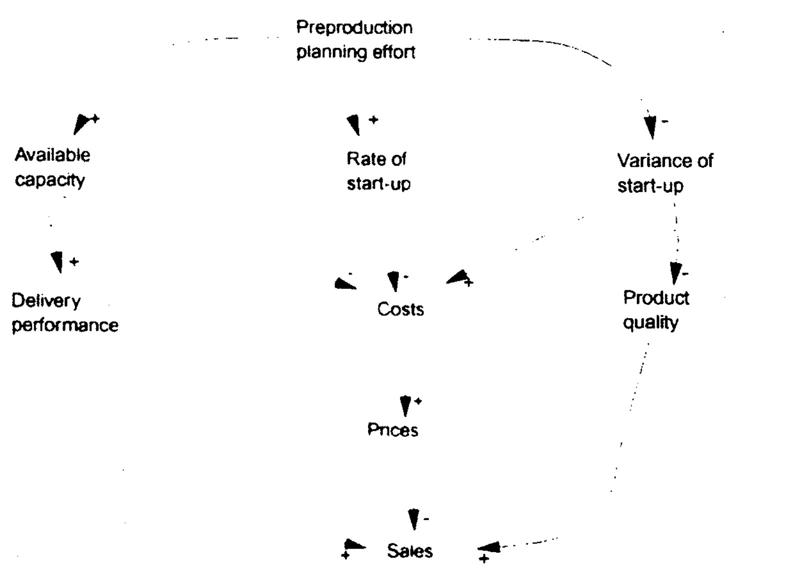

Figure 4. The causal relationships between effective start-up management and product diffusion

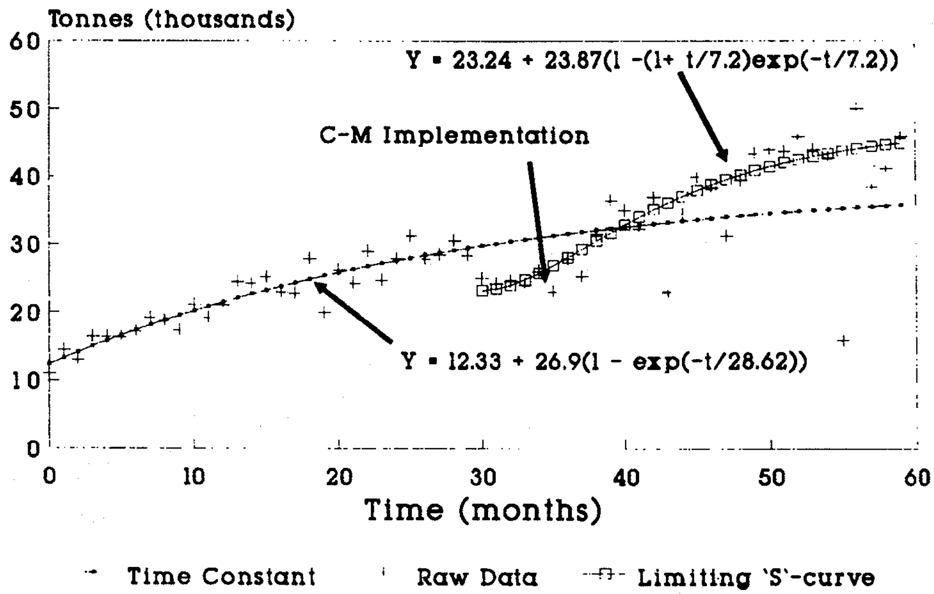

Figure 5: CONCAST start-up performance

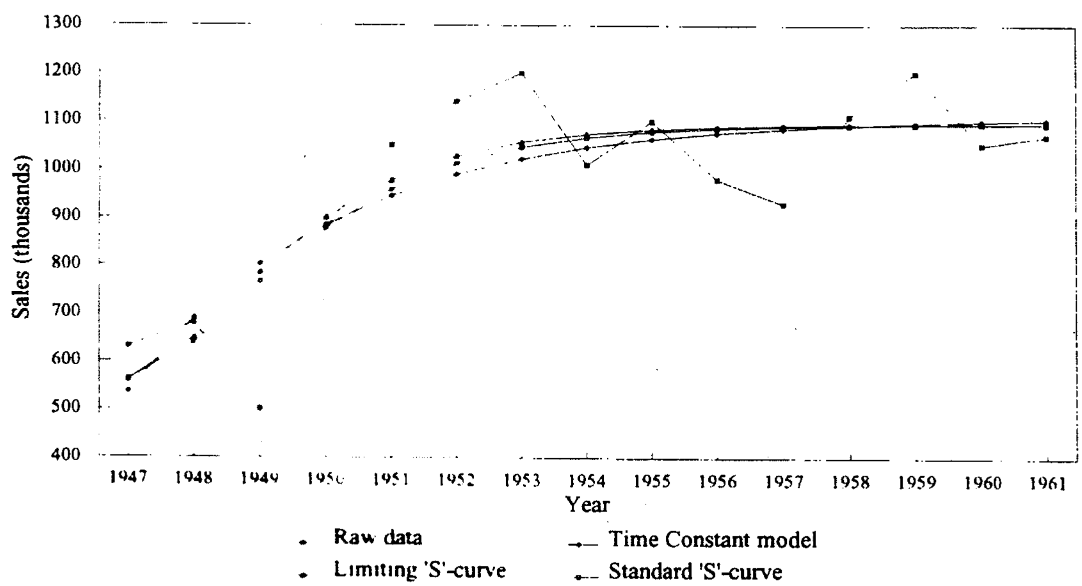

Figure 6: Sales of home freezers (1947-61)

reached by Month 57. Thus the introduction of C-M techniques has led to a predicted 20% improvement in final production output with an associated 24% reduction in learning rate as defined by τ.

Modelling Sales Data

Figure 6 shows the application of the System Dynamics learning curve models to the diffusion of products into the market place. For the purposes of comparison the sale of home freezers [Bass, 1973] has been modelled by the generalised 'S'-curve, the limiting case of the 'S'-curve and the Time Constant model. Table II shows the residual mean square (RMS)[1] value for each of the models.

Model ⇒ / Sales application ⇓	Limiting 'S'-curve RMS value ('000s)	Generalised 'S'-curve RMS value ('000s)	Time Constant RMS value ('000s)
Home freezers	15	16	18

Table II: Summary of RMS values for home freezer sales

Using the sum of errors squared as a performance measure there would have been little to distinguish between the generalised 'S'-curve model and its limiting case. But biasing the measure towards the simpler models using the RMS value indicates the Limiting 'S'-curve as the better modeller of the trend. At the same time it can be seen that the Time Constant model also gives a reasonable fit to the data although not accurately representing the infancy stage of the trend. Utilising the Time Constant curve as the trend model, corresponding residual time series and correlogram have shown a cyclical component within the data although the significance is limited by the lack of sufficient data points. There is therefore a case for utilising the Ripple curve to model the trend.

CONCLUSION

The paper has highlighted the development of a suite of learning curve models, based on the servomechanism principles of System Dynamics, that may be utilised as a management decision support tool for monitoring and controlling the start-up phases of new products and the associated manufacturing processes. The family of transients has been applied to a variety of industrial systems of which only two have been given as example.

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[1] $RMS = \dfrac{Sum\_errors\_squared}{No.\ datapoints - No.\ parameters}$

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