MODELING FOR STUDY OF THE PROSPECT OF A METROPOLIS

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ABSTRACT

The study of a metropolis becomes more difficult today since, with the development of the world, a metropolis performs more functions than just as an economic center. It is hardly possible to study the long-term behavior mode of the metropolis by using a single approach. Therefore, the model here uses not only qualitative analysis, but also statistics, econometrics, input/output theory and other quantitative methods, incorporated and integrated in a system dynamics (SD) model. The model can be used to study the limits to the economic growth, the interacts among the different sectors in a complex system, and so on. The modeling framework is very helpful to model the socio-economic-ecological complex system.

INTRODUCTION

From a historical viewpoint, large cities have a general dynamic behavior mode. Nowadays, the mode becomes much complicated, because the original single role of a metropolis as an economic center has been replaced by a modern multiple-functional role of economic, financial, trade and cultural center; and the inner structure of a metropolis becomes more complex and its relations with the outside world are strengthened. In that case, qualitative and quantitative methods and modelling are required to give a relatively clear description of the metropolis' future. Modeling is a hard task, since a metropolis usually has the following characteristics:

a. The development of a metropolis does not just refer to economic growth, it also includes the development of society, science and technology, population, and environment, etc., which are closely related to each other.

b. A metropolis, an open system, has many exchanges and relations both in material and in information. The former includes commodity, labor, funds, energy, raw material and so on; the latter includes various economic and political information.

c. About its inner structure. First, the inter-industry relations and connections are rather complex: how to seek coordination between primary, secondary and tertiary industries, and the coordination between basic sectors and other sectors. Secondly, what is the proper relationship between consumption, investment and saving. Third, how do the economies of different spatial parts interact. forth, all the relations above are nonlinear.

d. Urban infrastructure and other non-economic factors that limit the economic growth are playing an increasingly important role today.

e. The dynamic features: The inter-sector technical connections, technological factors, and the structures of consumption, investment, import and export and the like are all time varying. And the dynamic characteristics are also embodied in the time lags between different parts and different variables, so we should consider the inconsistency in time of a dynamic system.

In the light of the above characteristics of a metropolis, we might conclude: To do the quantitative study of such a metropolis, it is hardly successful by means of any single method. For example, we can create a macro-economic
model with econometrics method, but it is not very competent in solving the dynamic and nonlinear problems. Moreover, the econometricists seem not to pay much attention to the over-all structure which is critical when long-term forecast is launched. We may overcome these shortcomings by using SD model as a framework, but SD also needs other approaches to aid. For the considerations above, we use multiple approaches, pull their strengths together, kick off their weakness, and integrate them into a quantitative model that can fully represent the system itself.

Based on macro- and micro-economics theories, input-output theory and other related theories, the model is constructed in the framework of SD, into which econometrics method, statistics methods are incorporated to explore the prospect of the metropolis.

Figure 1. Main Relations of The Model
GENERAL STRUCTURE OF THE MODEL
To be close to the real world while modelling, the following two points must be taken into consideration:

First, demand orientation, i.e., the quantity and the structure of the outputs of the economy are directly or indirectly determined by the demands.

Secondly, disequilibrium. There is always some gaps between the demand and supply in most cases in the short run, but in the long run, there might exist such an adjusting mechanism as to decrease or to eliminate these gaps.

1) Fixed capital, labor and technology determine the normal or potential output of each sector, and together with the regulating rate and other limiting factors, determine the actual output. From the actual outputs, we have the gross domestic products (GDP) and national income.

2) Total national income minus the amount handed to the central government and the amount transferred to foreign countries and to the other parts of the country equals local income of the city. Local income, through consumption function, determines consumption. Income minus consumption equals saving.

3) From the desired fixed capital and the present capital stock, the desired investment can calculated. Meanwhile, local saving plus the sum of central government fund, foreign fund, and bank credit, and minus the capital outflow equals the total available fund of the city. The actual investment demand is a certain leverage of between desired investment and the capital availability.

4) Intermediate demands are determinated by the outputs through input-output matrix. Adding up consumption, investment, and intermediate and export and outflow to other parts of the country, we have the local demand of the products of each sector, local demand times the fraction that are produced by this city is the demand for each sector of the city.

5) Using the supply and demand of each sector, we can compute the supply-demand ratio, which, in turn affects the regulating rate and together with other limiting factors affect the actual output of each sector.

The above relationship can be illustrated more clearly by the following diagram as Figure 1.

SOME EXPLANATIONS OF THE MODEL

Production
The most widely used production function is Cobb-Douglas production function, which says the output is determined by fixed capital, labor and technology, i.e.:

\[ XP(t) = A(t) * K(t)^{\alpha} * L(t)^{\beta} \]  (1)

where, \( XP(t) \): potential output in period \( t \);
\( K(t) \): stock of fixed capital in period \( t \)
\( L(t) \): labor in \( t \)th period;
\( A(t) \): general technology;
\( \alpha, \beta \): parameters.

Under the assumption that the historical data fluctuated around the potential outputs, we can stimulate the needed parameters by using econometrics method (e.g., OLS).

The actual output of each sector does not always equal to the potential since there are many limiting factors.
that affect, they are:

a. Regulating rate. It is common knowledge that if social demand is more than we can normally produce, the workers will be asked to work overtime, then the regulating rate is greater than unit; but if the reverse happens, the regulating rate will be less than unit. But for some technological and some political reasons, regulating rate might not vary immediately. Usually, it has a certain varying range (different for each sector). When the desired regulating rate deviates the range the rate will take its extreme values, upper or low limit. (We will discuss it later.)

b. Shortage of intermediate inputs. There are several kinds of intermediate inputs. Some are zero-substitutable, one unit shortage of which will cause a linear cut in the outputs of the related sectors. Some are perfectly substitutable, the shortage of which can be totally offset by the increases of other inputs. But most inputs lie in between. To derive a standard formulation, we write:

\[ f_i = f_1i + f_2i + \ldots + f_{mi} \]  

(2)

and

\[ f_{ji} = \min (1, \text{fun}(i, Xs_j, Xd_j)) \]  

(3)

where

- \( f_i \): The effect of the shortages of intermediate inputs on the \( i \)th sector;
- \( f_{ji} \): The effect of the \( j \)th input on the \( i \)th sector;
- \( Xs_j, Xd_j \): Supply and demand of the \( j \)th sector.

\( \text{fun} \) means "function" which can refer to the coefficients of the input-output matrix, with some adjustment.

c. Other limiting factors. There are many other factors that very important to production but not included in the common I-O tabular:

First, there are some infrastructures, such as transportation and communication facilities which are indispensable for the production of the business. These are not included in the transportation sector or postal sector, but are regarded as public goods. They affect the outputs in the form of raw material tardiness or lack of information, etc..

Secondly, some natural resources and environmentary factors in some cases becomes limiting. For example, some industries have some requirements on the quality of water, and the output of excavative industry is almost totally determined by the richness of the resources.

Third, land limitation. Usually, land is excluded from the limiting factors, but as for a metropolis, it is not the case. For each project of investment, there is always some requirement for land input, but land is limited. With the development of the metropolis, land will become more and more a scarce resource. In the case of expansion, the limit of land may be found in the increase in transportation cost.

To summarize, we write down the actual production function:

\[ X_i = X_{Pi} \times d_i \times f_{i1} \times f_{i2} \times \ldots \times f_{im} \]  

(4)

where

- \( X_i \): Actual output of the \( i \)th sector;
- \( d_i \): regulating rate;
- \( f_{i1}, f_{i2}, \ldots, f_{im} \): The various limiting factors on the \( i \)th sector.

While calculating the effects of limiting factors, especially the off-the-table and non-economic ones, we can turn to experts in this fields for help, eg. we can launch a Delphi method. And the magnitude and the forms of
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effects (not necessarily in the form of multiplying them all up.) may be concluded in some table functions.

Income

After the output of each sector is computed, it is relatively easy to calculate the GDP and income:

\[ \text{GDP} = \text{SUM} \left[ X_i \cdot (1-m_i) \right] \]
\[ (i = 1, 2, \ldots, n) \]  \hspace{1cm} (5)

The parameter above, \( m_i \), refers to the fraction of intermediate inputs. If an input-output tabular is at hand, we can easily get it by summing up each column of the first quadrant of the tabular. One problem may rise here: the input-output coefficients are, in fact, time-varying. We will discuss it later.

Now we have GDP, subtracting the general tax from it, we are left the disposable income of the city under consideration. The general tax can be estimated with the following statistical formula:

\[ T = a + b \cdot \text{GDP} \]  \hspace{1cm} (6)

Sometimes, the tax is not linear with GDP, but graduated, i.e. the marginal tax rate \( b \) is varying with time or with the scale of economy, say, GDP. In those cases we may use table function. But linear tax function is often a good approximation.

The transfers to the foreign countries or to other parts of this country should also be subtracted from disposable income, but these parts are supposed to be exogenous.

Demand

1. Consumption demand

There are a variety of theories about consumption function, the one used here is somewhat similar to the relative income hypothesis. It is:

\[ C_t = \text{POPt} \cdot c_t \]  \hspace{1cm} (7)

where \( c_t = \alpha_0 + \alpha_1 \cdot \text{syt} \)

where \( \text{syt} = \text{SMOOTH} (y_t) \)

where \( y_t = Y_t / \text{POPt} \)

where \( \text{POPt} \): population;

\( Y_t \): disposable income of the city.

The consumption determined by the description above is an aggregate consumption. To derive the consumption demand for each sector, consumption structure should be introduced, so we have:

\[ VDc = \text{STRc} \cdot C \]  \hspace{1cm} (8)

where \( VDc \): vector of consumption demand;

\( \text{STRc} \): consumption structure vector.

2. Investment demand
Investment can, according to present statistics, be classified into two categories: current asset investment and fixed asset investment. For the former, we simply assume that it is a function of total output of the sector, with some freedom to vary.

The investment demand of the fixed asset is relatively complicated. The method we use here is a kind of stock-adjustment method which includes several steps:

First, we should have a desired stock of fixed capital of each sector. We assume, the desired stock is a function of expected output (which is the inverse function of the production function). The expected output, we assume again, is the SMOOTH of the past outputs. For fear that the outputs series is not stationary, we used two-stage smoothing method, instead of just one stage.

Then, the desired investment is derived by comparing the desired and the actual stock of fixed capital, multiplied with a coefficient.

\[ li^* = ki^* (Ki^* - Ki) \]  \hspace{1cm} (9)

After that, we should make some further adjustment, because, as we mentioned above, there is a problem of capital availability. If capital is short, the actual investment demand may be less than the desired; if capital is abundant, additional demand may be stimulated. But it does not follow that the contraction or the expansion of investment demand is proportional. Because different sources of funds have different preferences and there usually are some industrial policies of this kind or that in every country. However, under some hypothesis, we can simulate such process as to have a dynamic adjustment mechanism.

In the end, like what we have done with consumption, we establish the demand of capital goods used for investment by multiplying the aggregate investment with a structure vector:

\[ VD_k = STR_k \times TI \]  \hspace{1cm} (10)

where \( VD_k \): demand vector for capital goods;

\( STR_k \): structure vector of capital goods;

\( TI \) : aggregate investment demand,

\[ TI = I_1 + I_2 + ... + ln \ (li: investment \ demand \ of \ ith \ sector) \]

3. Export and outflow

These are assumed to be exogenous, i.e. are estimated with other method, or using some information about GDP.

4. Intermediate demand

Intermediate demands are computed by the following formula:

\[ VD_m = A \times VX \]  \hspace{1cm} (11)

where \( VD_m \) : intermediate demand vector;

\( A \) : input-output coefficients matrix;

\( VX \) : output vector.
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We have left out one thing, that is: all the coefficients, including consumption structure, capital goods structure, input-output technical coefficients, which are assumed to be constant must be allowed to vary, since what we are studying is a long-run prospect. In doing so, we may use several editions of input-output tabular, trying to find some trend. But it is usually hard to use any regression or other time series method because the series are relatively short. However, also by comparing them with those of other countries or other cities, we can have a rough empirical estimation of them. At last, we have these parameters in the forms of some mathematic functions or table functions of time or of the scale of economy, say, GDP.

The sum of item 1 through item 4 is the total demand, but is not yet the exact demand for the outputs of the city, because it also includes some demand for other regions. (For example we can import some goods to consume.) To derive the actual demand of the city, we should distinguish the amount of others. Then there comes the supply of outside. If the outside supply of the goods of a certain industry is infinite, then the inflow of this industry is just the extra demand, so not supply shortage of this industry. If the supply of outside is somewhat limited, then the supply shortage possibly occurs. In most cases, there are some kinds of division and cooperation among regions, so there always are certain relations between import (or inflow) and the total demand, either linear or nonlinear, different for each industry. In short, we can eventually derive the demand in each sector of the city.

Supply-Demand Balance

After we have got the demand and the supply of each sector, we can use them to compute a supply-demand ratio or a supply-demand discrepancy. If supply is greater than demand, no shortage of the goods produced by this sector is threatening, but there may be unwilling inventory investment which, strictly speaking, is not realized GDP and should subtracted from the available capital. If, reversely, supply is smaller than ex-ante demand, then part of the demand cannot be satisfied. On one hand, the final use---consumption, investment, export, etc.---are cut down, on the other hand, the shortage of the intermediate inputs might cripple the production of the related sectors, which is reflected in the limiting factors in formula (4).

The regulating rate that we have mentioned above is also derived here. In fact, it is simply the supply-demand ratio of a particular industry, with some smoothing:

\[ di = \text{MAX} \left[ \text{MIN} \left( UL_i, Di/X_i \right), LL_i \right] \]  \hspace{1cm} (12)

Above \( Di \): the demand for ith sector of the city;
\( UL_i, LL_i \): upper and lower limit of regulating rate of ith sector.

The Formation Of Fixed Asset

It is easily done by SD approach, by letting the stock of fixed asset be a "state" variable. But what should be pointed out is that the investment doesn't convert to fixed asset simultaneously, there is always a delay of some period in construction. Moreover, sometimes not all the investment converts to asset, i.e. some coefficients are needed to make some adjustments.

Some Technical Concerns

If we directly translate the formulas or relations described above into SD equations, we will soon find that some of them are simultaneous equations, which are not consistent with the reality and not allowed in SD
simulation. To solve this problem, we need some kinds of delay or smoothing functions in practical modelling. One possible unfavorable result of such a process is that there may emerge non-convergency, where the supply and demand diverge from each other and that is not consistent with the reality either. That kind of problem can only be solved during simulation, by a trial and error process, with great care.

About The Spatial Structure Of a Metropolis

A metropolis might have more than one part in space, perhaps for geographical reasons, or perhaps because of the difference between new and old districts. They are actually of a whole, so it is unreasonable to separate them into different regions completely, but if they are studied as if are homogeneous, it also seems unsuitable. So we should take into account that while modelling.

By adding another subscript, district, to the original single-subscript (sector), we can write the equations without extra trouble. The key point of studying the spatial structure is the relations between the different parts of it (including the flows of goods, population, funds and so on.) and the constraints to these flows and the result of failing to flow. The most ideal situation is to have a complete list of these flows, but it is hardly possible to obtain such a list, because present statistics has reach that far. But while studying some particular issues, for example, transportation between the spatial parts, we can use some empirical methods. For example, we may use a formula similar to "General Attraction" where the quantity of the transportation between the two parts depends on the productions of them and the economic distance between them.

The importance of studying the spatial structure is that sometimes the exchanges among the different parts (eg. the inter-district transportation), is critical to the production of some industry in one of the districts or another. So the inter-district transportation is a limiting factor to the economy as whole.

SUMMARY AND CONCLUSIONS

In the first part of this paper, we explored the characteristics of a modern metropolis, and concluded that it is impossible to use a single approach to study the prospect of such a complex metropolis, so we tried to develop a comprehensive one of multiple-approaches.

The main structure of the model is outlined and the main relations are illustrated with a diagram in the paper.

The main body of the paper explained the major equations and relations. Production, demand, capital formation, limiting factors another sub-sector of the model are explained in some details. We hope that by means of multiple-approaches, people can create a model that can more acutely, precisely, and satisfactorily describe the structure and dynamic behavior of a large metropolis.

REFERENCES