System Dynamics Production Models.
A Qualitative Analysis.

Authors:

Adolfo CRESPO-MARQUEZ
Rafael RUIZ-USANO
Gloria Elena PEÑA-ZAPATA
Javier ARACIL

Escuela Superior de Ingenieros de Sevilla.
Avda. Reina Mercedes, s/n.
41012 Sevilla. Spain.
adolfo@pluto.us.es ; usano@obelix.cica.es ; gpena@pluto.us.es ; aracil@esi.us.es ;

Abstract

The result of the formalization of a production system SD model is a set of first order differential equations, which can be numerically solved using the computer. As a result of that, the time evolution of the model's variables can be determined. Sensitivity analysis can be then utilized to evaluate the impact that the variations in an element's numerical value have in system behavior.

The analysis of the structural stability of the production system offers a wider perspective of its dynamic behavior, allows a new approach to the model sensitivity analysis. This new approach is extremely helpful in order to study global aspects of the system's behavior, while sensitivity analysis is more appropriated to deal with local issues.

In this paper, we try to investigate the conditions under which important qualitative changes in production models' behavior can be expected.

Introduction

In this paper, the analysis of the structural stability of the system is applied to a very simple example. This example is a production system containing only two production blocks, one represents a manufacturing process that can be performed by one single machine (WIP), the second models the stock of end products (INV).

We assume that the lead-time (LT) of the manufacturing process depends on the amount of work-in-process (WIP) in the first block. The higher the WIP, the longer an item takes to be processed. LT can be represented as a function of WIP showing different non-linear behaviors.

We will analyze the phase state WIP-INV, at the same time that the sensitivity analysis, in order to determine the stability points of the system. Moreover, we will try to identify the system behavior topologies[1] leading, under certain conditions, to system stability (Aracil & Toro, 93).

Qualitative analysis will also allow us to know when a topology will lead to unstable equilibrium points and bifurcation, and therefore, the possibilities for a catastrophe in the production system (Hale & Kocak, 91; Kutnetsov, 95).

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[1] Topology: representation of the equilibrium points as a function of the system's parameters.

Causal Diagram

The causal diagram of the model (Figure 1) shows the existence of four feedback loops. The first two negative ones have to do with the control of the output flow of the level variables. The third negative one controls the production flow in order to reach a desired inventory. The last feedback loop and positive one is very important for our study once conditions the level of WIP according to the lead time.

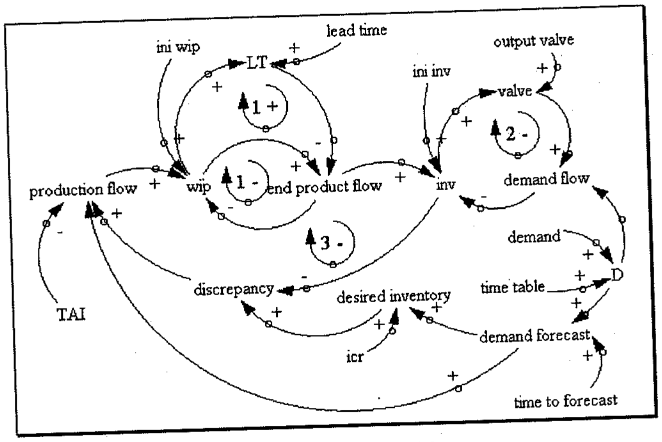

Figure 1. Causal diagram of the production SD model.

Stock & Flow diagram

In this diagram, a more detailed representation of the nature of the variables is shown (Figure 2).

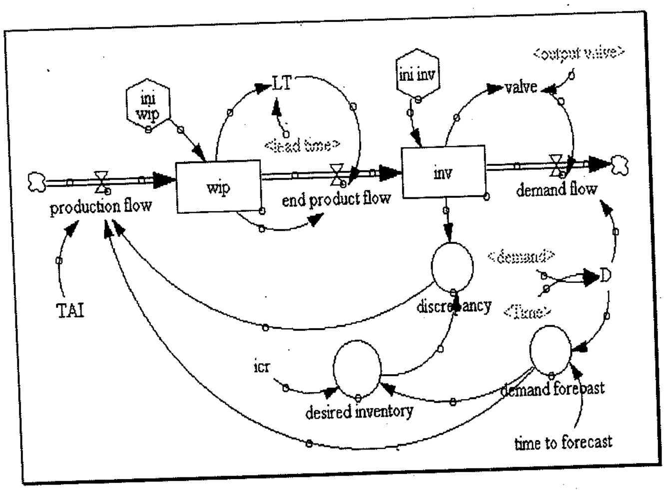

Figure 2. Stock & flow diagram.

Model differential equations and system stability

The equations of the state variables of the system are:

$$\frac{d(inv)}{dt} = [end\_product\_flow] - [demand\_flow] \tag{1}$$

$$\frac{d(wip)}{dt} = [production\_flow] - [end\_product\_flow] \tag{2}$$

Setting both of these equations to zero, we obtain the values for the variables under system stability:

$$\frac{d(inv)}{dt} = \left[\frac{wip}{LT(wip)}\right] - (D * valve(inv)) = 0 \tag{3}$$

$$\frac{d(wip)}{dt} = D + \left[\frac{D * icr - inv}{TAI}\right] - \left[\frac{wip}{LT(wip)}\right] = 0 \tag{4}$$

If the lead-time can be obtained, as a function of WIP, as follows:

$$LT(wip) = \left[\frac{1 + be^{-\beta wip}}{a}\right] \tag{5}$$

Then, we can plot (Figure 3) the values of WIP as a function of the market demand and for the system in stability conditions, just introducing (5) in (3).

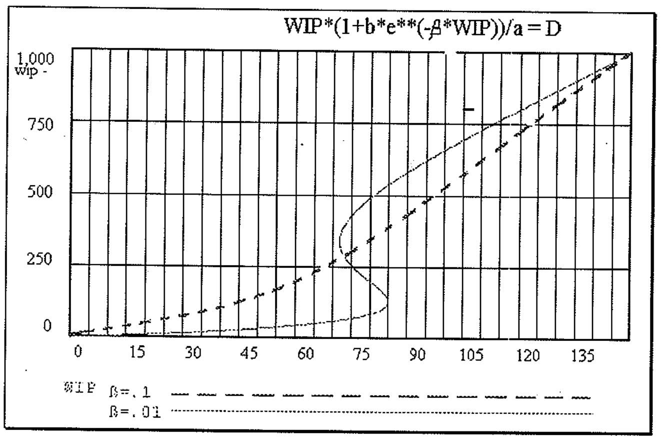

Figure 3. WIP stability points.

Figure 3 shows how, for different values of the parameters in (5) (considering *a=6, b=12.3, constants*), we obtain different representations of WIP meeting the stability conditions (3). The topology of these curves may allow in some cases the existence of several equilibrium points for WIP (E.g. three points for the interval $72 \leq D \leq 85$ aprox.). For instance, in case that the value of the demand is equal to 80 units/day, Figure 4 shows how WIP will reach two different equilibrium points depending on its initial condition. This gives an extra information about the unstability of one of the equilibrium points of WIP shown in Figure 3.

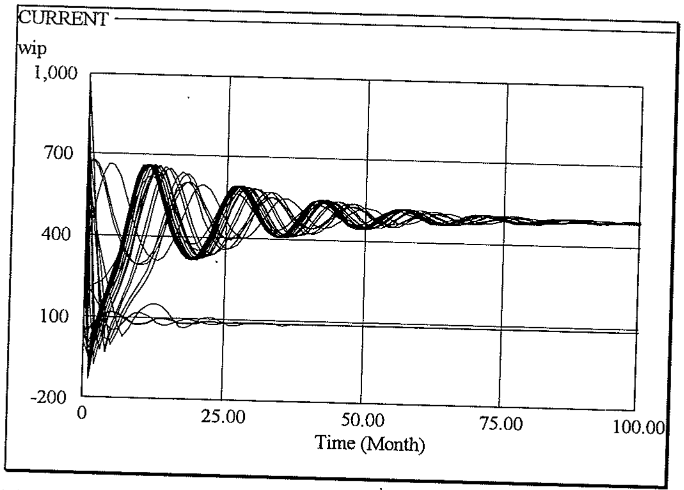

Figure 4. WIP stability points. Sensitivity analysis.

Conclusions

The evolution of a production system, where the lead-time is a function of the work-in-process, has been deeply analyzed. The qualitative analysis is a very adequate approach to identify possible behaviors of the system under stability, especially in cases where the shape of the function LT(wip), and the value of the market demand, change. From the analysis of the structural stability of the system, results a very wide perspective of its dynamic behavior that can be later, and for each particular aspect of interest, complemented with the sensitivity analysis, in order to gain further insights of specific issues.

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